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Allen L. Bernstein
1955
A STUDY OF REMEDIAL ARITHMETIC CONDUCTED WITH NINTH GRADE STUDENTS

A DISSERTATION

SUBMITTED TO THE GRADUATE COUNCIL OF WAYNE UNIVERSITY
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by

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FOREWORD

Studies in education are generally complex. When they combine large scale testing procedures with a classroom experiment in teaching methods in a large school, the administrative and methodological problems are enormous, and the success of the venture must be the product of the work of many of whom we wish to thank but a few.

The study was initially sponsored by Dr. C. L. Thiele, Supervisor of Mathematics and Science for the Detroit Public Schools, Mr. Herman Schumacher, Principal of Cody High School, and Mr. Raymond Agren, Assistant Principal of Cody High School. Without the encouragement and enthusiastic support of these gentlemen, whose authority made the final clinical study a reality, it would have been no study at all. The assistance and advice of Mr. Alton Hair, chairman of mathematics of Cody High School, and of many cooperating teachers, both in the mathematics and other departments of the school, was greatly appreciated.

The administration and scoring of almost two thousand test papers, and the coding and keysorting of about six hundred McBee Keysort Cards for the diagnostic analysis was a task which required much labor, far beyond the capacity of one individual. It was fortunate that the Future Teachers Club of Cody High School provided the nucleus for a staff of loyal, enthusiastic, and tireless assistants, whose efforts made it possible for the author to concentrate on arrangements and remedial teaching during the school day. The teaching profession will indeed be
fortunate if any of these young women care to pursue it as their life work. I wish particularly to thank the Misses Carolyn Bastian, Margaret Domeny, Joanne Jesky, Evelyn Kemp, Joan Mackie and Janet Watson for the many hours of yeoman duty which they put in at the Cody High School mathematics clinic.

I am particularly grateful to Dr. William Reitz, my adviser, for his patient, painstaking analysis of the procedures of the study and his subsequent advice in clarifying the logic and making the manuscript more readable. The long hours of conference spent with the committee members were also invaluable.

My wife, Gladys, was of great assistance when it was necessary to iron out the rough spots and find a clear mode of expression at many points in the manuscript. She also assisted in the proof reading and gave much needed moral support.

Allen L. Bernstein
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Chapter I

INTRODUCTION

1. A brief discussion and history of the problem:

The problem of remedial teaching has been with teachers for many years. Indeed, if John Dewey's notions concerning the nature of problem solving are applied in this situation, it is not difficult to realize that conditions, which would currently be described as problems for remedial teaching, must have existed long before educators recognized, defined and studied the subject. The review of the literature for this study makes reference to research studies in remedial arithmetic which go back to 1921. No doubt, some work was done by resourceful and understanding teachers before that time.

Remedial teaching has generally confined itself to the basic skill subjects of reading, writing, spelling and arithmetic. This is both natural and sensible. First, these areas are the most important for remedial teaching, since the skills acquired are basic to so many other learning activities in school. With all of the shift in emphasis in education which has taken place in the past generation, the 3 R's are still regarded by teachers, of all educational philosophies, as the core of a good school program. Second, growth in the basic skill subjects, in spite of the many difficulties involved, is easier to measure and evaluate objectively than growth in other kinds of learning. It is a commonly accepted definition that a student is classified as remedial in a particu-
lar phase of learning if he has not achieved up to expectations based on his potentialities. Such a definition is more easily translated into a practical, working rule in the skill subjects than in other studies. After definition, the next logical step is the search for causative factors. If a study of deficiencies in a subject other than the three R's is postulated, it is reasonable to expect deficiencies in the three R's to turn up among the rational causes. This analysis is not intended to convey the idea that remedial studies in other areas are not needed. A greater number of studies in the more complex areas of learning would cast much light on many problems of modern education.

The greatest volume of work in remedial teaching has been done in reading, and the related skills of writing and spelling. In the opinion of the author, the bulk of needed research in these areas has largely been completed. The most important educational problem remaining for specialists in these fields is the promotion of more widespread usage of the many techniques developed by this fine research.

Arithmetic has been far from neglected. This study will make reference to nineteen studies made in remedial teaching of arithmetic at all school levels. These studies can properly be called research in character. Many other arrangements are to be found in our schools in which remedial teaching is carried on, with little or no attempt to evaluate the results scientifically. An enormous volume of opinion is to be found written up in articles in various professional magazines, discussing the problem. Much of this opinion is apparently valid and in agreement with the findings of research studies, but no evidence is presented in which the results are properly evaluated.
2. Events at Cody High School, Detroit, which led to the design of the study:

The birth of this study was the result of the changing curriculum in mathematics in the Detroit Public High Schools and an accident of scheduling. In the fall of 1952, Detroit Public High Schools adopted a revised mathematics curriculum for the ninth grade. Mathematics teachers have debated the pros and cons of the "three track" program, in which students are placed in fast, average, or slow, classes, for many years. The term is indeed ambiguous, since some schools use a system of the same subject matter for each of the three groups, presented at varying rates and varying degrees of difficulty. Other schools describe "three track" programs in which the subject matter presented varies with the needs, as well as the abilities of the group. Detroit, previous to 1952, had been teaching algebra to both the 'fast' and 'average' groups, and a review arithmetic course to the 'slow' group. The new Detroit program offered Algebra to the 'fast' group, as before. A new course in General Mathematics was offered to the 'average' group. The 'slow' group was to consist of those students classified as remedial. By sheer chance, the author was scheduled to teach three ninth grade classes in Remedial Mathematics during the first semester of the new program. The problems encountered were such as to arouse a great deal of interest in high school arithmetic. Since the author's training has been largely in the guidance area, it was natural to view the problem from a clinical standpoint. The efforts of the first year of instruction led to a greater understanding of the kinds of diagnosis upon which a solution of these problems must be based, and to the firm conviction that some means of individualizing this instruction must be found.
Numerous conferences were held in the school, involving Dr. C. L. Thiele, Supervisor of Mathematics and Science for the Detroit Public Schools; Mr. Herman Schumacher, Principal of Cody High School; Mr. Raymond Agren, Chairman of the Mathematics and Science Department of Cody High School, and the author. At these conferences, plans were made for the various steps of the study:

A. Mass testing of incoming 9B students was planned for September of 1953. The instruments used were the Iowa Every Pupil Tests of Basic Skills, Test D, Form C; a diagnostic arithmetic test, devised by the author; and timed tests of the tables of fundamental facts in arithmetic. The tentative results of this testing, to be described in detail in a later chapter, led to the use of specific teaching techniques in the author's classes.

B. During the fall semester of 1953 the author taught three classes in Remedial Mathematics and devised diagnostic exercises based on the deficiencies found in the testing program. These exercises were presented as class lessons. An evaluation of this procedure is described in Chapter V.

C. The pilot study took place during the spring semester of 1954. The author was relieved of extra duties beyond his normal class load at the school, and organized two teaching clinics which met daily for one class period. These clinics were limited to six or seven students. Twenty-six cases were completed by the end of the semester, and will be reported in detail in Chapter V. The success of the pilot study led to the approval of the next phase of the project.
D. The major phase of the study was conducted during the fall semester of the 1954-1955 school year. During this phase, the author was given a half time teaching load, so that four periods a day could be devoted to the conduct of Mathematics Clinics. The fifty-nine cases completed are described in Chapter VI.

It was necessary to seek assistance for the enormous volume of clerical work done during the study. The most logical source for assistance was the Future Teachers' Club of the school. A number of very able students, who were members of the organization, volunteered their services. They were used throughout the phases of the study to check papers, work out diagnoses, compile and sort data, and aid in many of the routine tasks of operating the Mathematics Clinic. The large volume of clerical work included a good deal of duplicating and mimeographing. The expenses for the materials were defrayed by charging the students a fifty cent fee.

The first step in planning the study was the choice of methods for teaching and for the evaluation of the results. A variety of methods were used, each dictated by the circumstances of a particular phase of the study. This will be discussed in detail in Chapter II.

3. An appraisal of need:

The need for remedial instruction, in any subject, at any level, has been questioned by many educators, from two points of view. One group would challenge the need for such a program at all, and the other would regard it as a necessary, but temporary evil.

Those who would argue against the need for remedial teaching of any kind often do so on the basis of individual differences. The high
correlations between intelligence test scores and academic achievement scores are cited as evidence, and arithmetic is often singled out as an example, since arithmetic achievement tests generally have higher correlation coefficients with intelligence test scores than other subject matter test scores, and increasingly so when problem solving becomes a larger part of the arithmetic test. Why expand all this energy on the group with low achievement, the argument runs, since they cannot profit from it? So unsophisticated a point of view, in such raw form, is seldom expressed any more. The big flaw in such logic is the notion of "high" correlation. Even so unusual a figure as a correlation of .80, seldom attained in educational studies, leaves the scientist in an unhappy state when individual predictions are considered, since 36% of the variance of the predicted variable, in this case, achievement, is still unexplained. The second weak assumption in this thinking is the notion of the constancy of intelligence test scores. This concept has been seriously challenged in recent years, and large bodies of evidence exist which, if they do not settle the issue, at least show that, in our present state of knowledge, many cases have shown improvement in intelligence as a result of environmental influences. Who is to say, in the case of a particular individual who scores at the low end of the scale in both subject matter achievement and an intelligence test score, that this is an unalterable condition, and that it is a waste of energy to attempt instruction?

There are some educators who cite evidence to show that the differential effects of remedial instruction are usually lost within a year or two after such instruction has taken place. Such evidence is usually fragmentary. However, even when factually correct, the loss of
differential learning in a group which has had remedial instruction does not mean that such programs should not be attempted. The methods used to carry out the program, and the kind of situation to which the student returns are major factors determining the amount of retention. Such an argument is a refinement of that raised in the previous paragraph and suffers from the same weaknesses.

The second group of educators, questioning remedial instruction, are inclined to go to the other extreme and blame the need for remedial teaching on inadequate prevention in the form of good elementary school programs. This rationale is argued by many high school and college teachers. If the reader attends many teacher conferences and workshops and listens carefully to the feeling tones which accompany the discussion, it soon becomes apparent that these people are saying, in effect, "This is not my responsibility and I want no part of it." Such teachers are simply flying in the face of the raw facts. Students deficient in the basic skills are entering our high schools in large numbers every year and will continue to do so for many years to come.

A more thoughtful group advances the same argument on the grounds that school systems have limited funds for expanding teaching programs. If money must be spent, they argue, is it not better to concentrate on the lower grades, where instruction can admittedly be improved, than to spend it on repair, after the damage is done? Many leaders in the field of arithmetic, for example, argue that deficiencies in arithmetic exist solely because truly meaningful instruction is not pursued in the lower grades. That there is considerable truth in this argument is not difficult to show. Many examples in the literature will be quoted in
Chapter III, and some evidence will be added from this study. However, the weakness in this argument is the word solely. A careful examination of the factors causing learning deficiencies, in all subjects, leads inevitably to the conclusion that many causal factors exist beyond the control of the most perfect school program. Transiency is an excellent example. A child whose family has moved from city to city during his primary years, and may have attended as many as five or six different elementary schools, can hardly be expected to have achieved as well as less transient children of equal intelligence. Another weakness in the logic cited is the either/or quality of its thinking. Remedial teaching, from a research standpoint, has the additional responsibility of casting light on causal factors in learning deficiencies which may have gone unnoticed in other types of educational research. From the opposite point of view, teaching methods developed at the third grade level are often utilized with older children who were not ready to profit from them when they were in the third grade. Thus, the administrative choice is not to favor one level in favor of the other, but to find a reasonable balance.

Those who argue that eventual improvement of the elementary school program will eliminate the need for remedial teaching are indeed projecting themselves into a rosy, but far distant future. In the meantime, something must be done for the enormous number of educationally handicapped children entering the public high schools every year. Remedial teaching is here, in one form or another, to stay.
Chapter II
METHODOLOGY

Dissertations in the field of education make use of various research methodologies, each with its unique advantages or weaknesses. This study combines some of the elements of these methodologies. Indeed, the uniqueness of the contribution to the fields of arithmetic teaching and educational research depends not only on the data gathered in each of the various phases of the study, but also on the combination of varying research methodologies throughout these phases. The methodologies include a testing survey, a diagnostic study of test papers, an experiment in classroom procedure, an experiment in individualized teaching with concomitant case study data, and a critical review of the literature from several disciplines.

1. Choosing students for remedial teaching is a school dilemma:

Before presenting pertinent considerations regarding the methodology, it is necessary to define the problem in more specific terms than previously considered. Since remedial teaching is meant to correct learning deficiencies, it is necessary to select students for such teaching, and define what is meant by a learning deficiency. The definition found in much of the literature states that a student is considered a problem for remedial work if he has not achieved in a subject up to expectations based on his ability. At first glance, this seems quite a
reasonable definition, but any attempt to translate it into a practical, working rule for selecting students leads to the uncomfortable discovery that the definition, in a sense, is little more than a philosophic statement.

Consider the devices which are ordinarily used to predict achievement in school subjects, and to select students for various kinds of classes. Two of these are the previous achievement test score and the intelligence test score or intelligence ratio. The correlation between these two types of scores, where arithmetic is concerned, is generally high. However, the unexplained variance for a correlation coefficient of .8 is 36 percent of the total variance. Surely this leaves the educator unhappy about the efficiency of predictions in any individual case.

Schools generally choose between these two devices for classification of students. Both have severe limitations. Examining first the intelligence test score, the first problem which arises is the large unexplained variance in the predicted variable. The second is the serious question concerning the constancy of the intelligence test score in individual cases. Much evidence has been gathered to indicate that intelligence test scores can be favorably influenced by environmental changes. Schools are thus on tenous ground if a child with a low score in both intelligence and achievement in a subject were to be excluded from remedial instruction on the grounds that he would not profit from it. The problem is further complicated by the fact that intelligence test scores are often invalid. This is a serious charge, but it can be verified by clinical workers in many fields, who have other evidence concerning individuals, with which to make judgments. Numerous attenuating factors operate in any test
situation. Slight physical illness, nervousness, temporary emotional upsets, etc. may operate to produce lower test scores than one would ordinarily expect. Clinical workers are in a better position to observe evidence from which such conditions may be inferred than a proctor giving an examination to a large number of pupils. Reading handicaps may operate to give spuriously low scores in either group or individual intelligence testing. Group tests are often administered by teachers who have not been trained in test administration, and who may fail to live up to all of the instructions in the manual. These, and other factors, some of which are discussed in Chapter IV, lead to invalid scores in both intelligence and achievement tests.

The mental ratings available to the writer for Cody High School students are those on the Detroit General Aptitude Examination, customarily given in class size groups. It has been the writer's experience that many of these ratings, particularly at the lower end of the scale, are invalid, apparently for reasons such as those mentioned in the previous paragraph. It is thus clear that a ratio or a sum of low intelligence scores and low achievement scores could not be used to exclude a student from remedial teaching, even if it is assumed that some students in this group will fail to profit from it.

Subject matter achievement scores have precisely the same limitations as intelligence test scores, from the standpoint of validity and of predictability. However, the fact that remedial teaching is concerned with improving subject matter achievement, and not with improving basic intelligence, makes the use of achievement scores a proper tool for selecting students. To illustrate, let us consider the following combinations of scores:
a. Low intelligence score and low achievement score.

b. "High" intelligence score and low achievement score.

c. Low intelligence score and "High" achievement score.

If the scores are assumed to be valid, it is readily seen that using intelligence as a criterion for placing students in remedial classes would result in a wrong decision in both b and c. The possibility that either score may be invalid gives no further advantage to the intelligence score, and the arguments of the previous paragraphs indicate the desirability of selecting all students in case a. Looking at the problem another way, choosing students by intelligence scores could mean that some who need assistance would be bypassed. Using achievement scores, the worst that can happen would be that a student with a spuriously low score would be placed in such a class. If the time arrangements for instruction are flexible, as in this study, the student would be retained only for the few days needed to discover the more valid achievement level. Actually, the notion of low and "high" score is not a clearcut affair, but a matter of degree. This consideration does not change the logic of the preceding arguments.

A further limitation of intelligence test scores makes them less useful than achievement test scores for classification. By their nature, they are confidential, and very difficult to discuss with students or parents. Low achievement is more acceptable since it does not reflect, psychologically speaking, on the inner qualities of the student, or upon his heredity. Low achievement in arithmetic is so common that it may be treated as a matter of course. Thus the first question with which the study dealt was the selection of the most suitable achievement test for classification purposes.
With these beginnings of a definition of the problem, it is now appropriate to describe the methods used to conduct and evaluate the study:

2. The testing survey:

The Iowa Every Pupil Test of Basic Skills, Test D, Forms O and P, Advanced (Arithmetic); the Cody High School Diagnostic Arithmetic Test; and, the Detroit Public Schools Test on the Basic Subtraction, Multiplication and Division facts were administered to a large number of entering ninth grade students in September of 1953. These scores were studied for several purposes. One major purpose was to discover the soundest possible basis for classification of students. Correlations among the three tests were studied to observe similarities and differences.

The basic correlations will be shown in detail in a later chapter and are typical of achievement relationships as shown in educational testing studies. It is necessary, at this point, to discuss the nature of the sample and the limitations imposed by it. The attempt was made to test all of the entering ninth grade students with both the Iowa Every Pupil Test and the Cody High School Diagnostic Test. The practical problem of attendance in a crowded school building made this impossible. Many students, who were present for one test, were absent for the other. It can be quite reasonably postulated that the grouping attained on both tests was not representative because of the students who did not take both tests. A greater incidence of emotional and educational difficulties among children with poor attendance is well known. However, it is not difficult to argue that the relationships to be found in the "absent" group would be less clear cut and less predictable than those found in
the "present" group. Since it shall be argued that the correlations do not show a sufficiently high relationship among the sets of scores to use the tests interchangeably, this aspect of inadequate sampling does no harm. The tests of tables of fundamentals were not administered to all students. They were administered in three Remedial Mathematics classes and in four General Mathematics classes. Since one-third of the classes at Cody High School were Algebra, this is obviously poor sampling. However, it appears that deficiencies in this area are relatively lower in such classes. Even if this were not so, it would have no bearing on the conclusions drawn in this paper. Any conclusion drawn about the adequacy of knowledge of the fundamentals of students in the Algebra classes will necessarily be speculative. It is conceivable that testing all of the students, rather than some, would lead to slightly higher correlations, but would not give a sufficient relationship for efficient prediction.

Since the Cody High School Diagnostic Arithmetic Test is the basic tool for evaluation in this study, it is necessary to discuss the logic and construction of the test in some detail. The author's original intention was to give a three section test, which would evaluate many areas of arithmetic achievement, including applied problems and the structure of the number system. It was soon discovered that this would make the study quite unwieldly. The test was then limited to a single section in arithmetic computation. Limiting the study to this area has many implications, the most serious of which is that of opening the matter to the most fundamental criticism of any program in arithmetic instruction. What good are abstract computations if the student does not know how to use them? To some extent, this must remain an open question.
Consider the logic of the diagnostic test. (See appendix A) Each item in the test is designed to show a slightly different aspect of the diagnosis. For instance, problems C and D of Part I and problems E and F of Part II are designed to see if the student understands how to manipulate and write down decimal numbers for addition and subtraction. It is necessary to have at least two items in each section, in order to evaluate the possibility that an error is random, rather than a product of the student's misunderstanding. Grossnickle\(^1\) has shown that 80 per cent of the errors found on arithmetic tests are random. Postulating the probability that a random error will occur on a particular problem as .1, the probability that random errors will occur on two such problems is .01, a fairly safe risk.

Appendix A contains a manual and copy of the test, which describes in detail the types of diagnosis which can be made, based on this logic. Grossnickle has argued that it is necessary to have at least three problems of a given type, in order to be reasonably certain in diagnosis. While this may be so, it does not invalidate the use of the test, since the test is designed to give the clinical teacher an introductory diagnosis, rather than a final one. The student is going to receive instruction which will include interviews, and it is not difficult to find out when a diagnosis is in error.

It is now necessary to discuss validity and reliability. It can be argued that this test has face validity for the limited area tested. The

question of reliability imposes great difficulties. It is possible to get test retest figures for the purpose of checking reliability. However, in most testing procedures the next step is to do an item analysis of the test to determine whether elimination or addition of certain items will improve the reliability of the test. In a few achievement tests, where it is assumed that each item is a measure of the same thing, i.e., the same concept called "achievement", it is possible to eliminate or add items without changing the validity of the final outcome. (While this is common practice, it appears highly questionable.) However, in a diagnostic test, where each item has a distinct diagnostic purpose, the elimination of any item to improve reliability changes the nature of the diagnosis and may tend to defeat the purpose of the test. The addition of too many items would make it necessary to use more than 40 minutes for administration and would impose practical difficulties. There is also the possibility that a greatly lengthened test would introduce fatigue factors and increase the number of random errors. It has indeed been the experience of the author that fatigue factors do increase the number of random errors in the tail end of class exercises and tests. While information concerning test reliability is offered for the readers examination, the test was not revised for this purpose. Fortunately, the computed coefficient of correlation was .92, so that a favorable result was achieved without revision. (See Chapter IV, page 71).

3. The diagnostic test analysis:

The administration of the diagnostic test to 529 students led to the next technique used. A diagnostic check list was worked out for each paper by the student clerks assisting in the project. A mimeographed
list of forty-five possible diagnostic items was prepared for reference. One list was checked off for each test paper. (See appendix A) The percent of incidence of each item out of five hundred and twenty nine was reported. This is not a new technique, but has been used many times in research in arithmetic. The items were coded and a McBee Key Sort card was prepared for each paper. This permitted the examination, not only of the percent of incidence of each item, but of the interrelationships between paired items. The author was unable to find a more inclusive technique for examining these relationships, along the lines suggested by Thurstone's Factor Analysis. The difficulty is that the items under study are attributes, rather than variables, and the assumptions which are suitable for variables cannot be made. Since a comprehensive tool could not be found, simple statements about the significant relationships between paired items on the list must suffice. A fundamental theory regarding two areas of causation does appear to come out of these relationships. These areas are:

A. Gaps in instruction, particularly in the understanding of the number system.

B. Factors inherent in the personality of the individual.

The diagnostic analysis alone is insufficient to warrant the inference of such a theory, but other aspects of the study are consistent with it.

The reliability of the diagnosis, as well as that of the test score, was examined by computing the percent of agreement of diagnostic items from the first to the second test. The reliability of the coding was examined by having two clerks code a sample set of papers independently.
The results are described in Chapter IV.

4. Clinical teaching case study techniques:

The techniques described above led naturally to those used in the teaching phase of the study, namely, the completion of the clinical diagnosis and remedial teaching based thereon. This is a modification of the case study technique used in clinical psychology:

A. The student was interviewed and asked questions concerning his work and background in order to verify, or disprove, the diagnosis found on the check list accompanying his paper. The teacher did not attempt to probe deeply into the background and school history of the student, unless the data could not be explained without such information. Most of the time, students were agreeable to the instruction, and accepted the diagnosis. The teacher would take each item in the diagnosis, one at a time, and explain the nature of the student's misunderstanding. This was followed by a special practice designed to overcome the difficulty. Many of these practices were ordinary drill exercises taken from a standard textbook. For instance, a student with a deficiency in adding columns of numbers could only reveal the nature of the deficiency by adding aloud for the teacher. Once the difficulty (usually errors in carrying) was discovered, the student could use any addition exercise in any book for practice. In some deficiencies, special exercises were written. For instance, a student with a misunderstanding of zero as place holder in multiplication needed a special exercise in multiplication, in which all of the numbers contained zeros. A set of these exercises with a brief explanation of the need and purpose of each is included in appendix B.
When this type of instruction was effective, the teacher did not make further inquiries regarding the personal background of the student. More detailed clinical information was sought only in cases where this type of instruction did not work. It was usually possible, in such instances, to arrive at intelligible conclusions regarding the causes of the difficulty and operate on the basis of them. This is contrary to ordinary clinical practice, where it is customary to make more exhaustive investigations of each case, especially when the case is to be used in a research study. This did not seem feasible for several reasons. First, the purpose of educational research is to discover techniques usable by classroom teachers. It is unrealistic to expect that thorough training in clinical psychology should be part of the training of high school mathematics teachers or grade school arithmetic teachers. Fortunately, the second phase of the study revealed that this kind of training is unnecessary for dealing with all but a few unusual cases. The technique of dealing with the symptoms rather than the cause often served the teacher's purpose in the situation. This is not meant to imply that the student who responded to this type of instruction did not have other problems. It can be argued, that while these may have been present, they had little bearing on this phase of instruction. While this may not be a completely satisfactory philosophy from the standpoint of the mental hygienist, the teacher must limit his functions in some way.

A case folder was kept for each student containing his test papers and corrective exercises. Before the student was discharged from clinical instruction, a second form of the diagnostic test was given to observe changes and gains. The author wrote a short paragraph concerning each student, going into greater detail in somewhat more involved cases. For
the second phase of the teaching project, a progress chart form was devised. In place of the mimeographed check list, the diagnosis was written on the progress chart, so that the check list and a calendar of improvement could be kept on the one form. This form served another purpose, in that the diagnosis was read aloud in the presence of the student. This technique turned out to be a time saver, since the other method resulted in doing the diagnosis twice.

Many conclusions of a qualitative nature were made on the basis of the author's observations during these case studies. Another justification for the more limited approach to the case study aspect of the situation is that the technique made possible the completion of a large number of cases in a short time. It is unlikely that many of the insights gained would have been so clearly seen in twenty or twenty-five cases. Naturally, this type of conclusion is subject to the limitations of the biases and subjective judgment of the clinician, with the consequent danger of reading into the observations conclusions which may not be there.

5. Statistical evaluations:

The classical technique of before and after testing was used in both the class room and clinical teaching phases of the study. At the end of the fall semester of 1953, the Remedial Mathematics classes taught by the author were again tested with the Cody High School Diagnostic Arithmetic Test, and the Iowa Every Pupil Test of Basic Skills, Form C. As a partial control, the Diagnostic Test was given to two General Mathematics classes. With due account of the loss of comparison because of attendance, a sample of General Mathematics students was achieved in order to compare growth. No attempt was made to equate the groups for
intelligence, or other factors. While this may seem a serious deficiency in research procedure, the results indicate a significance which is remarkably high, so that such refinements become unimportant. One would hardly argue that the remedial group would be expected to have higher intelligence ratings than the control group. Therefore, the differential of growth in their favor must be significant.

Before and after testing was also used during the clinical teaching phase. No attempt was made to set up controls during that time. It appeared that the data gathered during the class room phase of the study gave a sufficient basis of reference for safe comparison:

A. The mean growth of the students during clinical instruction could be compared to the control group of the class room phase.

B. The mean growth of the students in clinical instruction could be compared to the growth of students in the remedial classes during the class room phase, so that the relative efficiency of the two methods of administration could be compared. At this point a difference between the students taught in the clinic during the pilot study and those taught in the clinic in the second phase should be noted. The former group were largely students who had completed Remedial Mathematics I during the class room phase and already shown some improvement. The growth that appeared as a result of clinical work was an increment. In addition, these students were members of the author's classes in Remedial Mathematics II, at the same time that they were attending the Mathematics Clinic. The students in clinical instruction in the second phase were taken from the classes of other teachers, and the growth they showed could be attributed largely to clinical instruction alone.
Other minor evaluations were made. The mean growth in months, based on the Iowa Every Pupil Test of Basic Skills, was tabulated for students in the classroom phase. Due to attendance problems, a limited number of scores were available. In the second clinical teaching phase, a follow up was made by studying the classroom grades of the students after they returned to the classes of other teachers in General or Remedial Mathematics.

6. Critical Review of the Literature:

To supplement the techniques described above, a critical review of the research literature was made. While this is customary in all research, the particular viewpoint of the author was instrumental in perceiving some relationships in the research data, not previously noted by other authors. While these conclusions might not have been drawn, without conducting a new study, many of them are quite independent of the results of this research, and can be drawn from the literature alone. This is explained in detail in the summary of Chapter III.
Contributions to our knowledge of remedial arithmetic have been made by many individuals, working in differing positions and trained in several disciplines. They can be categorized roughly into three groups, each with its unique viewpoints and techniques. One group consists largely of staff members of universities and colleges of education, and of doctoral students working closely with them. The members of this group, by both inclination and training, are most likely to examine specific variables in an educational pattern, and to make inferences about cause and effect relationships from the data in an experiment, whether statistical or otherwise.

A second group consists largely of teachers employed in public or parochial schools, working directly with children. Their major concern is with methods of classroom organization and teaching techniques, which will result in improved achievement. Many fine studies have been done by the people in this group, which point the way to better teaching, often without shedding much light on the causative factors more closely examined by the group described above.

A third group consists generally of clinical psychologists, guidance workers, or remedial teachers employed by case work agencies or remedial teaching agencies in universities or public school systems. Such workers generally operate on a case study basis, working closely with individual children and summarizing their findings in qualitative reports, with
some limited quantitative data when the number of cases is sufficiently large. From the standpoint of this study, this last group is particularly important. Their work has clarified much of the thinking regarding basic factors related to deficiencies in arithmetic, particularly where personality factors are concerned.

Many fine articles are to be found in professional magazines, such as: *The Mathematics Teacher*, *Instruction*, etc., which must be classified under the heading of Authoritative Opinion. The writers refer to classroom procedures which have been tried with apparent success, but fail to present evidence to evaluate their accomplishments. Since the literature in this field is quite extensive, there will be little loss in confining this report to studies in which actual evidence is presented.

The studies reported below can be classified in four general areas, each uniquely important in preparing this experimental study, and each contributing to our general knowledge of remedial arithmetic from its unique standpoint:

I Remedial Teaching Projects: Such projects have been carried out at all educational levels from the third grade to the first year of college, with a variety of workable methods, varying from strictly individual teaching to large class procedure.

II Error Diagnosis Studies: Much valuable evidence has been compiled about the kinds of mistakes made in arithmetic computation, at all educational grade levels. These studies have generally been limited to the systematic analysis of test papers, with inferences made about teaching procedures from the results. These inferences are open to serious question, but the factual description of the errors has consider-
able merit, without regard to its implications.

III. Studies in Learning Theories and Personality Factors related to the problem of arithmetic deficiencies: The coverage in this area is selective, in the sense that all studies of learning theory in arithmetic have a bearing on remedial teaching.

IV. Studies of Specific Techniques and Specific Factors used in Diagnostic and Remedial Teaching.

I. Remedial Teaching Projects

Blair\(^1\) conducted a survey of 1,090 secondary schools. Of 379 schools returning the questionnaire, 166 described greatly varied teaching programs concerned with the problem of remedial arithmetic. While the data is imperfectly described, it indicates the growth in awareness of the problem among secondary school people today.

To aid the reader, the teaching projects reported will be classified by grade levels:

A. STUDIES AT THE PRIMARY LEVEL:

The most outstanding work in the field of remedial teaching generally, and specifically in the field of arithmetic, has been done by Fernald\(^2\) whose clinic at the University of California has been in operation for 30 years. Her work is on a case study basis, and a short review of the theory of remedial teaching as presented in her book will be given in a later section. Books on remedial teaching have also been written by

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Brueckner\textsuperscript{3} and Blair\textsuperscript{4}. These differ little in general principle or detail. Fernald's work has been used as a principal guide in the conduct of this study because it contains many practical detailed suggestions. At a more advanced level, particularly when dealing with decimal numbers and per cents, Brueckner can be found of great value.

Plank\textsuperscript{5} reports a summary of twenty-four individual cases of arithmetic failure. The children were taught with the Montessori method and showed significant gains in achievement in a relatively short time. Plank recognizes that the number of cases is too small for sweeping conclusions, but her observations lead to several interesting hypotheses, which appear to contradict the findings of other workers:

a. Personality factors are pre-eminent as compared with intelligence or school experience in cases of arithmetic deficiency. Such factors as over-protection by parents, loneliness in the home, parents or teachers with rigid personalities, are submitted as possible factors. Plank disagrees with Hildreth\textsuperscript{6} (see page 43). She says that the correlation between intelligence and arithmetic deficiency is less than the correlation between intelligence and reading or spelling deficiencies. This appears contradictory to other evidence. Plank says, "Insecure children cannot stand the


\textsuperscript{6}Hildreth, Gertrude, \textit{Learning the 3 R's}, Educational Publishers, 1936, p. 814.
competitive atmosphere, nor the emphasis on speed, in arithmetic computation classes.

Schmitt\(^7\) reports a summary of thirty-four cases of children normal in their achievement in all subjects but arithmetic. No case of this type has shown a mentally defective child. All cases were reported to have understood basic number concepts, such as the ability to count up to twenty. Deficiencies were attributed to such factors as ill health (leading to absences), diet, mirror writing due to left handedness, gaps in instruction, and the attitudes of the children. While this study was done in 1921, it is remarkably consistent with the reports of recent remedial teachers. While Schmitt said nothing about basic personality factors, it can be inferred from her data that they were little different at that time than they are today. Instruction with the Montessori materials brought significant changes in the achievement of these children, who were third and fourth grade pupils.

Brownell\(^8\) reports four cases conducted by four workers under his supervision. Using techniques generally consistent with those reported by Fernald, the children showed gains of 1.2 to 2.6 years in achievement in six weeks of individual instructions.

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Becker describes an experiment in remedial work in addition of fractions with a sixth grade class of twenty-two pupils. Diagnostic tests revealed individual weaknesses in fifteen different steps in procedure; remedial teaching was carried out. Large gains in achievement were reported. Nothing is said about the precise method and the reader may wonder whether a meaningful procedure was used.

Tilton reports an experiment in which eighty minutes of remedial work was done with each of nineteen fourth grade students. The results indicated that even so short a time, significant results in achievement can be shown.

Fogler reports an experiment in which remedial teaching was done in classes of six. He notes that while the children showed great improvement in their arithmetic achievement, that the factor of greatly improved morale could be rated equally, if not more important.

Wilson reports an experiment by Sweeney in which six children in a fifth grade elementary home room were set aside as a special remedial group. Arrangements were made to teach the arithmetic lesson to this group right after recess. This continued for a whole year. Great gains were shown by all of the children, most of whom could be considered normal by the end of the year. Wilson also reports a great build up in morale.

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Bemis and Trow\(^1\) report an experiment in which a remedial class of 18 sixth grade students was compared with a carefully picked control class. The emphasis was on computation alone. The final results showed a small, but significant advantage for the experimental group. The children in this group showed an average growth of nine months instead of the expected five or less. The report doesn't describe specifically how the teaching was done.

Harvey\(^2\) describes an experiment with sixth grade classes with group instruction. Four diagnostic tests were administered to these classes. Examination of the types of error led to the construction of special remedial exercises. For example, a large number of students were found to be making errors in the use of zeros as a place holder in multiplication. Therefore, special exercises were constructed with practice in this technique. The method of teaching was left up to the individual teachers and was not supervised. Intensive reteaching was not done. (There is difficulty in seeing how such a statement can be made if the teaching was unsupervised.) There were 517 cases in the experiment and the results of reteaching were reported as excellent. It is worth noting that this experiment was done with heterogenous classes, rather than with students pre-selected as remedial.

B. STUDIES AT THE JUNIOR HIGH SCHOOL LEVEL:

Sister Mary Jacqueline\(^3\) reports an experiment done with eleven


students in a seventh grade class of twenty-three. These children were classified as remedial on the basis of a diagnostic test. The remedial instruction was independent of the normal classroom activity and was given before or after school. Attendance on the part of the children was voluntary. The children accepted the special help willingly. A remedial teacher worked on both basic computation and problem solving. Oral drill with flash cards and with fundamental facts was one technique reported. Success was reported for eight out of the eleven children and some progress for two others. Sister Mary Jacqueline shows excellent awareness of the emotional undertones of better school achievement, and values changes in attitudes as part of the description of success.

Addleston reports a study done with 7A, 8A, and 9B classes in which special drills were keyed to specific problems in a diagnostic test. The report is not clear but it indicates these drills were given interspersed with the regular class work. Significant change in the achievement of the pupils was reported. However, nothing is said about how the materials were taught, if at all, and it leaves some doubt as to whether the gains shown by the pupils were temporary or permanent. However, this can be said about most studies.

Fishbach describes a junior high school program of informal, individual work in reading and arithmetic. The arithmetic was taught in classes of twenty-four. The study reports an average of one year's growth per semester. However, little evidence is quoted.

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Thompson describes a two year experiment in remedial teaching which began in the 7th grade. A series of pre and end test units were prepared in many areas of arithmetic. Work for the pupils was individualized. No student was required to do the work of a unit if his pre test score showed mastery of the subject matter. If he did not show mastery, a series of drills were required of him, with appropriate coaching from the teacher. The experimental group showed a gain of 1.4 years in achievement in 10 weeks, compared to .4 years for the control group. The group also showed 2.6 years of gain in one full year. A followup of the pupils in the ninth grade showed that the mean achievement on an algebra test was equivalent to the 95 percentile in the public school population. It must be noted that this group was not a preselected set of students with deficiencies, but a heterogeneous group of all levels of achievement. The school was an experimental school, not representative of normal public school populations. The data must still be considered highly significant.

C. SENIOR HIGH SCHOOL LEVEL:

Guiler and Hoffman\(^\text{19}\) report an experiment in remedial instruction of 9th grade pupils. These pupils were not placed in special classes but were taught by individualized techniques in the regular mathematics classes, which contained 108 'remedial' students and 130 who were listed as a control group. The experiment followed the usual group of diagnostic


tests and individual practice exercises, based on the deficiencies which showed in the testing. The authors say nothing about remedial work on the basic tables (multiplication, division, etc.). However, this was an experiment with very excellent results. The authors recommend remedial instruction at all teaching levels.

Guiler and Hoffman also report a comparison of four different mathematics classes, which were compared for their achievement in basic computation. The four classes were:

1. Algebra
2. Junior Business Training
3. Applied Mathematics
4. Special courses in which Algebra was taught three days a week and remedial arithmetic techniques were used two days a week.

Algebra and junior business training had no effect on arithmetic achievement. The students in applied mathematics showed some improvement. The combination of algebra and remedial techniques showed the greatest improvement in arithmetic computation and the same achievement in algebra as did the students who took five days a week of algebra.

Griffith reports an experiment in which remedial instruction was given to two ninth grade groups, one by customary instructional methods, the other by a special drill card method. Both groups reported gains, and no significant difference between the two methods was found.

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Braverman reports a remedial arithmetic course in which the students were given individualized lesson plans with instructions. This was a self-work plan and the teacher's role became that of a coach. Braverman reports absenteeism as a major problem, limiting the success of many students.

Burton describes procedures in remedial teaching in both junior and senior high school. The senior high school offered a ten week class program for students selected as remedial. The junior high school did not remove the 'remedial' students from the regular program but introduced more individualized techniques into the regular program. While the evidence is not conclusive, it indicates that both systems are helpful, but that the senior high school system was better. Burton also notes that training in the more advanced subject matter of high school has no effect on arithmetic achievement.

Richter describes a city where high school mathematics classes were broken up into groups but kept within the same classroom, with differentiated teaching for each group. While the evidence presented is limited it indicates a more workable method than ordinary instruction.

Caporale describes an instructional program at the Bok Vocational Technical School in Philadelphia. Slower learning pupils were segregated

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and placed in smaller groups with understanding teachers. The procedures and instructions described were similar to those in other studies. It is important to note that this type of work has resulted in higher retention of students above the mandatory school age.

D. COLLEGE STUDIES:

Kinzer and Fawcett\(^{26}\) report a procedure with college chemistry students who were deficient in arithmetic. The students were given a basic computation test. Those who appeared deficient were offered the opportunity to attend voluntary remedial classes. Between three-fifths and three-fourths of the students took the opportunity to attend. Instruction appeared to succeed with some but not with others.

San Francisco City College\(^{27}\) reports a system in which students below certain test scores were offered remedial courses. These classes were crowded so that little time was available for individual instruction. Two out of three students passed the course. No research evidence is offered and it is impossible to tell whether the deficiencies were due to lack of understanding or a need for brief review.

Habel\(^{28}\) reports diagnostic testing and remedial teaching for college freshmen in both basic algebra and arithmetic. He reports gains for some students.

While the evidence on college studies is fragmentary, it indicates


that this kind of work has limited success at the college level. However, it must be noted that college teachers have quite different training and orientation than secondary teachers and it is possible that better work could be done at this level.

II. Error Diagnosis Studies

As previously stated, studies in error analysis have largely been done by Brueckner. His work can be found in his book, "Diagnostic and Remedial Teaching in Arithmetic", and it is scarcely necessary to review the many individual reports which are to be found in the journals under his name. Most of the other studies have followed the techniques which Brueckner has laid down. The summary of this chapter mentions the most important areas observed.

Grossnickle reports what we consider to be the most important single error study so far. Grossnickle distinguishes between two types of errors:

A. Chance errors

B. Systematic errors

He argues that a wrong problem on an arithmetic test does not necessarily mean that the pupil doesn't know how to do the problem. All people make mistakes on operations they understand. What we call systematic errors would lead to wrong results constantly on the same type of problem. Grossnickle argues, for example, that a student who makes an error in

\[ \text{\footnotesize \cite{29}} \]

\[ \text{\footnotesize \cite{30}} \]

\[ \text{\footnotesize Brueckner, op. cit.} \]

carrying on a multiplication problem may know how to carry. If the student makes an error in carrying on three successive multiplication problems, then he needs qualified instruction in this area. Grossnickle's analysis indicated that most of the errors on a particular arithmetic test are chance errors. This is a very important finding, since many workers doing research of this type have not even attempted to separate two types of errors in their analysis. This being the case, they have made erroneous conclusions about what the teacher should do. Grossnickle claims that three tests on a process are not enough for reliable diagnosis. This is true only if the test paper is the only diagnostic evidence. If other evidence is added, a reliable diagnosis may be had with two tests, and sometimes with one. A very fundamental question comes out of this study. If most of the errors on a single paper are chance errors, why the low score in arithmetic? This problem will be discussed in a later chapter.

Grossnickle reports a study of errors in division of decimals with 200 test papers of pupils in grades six through nine. He found two major sources of error peculiar to decimal problems:

1. Placing the decimal point.
2. Problems of the type $\frac{54}{6}$.

This is consistent with the findings of other observers.

Buswell and John, in a pioneering monograph on arithmetic diagnosis


discuss four methods of diagnosing the behavior of children leading to poor achievement in arithmetic:

1. The study of eye movements during computation with the use of a special tachistoscope.
2. Timed tests on fundamental operations.
3. Observation of the student during computation, with encouragement to "think aloud" while working.
4. Inferences from the work on test papers.

The monograph describes statistical results for each of the four types of diagnosis for a large number of cases. The results are similar to those of Brueckner and other observers. We believe the special significance of this study lies in the first method, since the others have been used by other researchists. While tachistoscopic observation is too slow and expensive for ordinary clinical teaching, the close correlation of irregular eye movements and poor achievement is an important verification of the findings of many workers. Buswell and John properly regard the eye movements as symptomatic, rather than causative.

Schane reports a study in which test papers were separated into three I.Q. classes, to see if the kind of errors made by low I.Q. students were different in character than those made by average or high I.Q. students. While Schane noted some differences, the differences were largely of degree rather than kind. The biggest difference between the superior and the slower group was in changing to a common denominator in the addition and subtraction of fractions.

Wilson\textsuperscript{24} tested Boston University students on tables of fundamentals, for both speed and accuracy, with results poorer than other observers have reported with 8th and 9th grade students. This raises a serious question about the retention of learning and the validity of much remedial evidence. Most studies in this area do not have sufficient follow up to indicate how much of the gain is permanent.

Guiler\textsuperscript{25} observed the work of 937 ninth grade students in computation with fractions. The errors observed are similar to those described by Brueckner for younger students, with differences in frequency, but not in kind.

Guiler\textsuperscript{26} also made a study of the work of 936 ninth grade students in computation with decimals, with results similar to those in the fraction study. Many of the errors which he observed were difficulties with whole number operations rather than errors in understanding decimals. One question appears to be unanswered in decimal studies. Fernald has indicated that placing a dollar sign on decimal numbers will enable some students to do the problem right who have previously handled the decimal question incorrectly. The same observation has been made with cases in this study. What effect does the unfamiliar context of a problem have on the achievement of students on that problem?

\begin{itemize}
\end{itemize}
Guiler \(^{37}\) made a study of the same pupils with per cent problems. He observed large areas of failure for the majority of students. His conclusion reveals the belief that school administration and teaching techniques are at fault. The phenomena he has observed are so universal in the experience of classroom teachers, that it is possible teachers of arithmetic are dealing with something more fundamental than Guiler has indicated.

Guiler \(^{38}\) also reports a study of college freshmen with problems in common fractions with approximately the same results that he reported for high school freshmen.

Arthur \(^{39}\) studied the papers of 400 pupils. He noted a discrepancy between the ability to compute and the ability to solve problems. While it is true that the poor computer will be handicapped in solving problems, it indicates that the two types of skills are relatively independent thought patterns. He also noted that type two and type three per cent problems gave great difficulty, a finding consistent with the work of Brueckner and Guiler. He also states the not too surprising conclusion that decimals and fractions give more trouble than whole numbers.


Phillips made a study of the errors made in the four fundamental processes on specially designed computation tests. These tests contained every item in the basic tables of fundamentals, suitably repeated. The tests were given to children in grades II through VI and contained as many of the basic processes as they had learned up to that point. He found that many errors were "chance" errors and did not reflect lack of knowledge, a finding consistent with Grossnickle. He found that many pupils could not respond automatically to specific items in the tables, and the tabulations indicated a prevalence of error in certain patterned areas. Zero errors, errors in borrowing in subtraction, and errors in reducing fractions were most common. Specific items in the multiplication table gave more trouble than any others, such as 7x8, 9x6, 9x7, and other items in the tables of 8's and 9's. He also came to the conclusion that the only effective diagnostic test for column addition is oral testing. While this data is consistent with the work of many other investigators, Phillips's conclusion that the solution lies in specially prepared drills appears quite shallow, and would be seriously challenged by the best thinking in the field of arithmetic.

Swenson studied the achievement of second grade students on the basic addition facts. She found that certain items in the table were more difficult for the children than others, but that the differences in difficulty varied with the method of teaching previously used. This poses


a difficult problem for the teacher at a more advanced level, who must deal with students taught by different teaching methods.

Davis and Rood\(^4^2\) reported a study in which 56 pupils were re-tested five times in two years on the fundamentals of arithmetic. Testing began in the 7th grade. They noted that the scores increased in general, but showed a tendency to level off. They noted certain types of errors on the second test which had not appeared on the same papers on the first test and made careful observations about the reappearance of these errors on subsequent tests. The recovery from the "loss of learning" described does not take into account what Grossnickle has said about random errors.

Readers will note that this report does not contain the actual frequencies of the various types of error described in the studies. This has been a deliberate omission. It can be noted that the types of errors are strikingly familiar from study to study and from educational level to educational level, especially above the sixth grade. It would only be of value to report the precise percentages if random and systematic errors could be clearly distinguished.

III. Studies in Learning Theories and Personality Factors Related to Arithmetic Deficiencies

As stated earlier, the most comprehensive work done in the area of remedial teaching has been presented by Fernald\(^4^3\). Since the findings of other observers appear quite consistent with her theories and methods,


\(^4^3\) Fernald, op. cit.
it seems wise at this point to make a brief summary statement of them.

Fernald lists the following causes of deficiency in arithmetic:

1. Mental deficiency - this is the only reason for unteachability. Conversely, it is argued that the student who cannot learn arithmetic is, therefore, feebleminded.

2. Reading disability - a primary cause of failure for many types of arithmetic problems:
   a. Inability to read problems.
   b. Lack of what may be loosely described as "background". Many mathematics teachers have found the meaning of a particular word to be a stumbling block in specific problems.
   c. Characteristics making difficulty in school adaption.
   d. Emotional blocking (a factor discussed far more adequately by Fernald than other writers)

3. Lack of number concept.
   a. Skill in the basic tables of fundamentals.
   b. Problem solving.

4. Blocking of adjustment by ideational or habitual factors or by emotional responses. It is noted that it is far more difficult to cure a bad habit than to teach where no bad habit exists.

Fernald recommends certain testing and teaching procedures with these causes in mind. It is unnecessary to reproduce these in great detail in this work. Suffice to say that these principles have constituted a majority of the procedures used in this study.
Hildreth has also written extensively on elementary methods. In addition to factors discussed by Fernald, she described "loss of learning over the summer" as a factor in arithmetic deficiency. This appears a rather controversial idea. She also reports that the correlation between intelligence test and arithmetic achievement test scores equals approximately .80 to .82.

Sweeney argues for a high standard of achievement in the tables of fundamentals. She reports that any child with an Intelligence Quotient of 80 or above can be taught 100 per cent mastery of the tables. She indicates that a partial cure for our errors in arithmetic instruction would be moving up the curriculum a grade. Plank, quoted earlier, refers to this argument for standards of Sweeney as a sign of a rigid teacher.

Brueckner, in discussing educational diagnosis and cures, is in agreement with Fernald. He reports a study in which achievement in four phases of arithmetic (computation, problem solving, vocabulary, and quantitative relationship) are correlated with intelligence. All the correlations were found to be positive and to vary directly as the complexity of the process studied.

\*Hildreth, op. cit.


\*Plank, op. cit.

Bruin reports an experiment in individualized instruction in arithmetic. This was not a remedial study, but an experiment done with seventh grade classes using children of all levels of achievement. Each child progressed at his own rate and level, kept a progress chart and had access to answer pages so that he could evaluate his own work. Bruin claims that there is great advantage in the child knowing exactly how much is required. While significant data to evaluate this procedure are not available, the limited data provided point to a promising development.

Scheidler and McFadden report an instructional plan in which seventh grade children were given strictly individualized instruction, with a large amount of self-checking and self-appraisal. They used the Strathmore Plan, which provides 500 tests and work sheets. The mean gain showed two years growth in five months of instructions.

Dahle studied sixteen children while they were learning long division through the use of motion pictures, the psycho-galvanometer, and other devices designed to record the many responses that were going on while learning took place. This study led to many tentative conclusions. Only those not mentioned in previous studies will be quoted:

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^48 Bruin, M. E., Experiment in Individualized Arithmetic, School (Elementary Education), Vol. 30, pp. 704-707, April, 1942.


^50 Dahle, C. O., Verbal Thoughts and Overt Behavior of Children While Learning Long Division, Vol. 9, pp. 1-8, September, 1940.
1. Instructional procedure is satisfactory for median intelligence or higher.

2. Special remedial aid does not insure mastery for below median students.

3. Solving one type of problem does not insure ability to solve another type.

4. Failure to judge and compare answers does not prevent a constant, steady attack upon problems.

5. Errors by the superior group are not of the same type as the inferior group.

6. Success and Failure have a marked effect upon stability and achievement.
   a. The students showed more electro dermal responses to failure than to success, if aware of the failure.
   b. Success creates success. Failure creates failure, if the student is aware of failure.
   c. An impasse in learning leads to infantile regression.
   d. Sometimes stresses and strains do not yield electro dermal responses and sometimes what appears to be instability is not substantiated by the psycho-galvanometer.

7. A school vacation makes possible the re-emergence of difficulties thought to be cured.

Gottsegen and Gottsegen\(^5^1\) describe a case in remedial reading in which a learning plateau is seen as resistance in remedial teaching. This case is mentioned because there is reason to expect the same behavior in remedial arithmetic.

MacLatchy\textsuperscript{52} reports a case study of a third grade boy whose wrong behavior in computation persisted for sometime after remedial instruction, even though he understood all his own mistakes.

Baker\textsuperscript{53} studied thirty-nine cases of arithmetic disability and studied the correlation of seven psychological tests with arithmetic achievement. He found that the Binet Free Association test was related to the choice of a wrong process in arithmetic. The Binet vocabulary was related to poor achievement in basic fundamentals and four other factors were related to disability in arithmetic. It does not appear profitable to discuss these findings at great length. The most important purpose of such a study is to provide tools for making accurate predictions, which does not appear feasible in a problem with so many causative factors. Baker does state that pupils will not improve if they do not have a favorable score on a majority of the factors involved.

Myers\textsuperscript{54} discusses the significance of systematic errors for elementary teaching, from the standpoint of prevention, as well as cure. Since this work is fairly extensive and important to all elementary teachers, it is appropriate to excerpt a few important conclusions:

1. The earlier systematic errors are corrected, the better.


2. After a child has learned correct responses, he may regress to an earlier error under some conditions.

3. Emphasis on speed results in a loss in accuracy. Emphasis on accuracy and real control of the subject matter will increase speed.

4. Current teaching is likely to consider a wrong answer as better than no answer, since the student tried. Diagnosis of much data reveals wrong responses made in order to try something. The risk is that of forcing a poor response pattern to become part of the nervous system of the individual, making future correct teaching more difficult. Children should be encouraged to say, "I don't know" and leave the space blank.

Anderson reports an experiment designed to compare meaningful methods of teaching to drill methods. Two groups were compared for achievement in the areas of computation and problem solving. While there was no significant difference in achievement in computation between the two groups, the group taught by meaningful methods had a distinct advantage in problem solving. A recall study made at a later time indicated that the group taught by meaningful methods exhibited superior recall in computation as well as problem solving.

Thompson reports the administration of a two year controlled experiment involving over one thousand students in remedial teaching in arithmetic, reading and language usage in secondary schools. No methods were prescribed at the outset, and a survey of the opinions of teachers

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and administrators regarding organization and methods of teaching was made. The consensus indicated the following opinions:

1. Remedial work should be scheduled, not left to chance arrangement. Attendance should be required.
2. Remedial classes should not be designated in a manner which will embarrass pupils.
3. A lighter work load for both teacher and pupil is necessary to make remedial instruction feasible.
4. Senior high school pupils below the tenth grade will usually profit from such instruction.
5. Teachers should be specially trained in remedial techniques.
6. Individualized work in groups is the best method.
7. Pupils should be chosen by achievement test scores chiefly, with intelligence test scores considered.
8. Students should be given high school credit for the work.
9. Time allotments for this teaching should be made by the semester, should be kept flexible, and should not last more than a year.

IV. Studies of Specific Teaching Techniques, and Specific Factors in Remedial Diagnosis

A. Harvey\textsuperscript{57} discusses an experiment concerning one particular factor in diagnosis, namely the lack of understanding of zero in multiplication. The diagnostic test revealed many students misunderstood the nature and use of the zero place holder. The misunderstandings observed were of four types:

\textsuperscript{57}Harvey, \textit{op. cit.}
1. Multiplying by zero.
2. Misunderstanding the nature of place value.
3. Errors in carrying.
4. Too few or too many zero place holders.

Reteaching resulted in a great reduction in the number of errors, and a great increase in the number of pupils with no errors.

Eaton describes a technique with the use of the dictaphone. Recordings were made of the voices of children while adding sums. The children were encouraged to do their thinking aloud. The recordings enabled the examiners to make specific diagnosis of particular mistakes.

Smith and Eaton also describe the use of a machine called the "Number Master". This machine enlarged the fundamentals of the basic tables and presented them to students for controlled periods of time. Students were observed closely and records made of immature or incompetent procedures. They observed eight kinds of difficulties, such as: counting, tapping, etc. The same behavior has been adequately described by Fernald.

It is difficult to see the need for the relatively expensive procedures described by Smith and Eaton. It seems that the same diagnosis can be made with paper and pencil testing and a stop watch.

Morton describes the use of pocket charts in the teaching of place value. His work was directed primarily to testing of decimals, but

58 Eaton, M. T., Value of the Dictaphone in Diagnosing Arithmetic Difficulties, Indiana University School of Education, B 14, pp. 5-10, September, 1938.


should be equally useful for teaching whole numbers to younger children. The pocket chart resembles a cloth shoe bag with strips of paper in the pockets, representing numbers in multiples of 10 as we look from right to left. Variations of this technique have been described elsewhere, but this one has the advantage that the students handle the strips of paper and have concrete experience in dealing with numbers.

B. 1. Physical and emotional factors.

Schonell reports that clinical observations indicate normal emotional reactions are more necessary than normal intellectual ones in the teaching of arithmetic.

Levy discusses maternal overprotection as a factor in arithmetic deficiency. He reveals, from clinical observation, that the overprotected boy of the aggressive type takes less kindly to a more disciplined subject. He noted several mothers who coached their boys on school work and who could not do the problems in upper grade arithmetic. This resulted in poor achievement on the part of the child. He noted further that the overprotected child showed a tendency to be advanced in reading as well as retarded in arithmetic.

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Rochlin\textsuperscript{63} reports a study of non-intellectual factors in learning general mathematics in college. The study was made by administering several projective type testing instruments to the students and comparing the responses to their achievement in general mathematics. The students were classified as underachievers and overachievers by comparing predictive ratings with achievement in the course. Both groups were found to be significantly different than normal achievers in several respects. The underachievers were significantly different in:

1. Non-positive cathexis (direction of psychical energy) for non-interpersonal activity.
2. Failure to make predominant use of a deductive mode of approach.
3. Predominant use of an intuitive mode of approach.
4. Lack of self-confidence in dealing with non-interpersonal activity.
5. Lack of self-confidence in relations with instructors.
7. Conflict or emotional disturbance associated with awareness of inadequacy.
8. Non-flexible organization of non-interpersonal activity.
10. Unfavorable outcomes projected for peers in school situations.

The overachievers were significantly different in:

1. Inclusion of abstract considerations.
2. Persistence in the face of obstacles to personal growth in the direction of greater social and emotional maturity.
3. Persistence in the face of obstacles to school achievement.
4. Self confidence in dealing with non-interpersonal activity.
5. Flexible organization on non-interpersonal activity.
6. Favorable outcomes projected for self in school situations.
7. Absence of dislike for mathematics.

Rochlin is careful to point out that the population studied, of University of Chicago students, is a highly selected group and may not be at all representative of other schools. However, the results reported will not be surprising to anyone who has done clinical work with younger children, when the operation of many of these factors has been observed.

Werner and Carrison\(^6\) describe a finger schema test. This test is designed to measure the awareness of finger relationships, both spatial and numerical. They found the results of this test significantly related to arithmetic achievement. They also describe a condition known as finger agnosia, which has a pathological basis and similar symptoms to a low achievement on the finger schema test. They believe that schema awareness is a matter of development rather than pathology.

Cruickshank\(^6\) studied the arithmetic vocabulary of mentally retarded boys. He found them poorer in arithmetic vocabulary in all words related


to the various processes of arithmetic, as compared with a controlled group of normal boys.

Cruickshank\textsuperscript{66} reports another study in which he compared mentally retarded boys with normal children for a variety of factors. He found:

1. If extraneous material is introduced into a verbal arithmetic problem, it will create more confusion and poor achievement among retarded boys than among the normal.

2. Retarded boys were also poorer in verbal problems with no extraneous material.

3. Normal boys were more likely to use paper and pencil and calculate an answer, as compared with mental calculations.

4. In a more important part of the experiment, two groups were matched for intelligence, a group with good arithmetic achievement and the other with poor achievement. Cruickshank concluded that reasoning ability was not the differential factor involved. He suggests that insight and the ability to formulate relationships are the important factors.

5. Retarded boys, in a test on choosing the right arithmetic process, exhibited a need to give a response of some kind, regardless of its appropriateness to the problem. They also chose a wrong operation, or changed operations in the middle of a problem. The members of the control group exhibited the same behavior, but in a considerably smaller proportion.

6. The experimental group had much poorer work habits (neatness, spacing, etc.).

7. The difference in achievement between the two groups was much smaller on more concrete problems.

Tilton reports an experiment which attempted to see if environment could change the place of arithmetic in the educational profile. Children were selected in the classes of six different teachers. The teachers were encouraged to give special attention and encouragement in arithmetic to these children. After several months, the achievement of these children was remeasured and the relative position of arithmetic was examined. If we look merely at the statistical average, it would appear that the results of the experiment were insignificant. However, careful examination of the data, by teachers, shows that some teachers exerted a positive influence on the relative achievement in arithmetic of their pupils, and others exerted a negative influence. The teacher most successful in this endeavor was already accustomed to a large amount of individualized instruction in her classroom. One of the teachers with negative results confined her "special attention" to writing notes to the parents about how poorly their children were doing. We draw the conclusion that environmental factors, particularly the teacher-child relationship, have a large influence on the relative position of arithmetic in the profile, although Tilton himself is more cautious.

Dutton made a study of the attitudes of teachers in training, toward arithmetic. The group studied was a class in the methods of arithmetic, at a teachers college. The great majority of the students exhibited attitudes which were highly unfavorable toward arithmetic. Some of the reasons given were:

1. I was afraid of making mistakes.
2. I always felt insecure with the subject.
3. Speed tests bothered me.

It was apparent with class discussion, that these young teachers were involved in a vicious cycle, in which their attitudes would influence those of their students if something were not done. Dutton therefore devoted the semester's work to helping the students work through their own attitudes, rather than the customary procedure of discussing arithmetic methods per se.

Wilson describes an effort to make the teaching of college arithmetic more interesting. The procedure was to teach about number systems with other bases than ten and to bring in material on the history of arithmetic. He reported considerably more interest on the part of junior college students than with previous classes.

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Alkire gave the Davis arithmetic test and a standard intelligence test to a large sample of 12th grade students. The sample was sorted over for various background factors which were examined for significance. The following conclusions were derived:

1. Boys were more likely to have higher achievement than girls.

2. Students who learned their elementary arithmetic in a rural school showed higher achievement.

3. Students from larger high schools (over 500 pupils) showed higher achievement.

4. Students with more experienced and more highly trained teachers showed better achievement.

Brownell studied the effect of practicing a complex skill upon previously learned simple skills. Students learning two place long division were examined for the effect on previously learned arithmetic skills. He concluded:

1. Learning two place division does not improve achievement on previously learned skills.

2. Retroactive inhibition may or may not exist and is more likely to affect the skills more closely related to the new skill.

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Glennon developed a test designed to measure the extent to which students understood the meaning of arithmetic. With all the discussions and controversies concerning meaningfulness in the teaching of the subject, there has been little attempt to measure how much students have achieved in this respect. The test was administered to a variety of groups from grades seven to fourteen, with the results clearly showing meager understanding of the meaningful aspects of arithmetic. The test was also administered to a number of teachers, who achieved a mean score of 55%. Glennon shows the tabulations for the various grades, indicating a gradual rising from 12.5% in the seventh grade to 44% for college freshmen. The picture is probably more serious than reported, since the sharp jump from the ninth grade to the twelfth grade is probably due to the large percentage of school drop outs in the interim years, rather than actual growth in understanding of those who remained.

Along the same lines, Johnson has discussed what we mean by meaning in arithmetic. He argues that we must first teach the pupil how to do a process, then why we do it that way. He claims that the time lag between how and why should be from a half a minute to four years, depending on the complexity of the process. He believes that the how of the four fundamental processes can be learned by the eighth grade, but that the why of common fractions and per cents is best deferred until high school. This is an area in which research is greatly needed.

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SUMMARY AND IMPLICATIONS OF RESEARCH FINDINGS

I. REMEDIAL TEACHING PROJECTS:

The reader will note that while significant results are reported in all of the studies described, the achievement tests used were not mentioned. The measurement of growth was seldom given. This omission has been deliberate, since it is rarely that the studies have used similar tests, and there is little basis for comparison from one study to another. Some writers have translated the gains described into years and months of growth. A careful study of the logic of this procedure would reveal that what constitutes a year's growth on one standard test, given in one part of the country, would represent something quite different on another standard test administered elsewhere. The logic of the normative approach can be seriously challenged, at least theoretically, since it is conceivable that dramatic changes in teaching methods might materially change the norms for a particular school. It is worth noting that many of the diagnostic tests used were not standard achievement test at all; in fact, no diagnostic arithmetic test could be found for which norms are given. With these considerations in mind, it still appears possible to summarize certain important aspects of these studies:

1. The causative factors in arithmetic deficiency as described by Fernald are consistent with the observations, procedures and results of

\[74\text{Fernald, op. cit.}\]
every study described. While some observers have said little, if anything, about emotional and physical factors, the data describing the results indicates that the factors were there and, quite possibly, were effectively dealt with. The only point of theoretical disagreement seems to center about the question of the correlation of intelligence and arithmetic achievement tests. This will be discussed in a later section.

2. A large variety of teaching experiments have been described, embracing a variety of organizational forms, ranging from individual case studies through work with pupils in small groups of four to seven, running the gamut up to group techniques with large classes. It is amazingly apparent that all of these procedures have produced good results. Through these diversified practices teachers should seek the elements which were common to all, and can be postulated as essential to successful remedial teaching. Two elements come out of this examination:

   a. Every experiment used individual diagnosis, and lesson plans, based on the diagnoses, as a basic teaching approach.

   b. The nature of more individualized teaching is bound to bring about a higher order of effective pupil-teacher relationships. The importance of this element will be demonstrated in this study. The same observation has been made in other teaching fields.

   One question remains unanswered at this point. Case study and small group teaching is expensive; some attempt should be made to compare the effectiveness of this type of organization with remedial teaching done in larger classes. Since, as previously discussed, the
various studies have used measures for evaluation which cannot be compared, no such comparison exists at this time. Such a comparison will be made in this study. (See Chapter V.)

II. ERROR DIAGNOSIS STUDIES

Most of the studies in this group do not have clear implications because of the factor of random error described by Grossnickle. For example, Davis and Rood, in discussing the results obtained when pupils were retested five times in two years, made numerous statements regarding loss of learning with recovery of learning, which may not have been loss of learning at all. The factor of random error might have provided a better explanation. Most of the other studies report but a single testing, so that the factor of random error cannot be effectively separated.

In spite of this difficulty, consistency in the kinds of error at all grade levels is apparent from one study to another. The percentage of incidence of a particular type of error will vary somewhat from one experiment to another. This may be largely due to varying populations, age and grade levels. Many of the errors are relatively rare, and important only for individual cases. However, a few areas are quite common, with important implications for all kinds of teaching situations. The following appear most important:

a. Errors in the use of zero, both as a place holder in multiplication and division, and other zero errors, such as $8x0$ equals 0.

b. Errors in borrowing in all kinds of subtraction.

c. Errors in the understanding and use of the decimal point in all four fundamental operations.

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75 Davis and Rood, op. cit.
d. Errors in carrying in addition and multiplication.

e. Errors in the tables of fundamental facts.

There is hardly any need to enlarge on this summary since Brueckner has described the various items in great detail. Since each particular type of error behavior has a small incidence and may have a variety of causes, we believe that there is no substitute for the clinically trained teacher who is thoroughly familiar with the kinds of analyses needed to make a diagnosis for a particular individual. No testing instrument exists which can do this. We believe further that none will ever be devised. The test discussed in a later chapter which was used in this study is a means of making an introductory diagnosis only.

Most studies have treated errors in whole numbers and errors in decimal numbers as separate categories. This is not correct procedure. With the exception of specific manipulations of the decimal point (for example, in problems like 3.16 plus 8.2 plus 18 plus .008), the errors found in decimal studies were the same ones found in whole number studies. Psychologically the two types of problems are much the same. Indeed, it is not surprising that most error studies report more errors in common fractions than in decimal fractions.

III & IV. LEARNING THEORY; PERSONALITY FACTORS AND SPECIFIC FACTORS:

The purpose of this section is to discuss certain specific controversies which appear in the literature.

A. Hildreth and others report a high correlation between intelligence and arithmetic achievement. On the other hand, Plank,

76 Brueckner, op. cit. (Diagnostic and Remedial Teaching of Arithmetic).

77 Hildreth, op. cit.
and other clinical observers, have expressed the view that emotional factors are far more important in arithmetic deficiency than low intelligence. If we examine the work of Cruickshank, it is observed that the difference in behavior of the normal and the mentally retarded boy is one of degree, rather than kind. This, and other clinical observations, added to studies which have been made in other fields on factors in intelligence, lead the writer to believe that the evidence is not in conflict at all. It appears rather that certain factors, emotional or physical, will lead to both low operating intelligence and retardation in arithmetic. This is not a clear cut affair, since some cases of arithmetic deficiency have high intelligence scores. This merits considerable further study.

B. The studies reported above by Bruin, Scheidler and McFadden on individualized instruction, are most important. These studies were not directed toward children who might be classified as remedial. The findings were consistent with the findings of the remedial teaching studies. It appears that some means of individualizing instruction so that the lessons are tailor made for the pupil, with the added promise of better (closer) pupil-teacher relationships, is most important for all teaching of arithmetic, not just for

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78 Cruickshank, op. cit. (Arithmetic Vocabulary of Mentally Retarded Boys & Arithmetic Ability of Mentally Retarded Children).

79 Bruin, op. cit.

80 Scheidler and McFadden, op. cit.
remedial teaching. Note, in addition, the conclusion of Alkire that children from rural elementary schools show superior achievement in arithmetic. Since it does not appear likely that rural teachers are more experienced or better trained, indeed the reverse, other reasons must be found for this superiority. It seems reasonable that instruction in the rural school house is more individualized than in the city school, a conclusion which speaks for itself.

C. Sweeney has argued that teachers can set the goal of 100 percent mastery in the fundamental processes for all children. Plank, commenting on this statement by Sweeney, expressed the view that insecure children would have great difficulty in dealing with a teacher who adopted so rigid a standard and viewpoint. It would appear at first glance that we are describing the work of two different kinds of personalities, whose impact upon a retarded or insecure child would be quite different. A more careful examination of what Sweeney has done indicates to the writer that this is not so. She has merely argued that deficiency creates deficiency, if uncorrected. The insecure child, who has not mastered the multiplication tables, will have his insecurities multiplied when he attempts more advanced work. However, a program designed to help him overcome the deficiency would be a great morale builder as well as a teaching program. This statement is not hypothetical since the

81 Sweeney, op. cit.
82 Plank, op. cit.
report of Sweeney's work given by Wilson describes the improvement in the emotional behavior of the children, as well as the gains in arithmetic.

D. Of the many people reported in this chapter, and in the larger general area of research in arithmetic, the number of writers who have concerned themselves with emotional and habitual factors, on a factual, rather than purely theoretical basis, is but a small minority. Considering the reports made by Dutton and Wilson on teaching attitudes and that by Tilton on the effect of teacher attitudes, it must be realized that this is a major area for consideration in teacher training and in the classroom. These factors have been known and dealt with by clinicians for many years. The implication for broader application appears consistent with studies in many other teaching areas.

E. A general observation which can be made of the various studies is that, in general, the things we teach are independent items. Note the study reported above by Brownell. Note also the conclusion that teaching algebra has no affect on arithmetic achievement.

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83 Wilson, op. cit. (Teaching the New Arithmetic).
84 Dutton, op. cit.
85 Wilson, op. cit. (Staking out New Ground).
86 Tilton, op. cit. (Experimental Effort to Change the Achievement Test Profile).
87 Brownell, op. cit.
Many studies have been criticized, as may this one, for limiting its area to computation in arithmetic. What about the social uses of arithmetic? What about using the social meanings of arithmetic for motivation and to promote meaningful understanding? The writer is inclined to the view that the factors mentioned in these questions are more independent than previously thought.
As stated in Chapter II, several tests were given to large numbers of entering 9B students at Cody High School in September, 1953. The program had two purposes:

1. To aid in establishing classification procedures for placing students in appropriate classes.

2. To aid in defining the problems and areas of remedial arithmetic.

With these objectives in mind, numerous procedures and results will be described. Criteria currently utilized by the Detroit Public Schools for classifying students, which led quite naturally to the testing program, are discussed. Details of the distribution of scores on the Iowa Every Pupil Test of Basic Skills, Part D, (Arithmetic) and on the Cody High School Diagnostic Arithmetic Test are described, followed by a discussion of the correlation between the two tests, and its implications for classification. Since the Cody High School Diagnostic Arithmetic Test is a new test, its reliability and objectivity are discussed in detail. The results of administering the Detroit Public School Tests on the Basic Subtraction, Multiplication and Division Facts are presented in detail. Finally, a detailed discussion of the diagnostic analysis done with test papers from the Cody High School Diagnostic Arithmetic Test is summarized in terms of theoretical and instructional implications.
1. Consider the first problem. Detroit Public Schools publish an algebra aptitude test used by many high schools for the classification of students. Cody High School was in the throes of organization of a new school (1953-1954 was the second full year of operation) and testing procedures had not been clearly established. Therefore some students were placed by the algebra aptitude test, but many others were placed in mathematics classes by eighth grade marks, intelligence and reading scores, teachers' ratings, or whatever other information was available to the school counselors. Often a grade point equivalent for mathematics appeared on a student's record, but the counselor had no way of telling if the score was based on Iowa, Stanford, California, or other standard test scores, which are not necessarily equivalent. The process of transfer from the elementary to the senior high school involves the use of temporary records until standard permanent record forms are filled out. Clerks often copy the score without the title of the test. To further complicate the issue, a number of parents with a great deal of middle class drive insisted that their children be given algebra, whether the records indicated the appropriateness of such an assignment or not. The testing survey began after the class assignments were made.

a. The Iowa Every Pupil Test of Basic Skills, Part D (Arithmetic), Form C, was administered in all classes. Since it is a sixty-eight minute test, exclusive of administration time, two class days were required for its administration. Six hundred and forty-three students completed the test. Table 1, following, shows the distribution of the raw scores.
Table 1

Iowa Every Pupil Test of Basic Skills, Part D, Advanced (Arithmetic) Achievement Test, Raw Scores September, 1953

<table>
<thead>
<tr>
<th>Score</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>91-95</td>
<td>2</td>
<td>.3</td>
</tr>
<tr>
<td>86-90</td>
<td>12</td>
<td>1.9</td>
</tr>
<tr>
<td>81-85</td>
<td>22</td>
<td>3.4</td>
</tr>
<tr>
<td>76-80</td>
<td>39</td>
<td>6.1</td>
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<td>71-75</td>
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<td>8.4</td>
</tr>
<tr>
<td>66-70</td>
<td>63</td>
<td>9.8</td>
</tr>
<tr>
<td>61-65</td>
<td>69</td>
<td>10.7</td>
</tr>
<tr>
<td>56-60</td>
<td>83</td>
<td>12.9</td>
</tr>
<tr>
<td>51-55</td>
<td>57</td>
<td>8.9</td>
</tr>
<tr>
<td>46-50</td>
<td>61</td>
<td>9.5</td>
</tr>
<tr>
<td>41-45</td>
<td>64</td>
<td>10.0</td>
</tr>
<tr>
<td>36-40</td>
<td>49</td>
<td>7.6</td>
</tr>
<tr>
<td>31-35</td>
<td>30</td>
<td>4.7</td>
</tr>
<tr>
<td>26-30</td>
<td>19</td>
<td>3.0</td>
</tr>
<tr>
<td>21-25</td>
<td>11</td>
<td>1.7</td>
</tr>
<tr>
<td>16-20</td>
<td>8</td>
<td>1.2</td>
</tr>
</tbody>
</table>

TOTALS 643 100.1
In Table 1, the grand mean score is 55.9, a raw score equivalent to a grade point score of seven years and two months. The test scores at which the lines for classification are drawn must be arbitrary until experience in the results of such a procedure has been accumulated.

b. The author constructed a diagnostic arithmetic test, which was given in seven Algebra classes, nine General Mathematics classes, and two Remedial Mathematics classes. While the purpose of the test was primarily to aid in the diagnosis of specific difficulties, a scoring system, related to the diagnosis, was devised. The test was limited in area to arithmetic computation, for reasons discussed at length in Chapter II. (See page 14) The maximum score possible was eighty-seven points. The test was revised slightly the following year, and the maximum score was then eighty-eight points. For complete details, see Appendix A.

Table 2 shows the distribution and the arithmetic mean on the Cody High School Diagnostic Arithmetic Test for each group, and for the entire group. Five hundred and eighty-two students completed the test.

One can readily see that many students were placed in General Mathematics and Algebra who might be considered remedial in arithmetic on the basis of this test. Later experience clearly showed that students scoring fifty-two or less were very likely to profit from remedial instruction. Indeed, many earning a higher score would profit. Using fifty-two or less as a criterion, one finds eighty-three 'misplaced' students, to say nothing of those 'misplaced' at the other end of the scale.

Note that the standard deviation of the overall group is 17.2. This is very important as a base of reference in future tables.
Table 2

Diagnostic Arithmetic Test Scores
September, 1953

<table>
<thead>
<tr>
<th>Score Group</th>
<th>Frequency</th>
<th>General Mathematics</th>
<th>Remedial Mathematics</th>
<th>Total Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>81-87</td>
<td>59</td>
<td>32</td>
<td>1</td>
<td>92</td>
</tr>
<tr>
<td>74-80</td>
<td>70</td>
<td>78</td>
<td>2</td>
<td>150</td>
</tr>
<tr>
<td>67-73</td>
<td>28</td>
<td>54</td>
<td>2</td>
<td>84</td>
</tr>
<tr>
<td>60-66</td>
<td>20</td>
<td>46</td>
<td>4</td>
<td>70</td>
</tr>
<tr>
<td>53-59</td>
<td>14</td>
<td>33</td>
<td>6</td>
<td>53</td>
</tr>
<tr>
<td>46-52</td>
<td>2</td>
<td>29</td>
<td>11</td>
<td>42</td>
</tr>
<tr>
<td>39-45</td>
<td>5</td>
<td>17</td>
<td>12</td>
<td>34</td>
</tr>
<tr>
<td>32-38</td>
<td>1</td>
<td>13</td>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>25-31</td>
<td></td>
<td>7</td>
<td>10</td>
<td>17</td>
</tr>
<tr>
<td>18-24</td>
<td></td>
<td>7</td>
<td>7</td>
<td>14</td>
</tr>
<tr>
<td>10-17</td>
<td></td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>TOTALS</td>
<td>199</td>
<td>318</td>
<td>65</td>
<td>582</td>
</tr>
</tbody>
</table>

Mean Scores: 73.8, 63.5, 41.9, 64.2

Note: Standard deviation for the Total Group was computed as 17.2.
Since the Cody High School Diagnostic Arithmetic Test was newly constructed, it became appropriate to question its validity and reliability. It has been previously argued that the goal of achievement in arithmetic computation is sufficiently valid, of itself, to permit the assumption of face validity for such a test.

To examine the reliability of the test, it was administered on successive Mondays to two classes in General Mathematics. Instruction during the interim period was concerned with the subject of graph construction, an area of subject matter only incidentally related to skill in computation. A sample of sixty-six was obtained with the results shown in Table 3.

Table 3
Reliability Test and Retest Scores for the Cody High School Diagnostic Arithmetic Test

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Coefficient of Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Test</td>
<td>69.1</td>
<td>14.8</td>
<td></td>
</tr>
<tr>
<td>Second Test</td>
<td>66.9</td>
<td>16.3</td>
<td>.761</td>
</tr>
</tbody>
</table>

Number of Cases = 66

The mean of the first test was found to be significantly higher than the mean of the Overall Student Body shown in Table 2, using the normal curve test of the null hypothesis. The mean of the second test does not differ significantly. The variances were tested for compatibility with the variance of Table 2, and with each other, by means of the Snedecor F test. No significant variations were found.
As explained in Chapter II, traditional means of improving reliability were unsuitable to this problem. However, an investigation was made of the test papers, particularly those pairs in which the scores were farthest apart. Sixteen pairs of papers were found, in which the discrepancy in score was traceable to the single factor of time. These students had finished from three to fifteen more questions on one test, than on the other. With these papers omitted, the correlation was recomputed for the remaining fifty cases. The results appear in Table 4.

Table 4

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Coefficient of Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Test</td>
<td>71.0</td>
<td>16.1</td>
<td></td>
</tr>
<tr>
<td>Second Test</td>
<td>69.0</td>
<td>14.7</td>
<td>.925</td>
</tr>
</tbody>
</table>

Number of Cases = 50

The same test of the variances, described under Table 3, was used for Table 4, with similar tests. Note, however, that both means are significantly higher than the mean of the Overall Student Body described in Table 2. The reader should be reminded that the cases in this sample are not taken from the Overall Student Body in Table 2. As some doubt existed, at this point, regarding the validity of the conclusion, one further step was taken. The sixteen pairs of papers removed from the
first sample, to prepare Table 4, were corrected by scoring only the questions which had been answered on both papers. The score of the paper with more answers was thus reduced. The sixteen pairs of papers were then added back to the sample with the results shown in Table 5.

Table 5
Reliability Test and Retest Scores for the Cody High School Diagnostic Arithmetic Test

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Coefficient of Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Test</td>
<td>66.3</td>
<td>16.6</td>
<td>0.917</td>
</tr>
<tr>
<td>Second Test</td>
<td>66.1</td>
<td>16.3</td>
<td></td>
</tr>
</tbody>
</table>

Number of Cases = 66

In this case, the tests reveal that both the means and the variances are compatible with the overall data of Table 2. However, some slight doubt exists regarding the sample. It seems reasonable to conclude that the test is highly reliable when used as a power test, rather than a timed 40 minute test, as originally intended.

A remaining question regarding this test has to do with the diagnoses obtained. Even if the score is reliable, will the diagnosis prove reliable? This will be discussed in a later section of this chapter.

c. The next step was to study the correlation between the Iowa arithmetic test scores and the diagnostic test to determine their relative merits as classification tools. The coefficient of correlation was computed to be \( r = 0.586 \). While this may be considered satisfactory
positive correlation for some theoretical purposes, it poses great difficulties for purposes of prediction and classification.

Table 6 is a scatter diagram showing the relationship between the two tests, from which the \( r = .586 \) was computed. It illustrates the problem neatly. The double lines indicate arbitrary points of classification for a three track mathematics program. Of the nine boxes formed, only three would indicate agreement of both scores. We can easily see that one hundred and fourteen students would be placed questionably, comparing one criterion with another. For example, nine students are in the range of fifty to seventy-nine on the diagnostic test, designated as 'average', who scored from 4.0 to 5.9 on the Iowa Every Pupil Test, designated as 'slow'.

Note that \( n = 326 \), a surprisingly small number, when it is noted that \( n = 643 \) in Table 1 and \( n = 582 \) in Table 2. Almost nine hundred students entered Cody High School that semester. It was intended to give both tests to all students, but the attendance problem brought about the limited results. It seems logical to the author that the correlation would be no better, and conceivably a good deal worse, in the "absent" group, compared with the "present" group.

At this point it seems important to remind the reader that this is written in retrospect. Some of the conclusions and theories advanced in the next few pages are the results of insight gained later in the study, which illuminated the path of logic in relation to a testing program.

Many would quarrel with the analysis of Table 6 on the grounds that the two tests do not measure the same thing. The Cody High School
Table 6
Scatter Diagram Showing the Relationship Between the Iowa Every Pupil Test of Basic Skills, and the Diagnostic Arithmetic Test, by cell frequencies

<table>
<thead>
<tr>
<th>Iowa Score (Grade Pt.)</th>
<th>4-4.9</th>
<th>5-5.9</th>
<th>6-6.9</th>
<th>7-7.9</th>
<th>8-8.9</th>
<th>9-9.9</th>
<th>10-10.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagnostic Score</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80-87</td>
<td>6</td>
<td>12</td>
<td>19</td>
<td>21</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70-79</td>
<td>2</td>
<td>24</td>
<td>35</td>
<td>28</td>
<td>12</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>60-69</td>
<td>2</td>
<td>28</td>
<td>14</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>50-59</td>
<td>1</td>
<td>4</td>
<td>19</td>
<td>14</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>40-49</td>
<td>3</td>
<td>15</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-39</td>
<td>1</td>
<td>2</td>
<td>12</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20-29</td>
<td>6</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-19</td>
<td>1</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of Cases = 326
Diagnostic Arithmetic Test is strictly a measure of computational skill, while the Iowa Every Pupil Test of Basic Skills examines many areas of arithmetic achievement. Indeed, it is divided into three sections, Part I for mathematical information and vocabulary, Part II for computation, and Part III for applied problems. To test this argument, a separate correlation was figured for the Diagnostic Arithmetic Test score with the grade point score on Part II of the Iowa Every Pupil Test. One might reasonably expect a greater degree of agreement. Table 7 is a scatter diagram, upon which the correlation was based.

Using the same arbitrary lines for classification as in Table 6, one again finds one hundred and ten 'misplaced' students. The coefficient of correlation was computed to be .638. Since the variable calculated by the transformation

\[ z = \frac{1}{2} \log_e \frac{1 + r}{1 - r} \]

is approximately normally distributed with mean \( \frac{1}{2} \log_e \frac{1 + r}{1 - r} \) and standard deviation \( \sqrt{n - 1} \), the null hypothesis can be established to test the significance of the difference between \( r = .586 \) in Table 6 and \( r = .638 \) in Table 7. The critical ratio was calculated as 1.06. Thus, the relationships of Table 7 are not significantly different than those of Table 6.

Had a third, independent test been introduced, and correlated with the results of either test described above, the author would predict similar results. Remembering that the students were originally placed in classes by other criteria, the reader can observe as much disagreement about the classification of students in Table 2 as in Tables 6 and
Table 7

Scatter Diagram Showing the Relationship Between Part II, Iowa Every Pupil Test of Basic Skills, and the Diagnostic Arithmetic Test, by cell frequencies

<table>
<thead>
<tr>
<th>Iowa Score (Grade Pt.)</th>
<th>4-4.9</th>
<th>5-5.9</th>
<th>6-6.9</th>
<th>7-7.9</th>
<th>8-8.9</th>
<th>9-9.9</th>
<th>10-10.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagnostic Score</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>80-87</td>
<td>5</td>
<td>12</td>
<td>29</td>
<td>14</td>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>70-79</td>
<td>3</td>
<td>19</td>
<td>47</td>
<td>27</td>
<td>11</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>60-69</td>
<td>2</td>
<td>9</td>
<td>29</td>
<td>14</td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>50-59</td>
<td>2</td>
<td>24</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40-49</td>
<td>1</td>
<td>7</td>
<td>13</td>
<td>7</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30-39</td>
<td>1</td>
<td>10</td>
<td>4</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20-29</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10-19</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Number of Cases = 333
7. The problem could be pursued further by studying the relationship of class placement to the Iowa Every Pupil Test of Basic Skills. It appeared that such a line of inquiry would lead to similar results and would be unprofitable. The clue to the problem lies in the analysis contained in the above paragraph concerning the relationship of Tables 6 and 7.

Logically, the correlation in Table 7 should be higher than that of Table 6. What has happened?

To aid in resolving the dilemma, let us now compare the correlation of Table 7 with that of Table 3, in which the reliability of the Cody High School Diagnostic Arithmetic Test was first computed. Since the variable

\[
z = \frac{1}{2} \log \frac{1 + r}{1 - r}
\]

is approximately normally distributed with mean \(\frac{1}{2} \log \frac{1 + \rho}{1 - \rho}\) and standard deviation \(\frac{1}{\sqrt{n - 1}}\), the null hypothesis that the two sample correlations come from the same population can be tested. In this case, the critical ratio is 1.79, not a significant value. It is conceivable that a larger sample in Table 2 would yield significant results for the same test.

Nevertheless, the difference between the two correlations cannot be considered to be large. It can now be argued that the testing situation itself imposes limits upon the predictions which can be made with test results, due to a variety of attenuating factors operating as powerfully as the relationships between the tests which have been studied. Test scores appear no more, nor less, accurate for placement purposes than numerous other criteria which have been used. It is possible that testing done under more carefully controlled conditions would not be subject to these factors.
to the same degree, but these conditions are not to be expected in normal public school operation.

At this point, the author wishes to postulate some of the attenuating factors which operate to hamper the validity and predictability of individual test results:

1. Items of administration:
   (a) Student misunderstands directions.
   (b) Student is late, and teacher forgets to allow him more time.
   (c) Miscellaneous factors such as broken pencils, etc.

Items in this group are easily corrected by retest, if someone is aware of the inaccuracy.

2. Emotional reactions to a test situation:
   (a) Student gets nervous and makes a large proportion of random errors, or student "freezes up". This factor may be exaggerated with students entering a new school for the first time.
   
   (b) Student misjudges time and proceeds to hurry, increasing error, or student misjudges time and works in too leisurely a fashion, thus not finishing.
   
   (c) Student does not care, or is in a state of rebellion toward the adult world, and deliberately gets a low score.
   
   (d) Student is aware of the purpose of the test, and deliberately gets a low score to be placed in an easier class.

None of the items described occur very often. In total, they occur often enough, and in sufficient degree, to lead to the conclusion that correlations between tests, and between test scores and achievement, will seldom be better than those described here, in practical school situations. Tests will generally have less predictive value at the lower end of the scale.

How, then, is this problem to be resolved? Placement in a "wrong" section, while serious, is neither fatal, nor inflexible. Examination
of Tables 2, 6 and 7 reveals very few instances where students would be misplaced more than one section above or below their ability based on the other criterion. Teachers must be trained to judge these situations and to spot discrepancies. Immediate transfer can be recommended, or promotion to a faster, or slower section for the following semester. Teachers must learn to recognize a situation as dynamic, be prepared to change decisions after a semester, and be prepared for surprising changes in students, both positive and negative. The student who is "average" this year, by all available criteria, may turn out to be "fast" next year, or vice versa. The answer lies, not in refining tests and controlling test situations, which is impossible (to the degree needed), but in training teachers to understand these situations, to acquire skill in making objective judgments, and to bring into the problem the degree of human understanding needed to complement the objective information in test scores.

Before leaving the problem of classification, the reader is offered a few qualitative comments concerning the use of the Iowa Every Pupil Test of Basic Skills, Part D, with ninth grade students. It is conceivable that the first of these might not apply in some high schools:

1. At Cody High School, a score lower than 5.5 (grade pt. equivalent) is probably not valid, and is due to other causes than low achievement.

2. The various sections of the test do give useful information, especially when the separate scores vary greatly.

(a) A girl scored 8.9 on part II (computation) and 6.1 on part III (applied problems). Subsequent class work revealed that she was indeed expert at computation, but had great difficulty in comprehending the application of these skills to a verbal situation.
(b) A boy scored 10.6 on part III, but only 6.9 on part II. He had a high intelligence rating, and did find his weakness in computation a stumbling block to high achievement.

It is also possible for such inconsistencies to reflect aspects of the test situation, rather than the actual learning problem. The comments offered concern individual students only. No attempt has been made to analyse such differences for large groups.

2. Defining the problems of remedial arithmetic:

As described in Chapter II (p. 14), the diagnostic portion of the program was limited to the study of computational skills alone. While this approach has many weaknesses, which are discussed in Chapter VII, it has the virtue of limiting the area of study and implementing more intense examination of the area chosen.

A. The first and most important part of the definition of the problem had to do with the Cody High School Diagnostic Arithmetic Test. A check list of common behaviors in arithmetic deficiency was prepared to go with the diagnostic test. (See appendix A, p.182) The list was based on the work of other investigators, principally Brueckner and on the past experience of the author. The student assistants were trained to make diagnoses, and prepared a check list for each paper, a voluminous task requiring many man hours of labor. A single diagnosis took about five minutes, after the student assistant had acquired some skill in the process. After the check lists were prepared, the data were coded on McBee Keysort Cards, with other pertinent information. Altogether five hundred and twenty-nine cases were coded on the cards.

---

1Brueckner, L. J., Diagnostic and Remedial Teaching of Arithmetic, John C. Winston Company, 1930.
Table 8, following, is a copy of the check list, each item followed by the number and percentage of cases in which each item was diagnosed.

In general, the results are little different than those of other investigators, although the precise percentages may vary somewhat.

Some of the low percentages reflect difficulties in diagnosis rather than an actual low incidence of the behavior described. For instance, item 35, "Does not change mixed numbers to improper fractions, but multiplies parts separately.," was shown by later clinical experience to exist to a slightly higher degree than shown, but the inference was difficult to make from the test paper. Item 30, "8 X 0 equals 8, etc.," is known to exist, but unlikely to show up on this test, due to an absence of suitable test items.

Two important theoretical questions arise concerning the technique just described. The first concerns the objectivity of the diagnosis. To obtain a measure of the objectivity of the work of the student assistants, a sample of twenty-one papers were diagnosed separately by two student assistants. The author then compared the paired check lists for agreements and disagreements. The percentage of agreement was 59.5 percent of eighty-nine items, a figure far too low for a satisfactory instrument. A careful examination was made to determine the causes of the disagreement. Two fundamental difficulties were revealed. One was the fact that student assistants would occasionally miss a diagnostic item that depended on a small visual cue. The other, and more important item, was the fact that student assistants would make a diagnosis based on a single problem, when the instructions were to check the item off only if the behavior were repeated at least once.

It is evident that more intensive training for personnel is essential.
Table 8
Diagnostic Check List for Cody High School
Diagnostic Arithmetic Test
529 cases

<table>
<thead>
<tr>
<th>No.</th>
<th>Item</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>General Comments:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Does not write in straight columns, leading to error.</td>
<td>13</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>Does the wrong operation. (adds when problem says subtract, etc.)</td>
<td>14</td>
<td>2.6</td>
</tr>
<tr>
<td>3</td>
<td>Sloppiness leads to error.</td>
<td>59</td>
<td>11.2</td>
</tr>
<tr>
<td>4</td>
<td>Copies problems or own work incorrectly.</td>
<td>33</td>
<td>6.2</td>
</tr>
<tr>
<td>5</td>
<td>Works slowly and does not finish.</td>
<td>101</td>
<td>19.1</td>
</tr>
<tr>
<td>6</td>
<td>Adding whole or decimal numbers:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Carrying numbers not there, or carrying wrong number.</td>
<td>8</td>
<td>1.5</td>
</tr>
<tr>
<td>7</td>
<td>Not carrying.</td>
<td>7</td>
<td>1.3</td>
</tr>
<tr>
<td>8</td>
<td>Leaving out decimal point.</td>
<td>14</td>
<td>2.6</td>
</tr>
<tr>
<td>9</td>
<td>Misunderstands nature of decimal point in problems like 7.1 + 3.15 + 16</td>
<td>133</td>
<td>25.2</td>
</tr>
<tr>
<td>10</td>
<td>Cannot keep sums in head with long columns of numbers.</td>
<td>17</td>
<td>3.2</td>
</tr>
<tr>
<td>11</td>
<td>Adding fractions and mixed numbers:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fails to reduce correct answer to lowest terms.</td>
<td>50</td>
<td>9.5</td>
</tr>
<tr>
<td>12</td>
<td>Changes denominator incorrectly.</td>
<td>40</td>
<td>7.6</td>
</tr>
<tr>
<td>13</td>
<td>Forgets the whole number part of mixed numbers.</td>
<td>13</td>
<td>2.5</td>
</tr>
<tr>
<td>14</td>
<td>Adds numerator without changing to a common denominator.</td>
<td>7</td>
<td>1.3</td>
</tr>
<tr>
<td>15</td>
<td>Subtracting whole or decimal numbers:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Does not borrow correctly.</td>
<td>41</td>
<td>7.8</td>
</tr>
</tbody>
</table>
Table 8
Diagnostic Check List for Cody High School
Diagnostic Arithmetic Test (cont.)
529 cases

<table>
<thead>
<tr>
<th>No.</th>
<th>Item</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>16.</td>
<td>Does not borrow correctly across 0's.</td>
<td>6</td>
<td>1.1</td>
</tr>
<tr>
<td>17.</td>
<td>Omits decimal point.</td>
<td>11</td>
<td>2.1</td>
</tr>
<tr>
<td>18.</td>
<td>Does not understand nature of decimal</td>
<td>69</td>
<td>13.0</td>
</tr>
</tbody>
</table>
|      | \[
|      | \[
|      | \[
|      | \[
|      | \[
|      | \[
|      | \[
| 20.  | Does not borrow correctly with denominate numbers.                   | 100       | 18.9    |
|      | **Subtracting fractions or mixed numbers:**                         |           |         |
| 21.  | Does not change fractions to a common denominator.                  | 8         | 1.5     |
| 22.  | Borrows 10 instead of the denominator of the fraction.              | 6         | 1.1     |
| 23.  | Subtracts top from bottom instead of borrowing.                      | 14        | 2.6     |
|      | **Multiplying whole numbers and decimals:**                          |           |         |
| 24.  | Does not use 0 correctly with numbers ending in one or more 0's.     | 62        | 11.7    |
| 25.  | Does not use 0 as a place holder, or omits them.                    | 17        | 3.2     |
| 26.  | Mixes up 7x8, 7x9 and 6x9.                                          | 33        | 6.2     |
| 27.  | Forgets to, or omits carrying.                                      | 6         | 1.1     |
| 28.  | Carries numbers not there.                                          | 2         | .4      |
| 29.  | Ignores the decimal point.                                          | 25        | 4.7     |
| 30.  | 8x0 equals 8, 2x0 equals 2, etc.                                     | 0         | 0       |
| 31.  | Places decimals incorrectly.                                        | 34        | 6.4     |
Table 8
Diagnostic Check List for Cody High School
Diagnostic Arithmetic Test (cont.)
529 cases

<table>
<thead>
<tr>
<th>No.</th>
<th>Item</th>
<th>Frequency</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>32.</td>
<td>Multiplying fractions and mixed numbers: Inverts fractions when multiplying.</td>
<td>20</td>
<td>3.8</td>
</tr>
<tr>
<td>33.</td>
<td>Multiplies fractions correctly, then inverts answer.</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>34.</td>
<td>Cancels numerator with numerator, or denominator with denominator, or both.</td>
<td>1</td>
<td>.2</td>
</tr>
<tr>
<td>35.</td>
<td>Does not change mixed numbers to improper fractions, but multiplies parts separately.</td>
<td>6</td>
<td>1.1</td>
</tr>
<tr>
<td>36.</td>
<td>Dividing whole and decimal numbers: Omits 0's needed as place holders.</td>
<td>34</td>
<td>6.4</td>
</tr>
<tr>
<td>37.</td>
<td>Subtracts incorrectly.</td>
<td>5</td>
<td>1.0</td>
</tr>
<tr>
<td>38.</td>
<td>Mixes up $\frac{7}{56}, \frac{7}{63}, \frac{6}{54}$ and/or $\frac{8}{64}, \frac{9}{72}$.</td>
<td>9</td>
<td>1.7</td>
</tr>
<tr>
<td>39.</td>
<td>Omits decimal point.</td>
<td>17</td>
<td>3.2</td>
</tr>
<tr>
<td>40.</td>
<td>Does not use decimal rule correctly.</td>
<td>143</td>
<td>27.0</td>
</tr>
<tr>
<td>41.</td>
<td>Dividing fractions and mixed numbers: Does not invert.</td>
<td>68</td>
<td>12.8</td>
</tr>
<tr>
<td>42.</td>
<td>Inverts dividend fraction.</td>
<td>12</td>
<td>2.3</td>
</tr>
<tr>
<td>43.</td>
<td>Cancels before inverting.</td>
<td>2</td>
<td>.4</td>
</tr>
<tr>
<td>44.</td>
<td>Cancels into fractions before changing mixed numbers to improper fractions.</td>
<td>1</td>
<td>.2</td>
</tr>
<tr>
<td>45.</td>
<td>Seems all mixed up.</td>
<td>32</td>
<td>6.1</td>
</tr>
</tbody>
</table>
The analysis made in Tables 8, 9 and 10 is subject to the limitations discussed in this paragraph. The teaching phases of the study are not affected, since the diagnosis in all cases for that portion of the study were made by the author. In any event, the diagnosis, from a teaching standpoint, is intended to be introductory.

The second important question deals with the reliability of the diagnosis. While the previous analysis revealed that the test is reliable when used as a power test, it is necessary to check the reliability of the diagnostic items. For this purpose, the same sample of sixty-six test papers used in the first reliability check were examined for agreement and disagreement of diagnostic items.

To minimize the problem of objectivity of diagnosis, the papers in this sample were checked by the author personally. Since the author's objectivity remains unchecked, the results obtained and the inferences drawn are subject to similar limitations. However, the fact that the percentage of agreement in the reliability check is higher than in the check of objectivity indicates greater objectivity in diagnosis on the part of the author. The sample of sixty-six papers revealed one hundred and thirty-six diagnostic items, 68.4 percent of which appeared on both test papers. This appears to contradict the high coefficient of reliability obtained from the same papers. The papers were checked only for questions which appeared on both papers, as in the reliability check. A careful examination was made to determine the causes of the low percentage of agreement. The principal cause was the fact that behavior diagnosed from two or three wrong questions on one paper would appear on the paired paper, but only once, not enough under the procedure established to check off that item in the diagnosis. Several pairs of
papers were examined in which the scores were remarkably close, but the diagnosis differed appreciably. This indicates the care the remedial arithmetic teacher must take in making diagnoses, since the test paper, at best, gives introductory information.

Subject to these limitations, the coding of the items in Table 8 on the McBee Keysort Cards made possible a check of the interrelationships between items. Thirteen of the most frequent items were chosen for study, resulting in seventy-eight paired relationships which were examined for significance. The frequencies are listed in Table 9, below. The items followed by the letter s were found to be significantly higher than chance expectation.

The statistical technique used was the Chi-square contingency test for paired attributes. For example, consider numbers 3 and 5 from Table 8 in the following table:

<table>
<thead>
<tr>
<th></th>
<th>5 (works slowly)</th>
<th>Not 5</th>
<th>sub-total</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 (sloppiness)</td>
<td>19</td>
<td>82</td>
<td>101</td>
</tr>
<tr>
<td>not 3</td>
<td>40</td>
<td>388</td>
<td>428</td>
</tr>
<tr>
<td>sub-total</td>
<td>59</td>
<td>470</td>
<td>529 (grand total)</td>
</tr>
</tbody>
</table>

This table fits the theoretical pattern for the Chi-square contingency test with one degree of freedom.

Of the seventy-eight paired items studied, forty-two show significant relationships. Eight of these are significant at the .05 level, the remainder at the .01 level or better. One of the eight items,
Table 9
Cross Frequencies of Selected Diagnostic Items in Table 8

<table>
<thead>
<tr>
<th>Item</th>
<th>No.</th>
<th>3</th>
<th>5</th>
<th>9</th>
<th>11</th>
<th>15</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>24</th>
<th>36</th>
<th>40</th>
<th>41</th>
<th>42</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sloppiness</td>
<td></td>
<td>3</td>
<td>19s</td>
<td>20</td>
<td>7</td>
<td>6</td>
<td>3</td>
<td>17s</td>
<td>14</td>
<td>11s</td>
<td>9s</td>
<td>9s</td>
<td>13s</td>
<td>8s</td>
<td>59</td>
</tr>
<tr>
<td>Slow worker</td>
<td></td>
<td>5</td>
<td></td>
<td>48s</td>
<td>11</td>
<td>4</td>
<td>19</td>
<td>28s</td>
<td>25</td>
<td>15</td>
<td>13s</td>
<td>24</td>
<td>13s</td>
<td>4</td>
<td>101</td>
</tr>
<tr>
<td>Decimal addition</td>
<td></td>
<td>9</td>
<td></td>
<td>26s</td>
<td>16s</td>
<td>46s</td>
<td>29</td>
<td>45s</td>
<td>16</td>
<td>19s</td>
<td>39</td>
<td>20</td>
<td>3</td>
<td></td>
<td>133</td>
</tr>
<tr>
<td>7.1 + 3.15 + 16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Reducing fractions</td>
<td></td>
<td>11</td>
<td></td>
<td>7</td>
<td>17s</td>
<td>9</td>
<td>20s</td>
<td>12s</td>
<td>5</td>
<td>29s</td>
<td>12s</td>
<td>3</td>
<td></td>
<td></td>
<td>50</td>
</tr>
<tr>
<td>Borrowing in subt.</td>
<td></td>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>41</td>
</tr>
<tr>
<td>Decimal subtract</td>
<td></td>
<td>18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15.55 - 3.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Brings down inst. of</td>
<td></td>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>subtracting</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Borrowing with</td>
<td></td>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>denominate numbers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplying with 0's</td>
<td></td>
<td>24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Place holders in div.</td>
<td></td>
<td>36</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decimal rule in div.</td>
<td></td>
<td>40</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Does not invert -</td>
<td></td>
<td>41</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dividing fractions</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inverts wrong fraction</td>
<td></td>
<td>42</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>12</td>
</tr>
</tbody>
</table>

s - Significantly related by Chi-square contingency test
number 3 paired with number 40, shows a frequency significantly lower than the expectation, at the .05 level. All of the remaining 41 show a frequency significantly higher. For the following discussion, number 3 paired with number 40 will be ignored.

It appears that error patterns in arithmetic are not independent items, but part of more comprehensive behavior patterns. The limited information offered by this sort of analysis could not, of itself, aid in making clear inferences regarding the nature of these patterns. However, it is reasonable to argue the theory that the patterns shown in Table 9 conform to the theories advanced by Fernald\(^2\), described in Chapter III of this study, and reiterated in slightly different form throughout this thesis.

Table 10, below, recasts Table 9 into a more verbal form, omitting factors not significantly related. Comments included are attempts to illuminate the information in terms of the theory mentioned above.

For purposes of simplification, two primary causal areas for error patterns in arithmetic are argued:

1. Lack of understanding of the number system.
2. Basic personality factors.

In the descriptions of Table 10, the single words number and personality will be used in place of the statements made above.

B. The next step was the administration of the Detroit Public Schools Test on the 100 Subtraction Facts, the 100 Multiplication Facts, and the 90 Even Division Facts to two classes in Remedial Mathematics and to five classes in General Mathematics. The test on the 100 Addition Facts was not given, since past experience had shown very few students with deficiencies in these.

<table>
<thead>
<tr>
<th>Item (with descriptive comments)</th>
<th>Significantly Related To</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 3. Sloppiness:</td>
<td>No. 5. Slowness</td>
</tr>
<tr>
<td>A basic personality factor.</td>
<td>Indicating that all items</td>
</tr>
<tr>
<td>Indicates that all items shown, except 5, can be symptomatic of personality as well:</td>
<td></td>
</tr>
<tr>
<td>as, or rather than number factors.</td>
<td>19. Brings down instead of subtracting:</td>
</tr>
<tr>
<td></td>
<td>51. - 1.3</td>
</tr>
<tr>
<td></td>
<td>50.3</td>
</tr>
<tr>
<td>No. 5. Slowness:</td>
<td>3. Sloppiness</td>
</tr>
<tr>
<td>Same comment as 3, but with additional complication, since slowness may be due to lack of mastery.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>19. Brings down instead of subtracting:</td>
</tr>
<tr>
<td></td>
<td>51. - 1.3</td>
</tr>
<tr>
<td></td>
<td>50.3</td>
</tr>
<tr>
<td></td>
<td>36. 0's in division.</td>
</tr>
<tr>
<td></td>
<td>40. Decimal rule in division.</td>
</tr>
<tr>
<td></td>
<td>41. Does not invert.</td>
</tr>
<tr>
<td></td>
<td>42. Inverts wrong fraction.</td>
</tr>
<tr>
<td>No. 9. Decimal Addition:</td>
<td>No. 11. Failed to reduce fractional answers.</td>
</tr>
<tr>
<td>7.1 plus 3.15 plus 16</td>
<td>15. Borrowing in subtraction.</td>
</tr>
<tr>
<td>Almost purely a number factor.</td>
<td>18. Decimal subtraction 5.1-2</td>
</tr>
<tr>
<td>Only number 11 casts some doubt, but see below.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>20. Borrowing denominate numbers.</td>
</tr>
<tr>
<td></td>
<td>36. 0's in division.</td>
</tr>
<tr>
<td>No. 11. Fails to reduce fractional answers:</td>
<td>No. 9. Decimal addition.</td>
</tr>
<tr>
<td>Almost purely a number factor.</td>
<td>18. Decimal subtraction 5.1-2</td>
</tr>
<tr>
<td></td>
<td>20. Borrowing denominate numbers.</td>
</tr>
<tr>
<td></td>
<td>24. Multiplying with 0's.</td>
</tr>
<tr>
<td></td>
<td>40. Decimal division rule.</td>
</tr>
<tr>
<td></td>
<td>41. Fails to invert divisor fraction.</td>
</tr>
<tr>
<td>Seems to be primarily a number factor, but note the #3 is related to three of the same factor. Some chance that this could be carelessness, etc. in some students.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>18. Decimal subtraction.</td>
</tr>
<tr>
<td></td>
<td>20. Borrowing denominate numbers.</td>
</tr>
<tr>
<td></td>
<td>24. Multiplying with 0's.</td>
</tr>
<tr>
<td></td>
<td>40. Decimal division rule.</td>
</tr>
<tr>
<td>No. 18. Decimal Subtraction: 5.1-2, etc.</td>
<td>No. 9. Decimal addition.</td>
</tr>
<tr>
<td>At first glance, seems to be reducing fractions. purely a number factor, but careful study shows the issue is not clear cut. Numbers 19 and 40 indicate personality elements which might operate.</td>
<td></td>
</tr>
<tr>
<td>No.</td>
<td>Item (with descriptive comments)</td>
</tr>
<tr>
<td>-----</td>
<td>----------------------------------</td>
</tr>
<tr>
<td>19</td>
<td>Brings down instead of subtracting: 51 or 16</td>
</tr>
<tr>
<td></td>
<td>$\frac{-1.3}{-9\frac{3}{5}}$</td>
</tr>
<tr>
<td></td>
<td>Personality and number factors well mixed in this one; could be due to either or both.</td>
</tr>
<tr>
<td></td>
<td>20. Borrowing Denominate Numbers: Primarily a number item, although 19 and 36 indicate that personality factor may be present.</td>
</tr>
<tr>
<td></td>
<td>36. Omits 0's in quotient: Both factors involved, but personality factors seems more important - a surprising conclusion.</td>
</tr>
<tr>
<td></td>
<td>40. Decimal division rule: Primarily a number item, but 19 and 24 suggest that personality can be important.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Cross Relationships of Diagnostic Items in Table 9

<table>
<thead>
<tr>
<th>Item (with descriptive comments)</th>
<th>Significantly Related To</th>
</tr>
</thead>
<tbody>
<tr>
<td>About 50% of each.</td>
<td>41. Does not invert divisor fraction.</td>
</tr>
</tbody>
</table>
The technique which follows was used for getting the approximate time of completion: The teacher started all students at the same time with a stopwatch; after the first minute, the teacher marked the time on the chalkboard, then erased and changed it every ten seconds. When the pupil completed all of the items on the test, he looked at the chalkboard and marked the time on his paper. This procedure yielded a power score based on the number of right answers, and a completion time. The completion time appears to be a more useful diagnostic item than the power score, since most students scored one hundred percent or close to it. Those students who made more than one or two errors generally had slower times as well as a pattern of error. These patterns are discussed in Chapter V (p. 113). Later clinical experience showed that those who scored one hundred percent but took four minutes or more to complete the table were often using immature methods, such as counting, to get the answers. Table 11 shows the distribution of completion times for the three tests. It is interesting to note that responses to the subtraction facts are somewhat less automatic than those to the other two tables. Clinical experience did show many students who subtracted from numbers larger than ten by counting.

Tables 12a, 12b and 12c show scatter diagrams of the completion times described in Table 11, compared with the corresponding scores on the Cody High School Diagnostic Arithmetic Test. While it is evident that these factors are positively correlated, the relationships are far from clear cut. Many students doing well on the arithmetic test were slow on the tables and vice versa. It is not at all clear when a deficiency in time of accurate response to the tables is a causative
Table 11
Completion Times on the Detroit Public Schools Test on the Basic Subtraction, Multiplication and Division Facts

<table>
<thead>
<tr>
<th>Time (min. &amp; sec.)</th>
<th>Frequency:</th>
<th>Subtraction</th>
<th>Multiplication</th>
<th>Division</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 - 2:50</td>
<td>128</td>
<td>154</td>
<td>123</td>
<td></td>
</tr>
<tr>
<td>3 - 3:30</td>
<td>31</td>
<td>43</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>3:40 - 4</td>
<td>32</td>
<td>24</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>4:10 - 4:30</td>
<td>21</td>
<td>12</td>
<td>13</td>
<td></td>
</tr>
<tr>
<td>4:40 or more</td>
<td>29</td>
<td>26</td>
<td>32</td>
<td></td>
</tr>
<tr>
<td>TOTALS</td>
<td>261</td>
<td>259</td>
<td>219</td>
<td></td>
</tr>
</tbody>
</table>
Table 12a

Scatter Diagram Showing the Correlation Between Diagnostic Arithmetic Test Scores and Completion Time on a Test of the 100 Basic Subtraction Facts

<table>
<thead>
<tr>
<th>Time (min. &amp; sec.)</th>
<th>0-2:50</th>
<th>3:30-4:10</th>
<th>4:10-4:30</th>
<th>4:40 or more</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diagnostic Test Score</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>81-87</td>
<td>6</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>74-80</td>
<td>28</td>
<td>10</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>67-73</td>
<td>7</td>
<td>7</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>60-66</td>
<td>.13</td>
<td>7</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>53-59</td>
<td>9</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>46-52</td>
<td>11</td>
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<tr>
<td>39-45</td>
<td>3</td>
<td>3</td>
<td>2</td>
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</tr>
<tr>
<td>32-38</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>25-31</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>18-24</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
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<tr>
<td>10-17</td>
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<td></td>
</tr>
<tr>
<td>Total</td>
<td>88</td>
<td>45</td>
<td>23</td>
<td>19</td>
</tr>
<tr>
<td>Grand Total</td>
<td>205</td>
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<td></td>
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</tr>
</tbody>
</table>
Table 12b

Scatter Diagram Showing the Correlation Between Diagnostic Arithmetic Test Scores and Completion Time on a Test of the 100 Basic Multiplication Facts

<table>
<thead>
<tr>
<th>Time (min. &amp; sec.)</th>
<th>0-2:50</th>
<th>3-3:30</th>
<th>3:40-4:10</th>
<th>4:10-4:30</th>
<th>4:40 or more</th>
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</thead>
<tbody>
<tr>
<td>Diagnostic Test Score</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>81-87</td>
<td>7</td>
<td>4</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>74-80</td>
<td>22</td>
<td>9</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>67-73</td>
<td>9</td>
<td>6</td>
<td>1</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>60-66</td>
<td>14</td>
<td>6</td>
<td>2</td>
<td>3</td>
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<tr>
<td>53-59</td>
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<td>3</td>
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<td>4</td>
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<td>46-52</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
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<tr>
<td>39-45</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td></td>
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<td>32-38</td>
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<td>9</td>
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</tr>
<tr>
<td>18-24</td>
<td>3</td>
<td></td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>10-17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Total</td>
<td>80</td>
<td>40</td>
<td>16</td>
<td>14</td>
<td>24</td>
</tr>
</tbody>
</table>

Grand Total 174
Table 12c

Scatter Diagrams Showing the Correlation Between Diagnostic Arithmetic Test Scores and Completion Time on a Test of the 90 Even Division Facts

<table>
<thead>
<tr>
<th>Diagnostic Test Score</th>
<th>Time (min. &amp; sec.)</th>
<th>0-2:50</th>
<th>3-3:30</th>
<th>3:40-4</th>
<th>4:10-4:30</th>
<th>4:40 or more</th>
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<td></td>
<td>10</td>
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<tr>
<td>74-80</td>
<td></td>
<td>28</td>
<td>8</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>67-73</td>
<td></td>
<td>9</td>
<td>7</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>60-66</td>
<td></td>
<td>16</td>
<td>4</td>
<td>3</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>53-59</td>
<td></td>
<td>6</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>46-52</td>
<td></td>
<td>9</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>39-45</td>
<td></td>
<td>3</td>
<td>1</td>
<td></td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>32-38</td>
<td></td>
<td>3</td>
<td>6</td>
<td>4</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>25-31</td>
<td></td>
<td>2</td>
<td>2</td>
<td></td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>18-24</td>
<td></td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>10-17</td>
<td></td>
<td></td>
<td></td>
<td>1</td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

Total 88 : 33 17 9 24

Grand Total 171
factor in low achievement in computation, and when it is not.

Whether a deficiency in the basic tables of fundamental facts can stand on its own merits as an instructional need is a matter of educational philosophy. What was done at Cody High School was to consider a combination of slow completion time and incorrect responses as a problem for remedial instruction.

SUMMARY

Conclusions from the Testing Survey:

A. No single test, of the three examined, could serve as a satisfactory instrument for classifying students. The coefficients of correlation between the Iowa Every Pupil Test of Basic Skills, Part D (arithmetic) and the Cody High School Diagnostic Arithmetic Test was .586. This shows a significant relationship, but hardly of the order needed to predict one from the other in individual cases. The coefficient of correlation between Part II (computation) of the Iowa Every Pupil Test of Basic Skills, Part D, and the Cody High School Diagnostic Arithmetic Test was .638, not significantly higher than the correlation mentioned above. Since the latter test is limited to basic computation, one would expect the correlation to be higher. From this and various qualitative observations, the author makes the following important inference:

Certain pupil behaviors, having roots in the nature of the examination situation, rather than the particular test, operate as limiting factors in the validity of examination scores. These include nervousness, poor timing, hurrying the work in order to finish, with a consequent increase in error, poor work as a result of headaches, etc.
These factors may be temporary or recurrent. These factors operate in all tests. Teachers must be trained to look for such behaviors, and make appropriate adjustments in classification and teaching procedures.

B. The coefficient of reliability of scores on the Cody High School Diagnostic Arithmetic Test, used as a power test, is .92. The diagnostic items checked off on the same papers used to compute this coefficient, had a ratio of agreement of 68.7 percent. The disagreements were most often of the type in which the student did a problem wrong two or more times on one paper, leading to a diagnosis, and only once on the other paper, giving an indication, but not a diagnosis. The objectivity of the diagnoses made by student assistants was checked, and the ratio of agreement was 58.9 percent. This unsatisfactory result indicates the need for more careful training of student assistants. Although the test score, when used as a power test, is reliable, the test should be used to obtain an introductory diagnosis only, to be improved by observing the student as instruction progresses.

C. The statistical summary of the error diagnosis made of five hundred and twenty-nine test papers is consistent with those of other investigators. Within the limitations of human judgment, random errors were separated from systematic errors, and only systematic errors were recorded. These patterns are generally consistent with clinical interviews and subsequent work performed by the student.

D. The error patterns described were studied in pairs, for significant relationships, by the Chi-Square contingency test. Of seventy-eight paired relationships, forty-one were significant. The interrelationships suggest patterns leading to a two category causal theory,
(a) misunderstanding the nature of the tens number system, and (b) general personality factors. This theory is consistent with the work of Fernald and others.

F. Many students were deficient in their mastery of the basic tables of fundamentals, particularly in subtracting from numbers larger than twelve, and in the 7, 8 and 9 facts in multiplication and division with higher numbers.
The pilot study was conducted during the school year 1953 to 1954 and consisted of two experimental phases. The first phase utilized the classroom situation as a means of presenting special practice material designed to aid in the correction of common errors found on the Cody High School Diagnostic Arithmetic Test. The second phase saw the establishment of teaching clinics, each with six students, who were instructed by wholly individualized methods.

1. The classroom teaching experiment:

Special exercises were prepared by the author, based on the common diagnostic items found on the Diagnostic Test. These were presented in the author's classes in Remedial Mathematics, and were supplemented by an arithmetic workbook\(^1\), typical of many available to schools today. These materials were the basis of approximately one third of the class work for the semester. The exercises appear in Appendix B.

It is appropriate, at this point, to describe the classroom teaching methods used to present these exercises. The general procedure was to ask the students to do the first three problems in the exercise, and check answers before proceeding. The purpose was to enable the student to find out if he had been doing the computation correctly. A large

\(^1\) *Mastering Basic Arithmetic*. Chicago: Lyons and Carnahan.
number of students in the class usually made errors in the first three problems. The teacher then demonstrated on the chalkboard both the common wrong procedures, which it was assumed some had been using, and the correct procedures. There were generally some students in the class who understood the correct method, to some extent, and who volunteered to assist in the explanation.

There are many teachers who would object to the procedure of showing a wrong type of solution on the chalkboard, on the grounds that negative methods are a poor approach to teaching. If the subject matter were being presented to fourth or sixth grade students for the first time, the author would agree heartily. The ninth grade students had been exposed to the subject matter many times, had expressed boredom and disgust with it, and in many cases, thought they had an adequate knowledge of it. Many students were quite oblivious of the fact that the procedures they were using were wrong, and attributed their wrong answers to errors in computation, rather than understanding and procedure. The device of showing the incorrect procedure on the chalkboard had two purposes. One was the attention getting value for the student who was surprised to find that his incorrect solution had been duplicated. The other purpose was to facilitate meaningful explanation of the correct solution.

Corrective exercises are no better than the teaching method used to present them. In general, the method of explanation was based on a meaningful approach to the problems of the number system. For example, it is possible to explain multiplication problems involving zero by saying that you place a zero under each zero in the multiplier,
and proceed to the next place. It is also possible to say that when you multiply three hundreds by six tens, the result is twelve thousands and, therefore, the two is put in the thousands place. In view of the vast volume of research which has been developed concerning a meaningful approach to arithmetic, the author assumed the superiority of the latter methods and used them as often as possible in presenting these materials. While the statistical results of the study could not be used to support this contention, many remarks made by students and other qualitative observations made by the author appear to justify the use of these procedures.

The two examples following are excerpted from Appendix B and illustrate this type of deficiency, and the corrective exercises constructed:

1. Diagnostic Check List item number nine: The student adds the problem 4.13 + 8.287 + 19 by placing the 19 after the decimal, or by placing the three in 4.13, the seven in 8.287 and the nine in 19 in the same column. A meaningful discussion of the nature of the tens number system is in order, in which the teacher makes it clear that placing the 19 after the decimal actually changes the character of the number radically by changing it to 19 hundredths. The deficiency in subtraction is psychologically the same. Therefore, practice units of ten addition and ten subtraction problems were constructed. (See Appendix B, pp. 190-193)

2. Diagnostic Check List item number twenty-four: The student does not use zero as a place holder correctly, i.e.
It is clear, upon examination of the wrong solution, that the student mistakenly placed the result of multiplying three hundred by five ones in the tens place instead of the hundreds place. Multiplication by the zero in the tens place of 308, was ignored. Special practices of ten problem units, in which all numbers contained zeros as place holders, were written to aid in this teaching problem. (See Appendix B, pp. 198-199)

In general, the type of special treatments described were written only when such material was not available in regularly used text books. Many aspects of arithmetic with common fractions, for example, were adequately treated in the available text book and workbook, from the standpoint of practice exercises, if not from the standpoint of meaningful explanation.

This description is not intended to give the reader the impression that the kinds of problems described do not appear in standard classroom materials. The difficulty lay in the fact that the problems were mixed into other exercises, generally labeled review. This type of text organization tends to mask the deficiency. A student with a particular deficiency may still have an achievement score of seventy to eighty percent on a classroom exercise. Since the teacher may consider the score to be "passing", the deficiency may never be dealt with.

The exercises in Appendix B were based on the most common deficiencies. Many behavior patterns, properly classified as deficiencies,
but much rarer in incidence, were not treated in this manner. In this classroom teaching experiment, these items were untouched, or were dealt with only incidentally. For example, an occasional student would do the following solution to a subtraction problem:

\[
\begin{align*}
7,891 \\
- 97,075 \\
\hline
- 825 \\
97,075
\end{align*}
\]

or 87,075

Note that the student's conception of borrowing was to reduce each successive place in the minuend by one more number.

The remaining two thirds of the semester teaching program for Remedial Mathematics classes was devoted to other phases of elementary mathematics. These included many of the social applications of arithmetic, such as problems of spending money wisely and installment buying, including such phases as calculating payments and obligations when unpaid for articles are repossessed by the seller. Other lessons included practice in estimating answers to arithmetic problems by rounding off numbers and using mental arithmetic. These aspects of the teaching program are entitled to fuller consideration. However, scientific evaluation of these activities was not attempted. The author can add the qualitative observation that some of these lessons were helpful in generating student interest.

To aid in evaluation of the classroom experiment, the Cody High School Diagnostic Arithmetic Test was given to the Remedial Arithmetic Classes as a final examination when the semester ended. Comparisons could not be made for some students, who had not attended both examinations. However, a sample of fifty-five students, who had completed both examinations, was obtained.
In order to establish a control, the same test was given to two classes in General Mathematics II at the beginning of the following semester. The students in these classes had come from the classes of six different teachers, thus eliminating the possibility that a differential between the experimental and control groups would be attributed to the teacher factor. A control sample of forty-eight was obtained.

No attempt was made to equate the control and experimental groups for such factors as intelligence, age, sex, etc. Such an omission has some shortcomings from a scientific point of view. It is hardly to be expected, however, that the experimental group as a whole would be found superior to the control group in factors of major importance, considering the nature of the selection. The validity of the conclusions can hardly be questioned on these grounds.

Table 13 lists the comparison of the mean scores before and after testing for both groups. The critical ratios show highly significant results for the experimental group and no significant gain for the control group.

The question now arises as to which part of the instructional program was responsible for the gain. The students in the control group also had computation drill. The text for the General Mathematics course was "Every Day General Mathematics", by Betz. Two chapters of this book are devoted to various phases of computation drill. The teacher has the option of omitting these chapters if desired. The author's professional conferences with the six teachers concerned with the control group led to the knowledge that they all felt that this computation drill was an important part of the course of study. It is
Table 13
Summary of Cody High School Diagnostic Arithmetic Test
Scores Before and After Special Class Instruction

<table>
<thead>
<tr>
<th></th>
<th>Mean Score Sept. 1953</th>
<th>Mean Score Jan. 1954</th>
<th>Increase</th>
<th>Critical Ratio</th>
</tr>
</thead>
<tbody>
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<td>Overall Student Body</td>
<td>64.2</td>
<td>x*</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>General Mathematics</td>
<td>69.7</td>
<td>70.9</td>
<td>1.2</td>
<td>.44</td>
</tr>
<tr>
<td>Remedial Classes</td>
<td>42.7</td>
<td>56.6</td>
<td>13.9</td>
<td>4.06</td>
</tr>
</tbody>
</table>

* No data available

n = sample size
s = sample standard deviation

Note: All variances were tested for compatibility with that of the Overall Student Body by means of Snedecor's F test. No variance yielded a significant ratio.
highly likely that their students did most of the drill work in the book. The evaluation indicates that this drill work, and much of the drill work which went on in the Remedial Mathematics classes of the author was largely a waste of time. The gains shown were probably due to the special corrective work and some parts of the drill program, specifically related to it.

Two possible objections may be made to this conclusion. One would be the argument that the control group mean was high enough to indicate satisfactory mastery of computation, and significant gains would not only be less likely, but hardly necessary. This notion can be tested by examining the range of scores in the control group, since the statement is correct regarding some of the students in the sample. The range of scores in the initial test was from thirty-eight to eighty-five and in the final test from forty-one to eighty-seven. Examination of the individual scores indicates that this argument will not stand close scrutiny. The other objection concerns the need for a maintenance program in basic skills. This is a controversial argument which the author prefers to leave open.

As a further means of evaluation for the experimental group, the Iowa Every Pupil Tests of Basic Skill, Part D (arithmetic), Form P, was administered in the Remedial Mathematics Classes near the end of the semester. A sample of forty students with scores at the beginning and end of the semester was obtained. The mean growth for the group was .56 months. Nine individuals in this sample showed an improvement of one year or more. Since the mean gain for an average group in one semester is expected to be five months, a mean gain of .56 months for
a "below average" group can be argued to have some significance. One student showed a gain of three years, but his initial score was probably invalid.

In addition to objective measurements of growth in arithmetic achievement, some students evidenced greatly increased morale, by expressing greater liking for the subject, and by indicating that they really understood processes for the first time.

2. The first instructional clinics:

At several conferences with Dr. C. L. Thiele, Supervisor of Mathematics for the Detroit Public Schools, Mr. Herman Schumacher, Principal of Cody High School and Mr. Raymond Agren, Department of Mathematics at Cody High School, the data gathered during the classroom phase of the study were examined, and instructional clinics for the semester beginning January, 1954, were approved. The author was excused from extra duties, so that two periods a day could be devoted to this effort. The plan was to work with groups of five or six students, and to expand the size of these groups if feasible. The author's teaching schedule included two classes in Remedial Mathematics II and one of Remedial Mathematics I. The students for the Mathematics Clinics were chosen from these classes, on a random basis. Since it was necessary to make special scheduling arrangements, and often necessary for the students to come to school earlier or remain later, attendance was placed on a voluntary basis. This places some limitations on the conclusions which can be drawn from the study, but these objections were eliminated in the final phase of the study. (See Chapter VI) Students were told that they could be excused from their regular classes while
attending the clinic, if they wished. In cases where this meant going home earlier, they generally exercised this option. If the choice meant attending a studyhall instead of the class, the students generally preferred to attend the class.

Since these students had generally shown some growth during the previous semester, the diagnosis used for instruction was derived from the second Cody High School Diagnostic Arithmetic Test, taken as a final examination the term before. A small number of the students had come from other classes. In these cases the initial test given in September, 1953, was satisfactory.

The instructional procedure was to work out with each student a detailed diagnosis of the reasons for his errors, and prescribe corrective measures. If the errors were due to gaps in understanding, instruction followed by a practice exercise was the method used. In this situation, it did not matter whether the student's deficiency was a common or an unusual one. Corrective exercises which did not exist could be written as needed. This was often unnecessary. While the student might exhibit an unusual deficiency, say in subtraction, and might require an unusual explanation to clear up the misunderstanding, the vehicle for practice could well be an ordinary subtraction drill from an ordinary workbook.

No set number of lessons was prescribed for any student. The number of classes attended for the cases completed ranged from five days to the entire semester. The mean attendance was 7.1 weeks. The other items in the instructional procedure should be noted at this point. If the student's deficiency was due to inadequate mastery of one or more of the tables of fundamentals, time was taken to conduct practice with
flash cards and written techniques. The procedure used is that described by Fernald. This procedure is advantageous in that no time is wasted drilling the student on facts over which he already has adequate control. This procedure was also excellent for morale. It was much better to show the student that there were six items in the multiplication table that he did not know, than to tell him that he did not know the multiplication table. This phase of instruction was not dealt with at all in the classroom teaching part of the study.

By the end of the semester, the author had worked with twenty-five cases, of which twenty-two could be considered completed, in the sense that the students had improved as much as was desired or possible. The other three included two cases of extreme truancy and one case in which no reasonable measures of achievement could be obtained. In all three of these cases, the author was able to establish friendly personal relationships with the student, but was unable to achieve the kind of working relationship in which learning takes place.

As the semester's work went on, certain principles, consistent with the work of Fernald with younger children, began to emerge. It is interesting to note that cases are sometimes recorded in which certain types of causal factors exist, uncomplicated by the presence of other factors. This is generally not the case, as instructional and personality factors are usually intermingled in the one individual. Several such cases are described to illustrate some of the principles derived:

a. Gaps in Instruction Responsible for Deficiencies:

If the case is not complicated by emotional or other personal factors, it is generally not necessary to examine the causes of the deficiency. It is enough to treat the symptoms by teaching the student what he does not understand. Consider the case of Lela W.:

Lela was a rather mature girl for her age. She was poised, reserved, but not to the point where one would consider her excessively shy and withdrawn. She was neat, well-groomed, and considered a good citizen by all of her teachers. In our contacts with her outside of class, she seemed to have her normal quota of friendships. In short, she appeared as a normal adolescent, with no apparent emotional problems which would complicate learning. Her initial score on the diagnostic test was fifty-three and she exhibited the following deficiencies:

a. Copies own numbers incorrectly.
b. Omits decimal point in addition.
c. Misunderstands nature of decimal point in problems like 7.1 plus 3.15 plus 16.
d. Fails to reduce fractional answers to lowest terms.
   (Lela told the teacher that she would know how to do this, but had misunderstood the directions on the test.)
e. In multiplication, does not use zero correctly with numbers ending in one or more zeros.
f. In dividing fractions, inverts the dividend fraction instead of the divisor.
g. Does not borrow correctly when subtracting denominate numbers.

Lela showed a very quick grasp of the principles involved in each of her deficiencies. It was only necessary to explain the items once and to assign a short practice of ten problems for each, which she immediately completed without error. Five days after instruction began, she took another form of the Diagnostic Arithmetic Test and scored eighty-one points out of eighty-seven. Examination of her paper reveals the other points can be safely attributed to random mistakes.

b. Cases Not Due to Low Achievement:

In this situation the deficiency is generally due to personality factors and has little or nothing to do with a lack of understanding of arithmetic processes. In such cases the approach is two-fold. The instructor must, in a calm, objective manner, show the student how his behavior is the cause of his low achievement and to utilize whatever
learning deficiencies may exist, however few, as a means of exhibiting
capacity and a willingness to teach. In such cases the pupil-teacher
relationship is the primary factor in the improvement. Consider the
case of Leonard L:

Leonard was a slender boy of fifteen, who presented to the casual
observer what could be described as a sloppy appearance. His hair
was unkempt, his shirttail often out. He had begun his high
school career by being involved in several mishaps which led to
disciplinary action. Some of his teachers regarded him as rather
difficult to deal with. At the same time the boy exhibited a
rather friendly attitude, and on occasion showed a genuine desire
to please, coupled with many likeable qualities. Past clinical
experience has indicated to the profession that a sloppy appearance
is often symptomatic of a conflict with the problems of growing
up. An examination of Leonard's diagnostic test (score forty)
tends to verify such a diagnosis. Consider the items checked
off:

a. Misunderstood nature of decimal point in problems like
   7.1 plus 3.15 plus 16.
b. Does not use decimal rule correctly in division.
c. Mixes up \( \frac{7}{56} \), \( \frac{7}{63} \), \( \frac{6}{54} \).
d. Fails to reduce fractions.
e. Changes denominator incorrectly.
f. Copies answer incorrectly.
g. Does not invert divisor in dividing fractions.
h. We note, in addition, that the student clerk wrote the
   following comment on Leonard's paper: "What a mess!"

Discussion with Leonard revealed that of the above items, only the
first two were due to misunderstanding. Other errors showed up on
his paper which could not be held to any definite pattern, and
could be explained only on the basis of sloppiness. Leonard is
left-handed, a partial explanation of the slanted columns of
numbers on his papers. We helped him devise a method of using a
3 by 5 white card to help him keep his numbers in line. After
two days, he discarded this crutch, and wrote passably neat work
without it. After seven days, he took the test again and scored
eighty-one points out of eighty-seven. We wish to note that
during the period of instruction, he asked the instructor the
following question: "Mr. Bernstein, do you come in extra early to
give us kids this special help?" Since we sensed what was on his
mind, we lied and answered his question in the affirmative. "Well,"
he said, "if you can do that for us, I guess we ought to put out
for you."

It would be an illusion to believe that this boy learned much
arithmetic in the seven days which he spent in the Mathematics Clinic.
It seems more accurate to believe that he was aided in resolving some of his hostilities toward the adult world. He was also aided in reappraising himself, since he genuinely believed that he was very deficient in arithmetic. His subsequent behavior in class and in school generally, indicated that in the struggle for maturity, maturity was winning out. We believe that this, too, is teaching.

c. Emotional Insecurity or Other Emotional Problems, Not Necessarily Influencing the Method of Instruction - To illustrate this situation, let us consider the case of Jean G.:

This girl had had a very unstable home situation for three years previous to her enrollment at Cody High School. Her parents had been divorced and she had been moved around from one situation to another, attending several different schools over a period of three years. The semester she enrolled in Cody High School was the first stable period in her whole life subsequent to the divorce. She was living with an aunt, who apparently showed genuine interest in the child. Her counselor reported to the instructor that Jean was extremely insecure, and had great fears about her ability. However, she was very anxious to succeed and her insecurity presented no obstacle to her teachers in either a classroom or a clinical situation. The procedure, therefore, was to analyze her weaknesses and faults in arithmetic and deal with these without discussing Jean's emotional problems with her. The relationship was primarily instructional, and assumed therapeutic aspects only in that the situation served to build confidence and self-respect. It is unnecessary to list her arithmetic deficiencies in great detail to illustrate this principle. These are noted in the case summary in Appendix C. It should be noted that she scored thirty points out of eighty-seven on the diagnostic test in September, 1953, and scored sixty-one on another form at the end of the semester in January, 1954. This improvement was achieved without clinical instruction in one of the Remedial Mathematics classes conducted in the study. Examination of her second test revealed that deficiencies still existed and clinical instruction was begun at this point. It's worth noting that unlike the previous cases used as illustrations, Jean also had deficiencies in her basic tables of subtraction, multiplication, and division. The technique used was similar to that described by Fernald. The Detroit Public School Test on the basic facts was given to Jean and the time of completion noted. The papers were then examined for patterns of wrong answers. For example, Jean made nineteen errors or omissions in the multiplication table. Analysis revealed that ten of these errors involved multiplying by zero (2 X 0 = 2, etc.) Further analysis revealed
that the other errors were paired, i.e., 7 x 8 and 8 x 7, or fitted into the table of multiplying by nine. This kind of analysis was conducted with Jean watching so that the instructor was able to convince her that she only had six facts to master instead of nineteen in order to gain complete control over the table. We were further able to show her that the facts that she had wrong on the division table were generally the same ones, i.e., 7 /56. This process of narrowing down the area of study was a boost to the girl's morale, a factor noted in many cases where the problem of the mastery of the tables was involved.

(It had been the writer's experience, verified by other workers in this field, that the bulk of deficiencies in multiplication and division were in the tables of 7, 8, and 9, particularly when multiplied by high numbers. Deficiencies in the subtraction tables generally involve subtraction from numbers higher than 12.)

Two programs were used to help Jean with the problem - one was the use of flash cards, with a student assistant giving her the drill. The other was a method of writing out the successive facts in the tables of 7, 8, and 9. Jean was also taught to speed up subtraction by the process of bridging tens. She showed very rapid improvement on the tables and was able to show reasonable mastery of them two weeks after this instruction was begun. Seven weeks after she entered clinical instruction, Jean was retested and showed a score of seventy-four points, losing only two points on what could be described as errors of misunderstanding. The remaining errors on her paper could be classified as random.

The initial emotional factors which helped in the situation were the excellent relationship which existed not only between Jean and her instructor, but also between Jean and her counselor. Her over-all adjustment to the school was excellent. She found a further means of bolstering her self-confidence by becoming a leading competitor on the girls' swimming team. The gains she showed in arithmetic achievement must certainly be given some credit for her over-all improvement. Follow up information revealed that Jean had moved to Walled Lake, Michigan, was in attendance at Walled Lake High School, and had achieved a grade of B in tenth grade commercial arithmetic there. She has taken the trouble, on two occasions, to visit Cody High School and report her progress.
d. Low Achievement as a product of the test situation, rather than a lack of knowledge:

Numerous students have reported to the writer that they become nervous in test situations. However, some who do are still able to perform satisfactorily on the test. The clinical instructor must learn to be sensitive to those cases which have mastery of the subject matter in question, but are unable to display this mastery under the pressure of a test situation. Consider the case of Jack S.:

Jack's examination, as of January, 1954, showed a score of fifty-seven points. A diagnosis revealed only two items of misunderstanding:

1. He tended to omit zero as a place holder in long division.
2. He misunderstood the decimal rule in division.

One wonders why Jack should have such a score with only two misunderstandings. His test paper revealed several items where points were lost by omissions, or by such carelessness as copying his own handwriting incorrectly. Subsequent work with Jack revealed that, although he was slow spoken and slow moving, he could do accurate work in a daily class work situation when not rushed. However, if not pressured somewhat, he would daily and take too long to finish the work. In addition to the two items above, he was found to have weaknesses in the multiplication and division tables. After practice, he showed improvement on these, but when retested, still took 4 or \(4\frac{1}{2}\) minutes to complete them. Since he could respond to flash cards automatically and correctly, we concluded that the poor time was a product of the test situation. When Jack took the arithmetic test after ten weeks of work, he scored sixty-eight. This time his paper revealed no patterns, and we concluded that the errors were random in character. A loss of nineteen points due to random errors is considerable and we encouraged Jack to practice a little further and try the test again. He later scored sixty-six points on another form, with a pattern of random errors similar to the previous test. Jack's inability to respond to the pressures of the examination must be deduced as the principle limiting factor in his achievement.

Many students have come to the author's attention who are greatly limited in school achievement for this reason alone. Indeed, some students will be classified as remedial problems, who are not remedial at all. This factor is also a major source of error in educational studies, in
which test results are correlated. Some students will testify that they
become nervous only on mathematics tests. A smaller number will testify
that they become nervous in all test situations. Jack was the latter
type. The former type can obtain help from clinical instruction. The
prognosis for a student like Jack for future examinations is poor. We
believe this is a major unsolved problem in education.

e. Some students are unable to bridge the gap from concrete to some
phases of abstract arithmetic. Consider the case of Betty S.:

The only aspect of this case which is discussed at this point is the
problem of understanding and using decimal numbers. For further
details see Appendix C. One of the deficiencies that Betty displayed
at the beginning of clinical instruction was a failure to understand
problems like 4.16 plus 6.23 plus 17. When a dollar sign was placed
on the numbers in such problems, Betty was able to comprehend the
nature of the problem and add it correctly. However, when we used
the language of arithmetic and talked about ones, tens, and hundreds
to the left of the decimal point and tenths, hundredths, thousandths,
etc. to the right of the decimal point, she had great difficulty in
understanding the explanation. She practiced this kind of exercise
many times in the course of the semester, (see Appendix C.)
Examination of her exercises reveals that she could master the
problem if the numbers did not exceed two digits on either side of
the decimal point. She was unable, even with considerable help,
to comprehend a number like 7,325 whether written with or without
a decimal point. We did not believe, at the end of the semester,
that her mastery of this situation had gone beyond this point. She
had similar difficulty in understanding the use of decimal points
in multiplication and division, although she could multiply satis-
factorily with whole numbers.

Cases of this type were rare in both this and the later phase of the
study. Only three cases out of both phases displayed this kind of re-
tardation. Fernald has argued that this inability to bridge the gap from
the concrete to the abstract is a symptom of low intelligence. While,
for practical purposes, this may be so, Betty's case may leave open the
possibility that the retardation is emotional in origin and a product of
fear and insecurity. She displayed many of the symptom patterns of a
disturbed child. The teaching efforts in this situation cannot be called a complete failure for two reasons:

1. Betty did improve in the handling of numbers in the more concrete situation of figuring with money, certainly a matter of importance.

2. The informal character of the clinical teaching situation appeared to relax some of Betty's fears.

There was a definite impression by the end of the semester, that the instructor was the first adult with whom Betty felt she could converse on equal, friendly terms. At the beginning of the semester she behaved like a very frightened person.

f. Cases where specific emotional problems may be considered a direct cause of arithmetic deficiency:

It was the author's experience that a direct relationship between specific personal factors and achievement in arithmetic is a rare thing. While personal factors are generally present, they are likely to be involved in any phase of low achievement in school, or are often products of the deficiency in arithmetic rather than causes of it. Previously discussed cases are ample illustrations. Consider now the case of Marlene P.:

Marlene's score had shown an improvement of seventeen points from twenty-seven to forty-four, on the basis of the previous term's class work. First examination of her paper revealed what appeared to be five routine matters for instruction, including such things as place holders in multiplication. (See Appendix C.) Nervousness and slowness appeared also as factors. Several weeks of instruction on a single item revealed that we appeared to be dealing with a pathological problem. Marlene is intelligent, and understood what was being shown her, but could not avoid a high percentage of what appeared to be careless errors, in spite of every effort, every warning, and encouragement.

We abandoned the instructional approach, and tried to examine the problem in terms of her basic emotional problems. On the basis of several interviews, including a long private interview with Marlene, in which she seemed to feel that our analysis of the situation was correct, we wish to hypothesize the following explanation:
Marlene was the youngest of several children, the others having finished school and entered adult life with reasonable success, including a brother who successfully finished engineering school. The parents teased her about being the baby, and she said she liked being the baby. The father wanted her to be good at mathematics, and both parents wanted her to be a nurse. Marlene was well aware that algebra and chemistry were essential to a nursing career. When asked whether she wanted to be a nurse, she responded in the affirmative, but seemed unsure of herself.

Marlene indulged in a good deal of bizarre behavior, including being outspoken at the wrong time (in class) with humorous remarks. She had the knack of imitating the mannerisms of Red Skelton, and her classmates found her quite funny (as indeed she was). When we discussed this, it turned out that her imitation of Skelton was quite unconscious, indeed a real facet of her personality.

On the basis of this and other material not described here, we concluded that the deficiency in arithmetic had something to do with unconscious hostility toward the choice of a profession for her, in which she had not been consulted. (Her parents also had discussed being an airline hostess - Marlene has pretty features, but is quite obese.) In addition, she was going through a pronounced struggle for maturity, in which the advantages of remaining immature were very great. We pointed out to her that it would be difficult for her to give her bizarre behavior up, because she enjoyed its advantages. She accepted most of my interpretations.

For the next few weeks, instruction was put on a permissive basis. She did what she wished, including nothing if she so desired. We made it a practice not to shower attention on the basis of mistakes, and sent Marlene back to class after seven weeks without a retest. She took the retest in class as a semi-final examination and showed a score of sixty-one, another seventeen point improvement. It is difficult to assess what this means. Her class behavior showed some improvement, but we must be cautious in gauging whether any real changes have taken place.

The author is well aware that much of the diagnosis made in Marlene's is speculative in character. It represents the best conclusions which could be reached in terms of our training and the facts assembled in the situation. We are humbly aware that other case workers might have found the diagnosis somewhat different. The relative failure in dealing with Marlene's problems is consistent with the diagnosis and the results of the case, but cannot be considered as verification.
g. Some general qualitative observations:

It is not surprising that many of the students who attended the Mathematics Clinic showed great gains in other areas than achievement in arithmetic. It can be safely concluded that students showed great gains in self-confidence, poise in test situations, attitude toward the subject, and other factors properly classified as morale. In the cases of Leonard L., previously described, Will C., below, and others not described, there is a great probability that the over-all adjustment to school was materially aided. While such gains are hardly enough to effect the retention of a potential school drop-out, without other positive factors operating, they can certainly serve as a major aid. There are a small number of children who cannot be aided, even in this relatively sheltered instructional situation, an observation reported by other workers in the field of remedial instruction.

Student teachers were used unsuccessfully in this situation. Some of the students who reported to the Mathematics Clinic were assigned to student assistants, designated as instructors, but the system soon broke down and the majority of instructional problems were eventually returned to the teacher in charge. The causes of this failure were examined and the following principles were formulated for future reference.

1. The student assistants, although honor students and future teachers, could not be considered expert in arithmetic. At first glance, this seems strange, since all of the students had achieved excellent grades in mathematics. They could, indeed, do arithmetic problems with a high degree of skill. It became apparent when they tried to act as tutors that they were teaching the mechanics of an operation without regard to its meaningful aspects. In retrospect, this is not surprising,
since these fine students were probably not taught by meaningful methods. Thus, any attempt to utilize such personnel for instruction must be proceeded by a short period of training. This was attempted later.

2. Student assistants cannot be expected to be sensitive to peculiarities of the relationship between pupil and teacher.

b. Consider the case of Will C.: We do not wish to discuss this case in its entirety, but simply to explain what happened, since a student coach was involved. Complete details are in Appendix C. When Will entered the mathematics clinic, he was "taken over" by Carol B., one of the student assistants. It should be noted that Carol is a very large girl and towered over Will by five or six inches. Will had normal height for his age. Carol was quite eager to give him a lot of assistance, and indeed did a rather excellent diagnosis of his difficulties, taught him how to read numbers meaningfully and, in general, organized and supervised a series of lessons for him. At several points in the instruction she complained that he was not trying. Carol was not conscious of the fact that she was actually dominating him. It appeared to the teacher that some of Will's failure to try was a polite way of showing his resentment of the situation. We tried to discuss the matter privately with Carol and tell her that Will was a boy who appeared to resent teacher pressure and preferred to be left alone. Carol did not accept this interpretation of the situation, and insisted that she was merely trying to help him, not dominate him. She insisted that she had not displayed impatience at his slowness, when it was obvious to the instructor and the other student assistants that she had.

The case had a very interesting conclusion in that Will was returned to a large class before he was thought ready for it, due to an accident in the school schedule. He proceeded to show a greatly improved adjustment to the class and began to do excellent work. He took the arithmetic test as a semi-final exam and scored seventy-one, a fourteen point gain over the previous score. Since the class work was not directly related to this phase of instruction, we must attribute much of the gain to the clinical instruction. It was evident that Will was greatly relieved to be rid of Carol's domination, and we are inclined to interpret his improvement as a delayed reaction which had not been in evidence while he was in the clinical group.

3. The clinical instructor should do the initial diagnosis of a remedial case before assigning a student assistant. The lesson learned is that many students can be helped by student instructors, but that more
difficult cases must be reserved for a professionally trained teacher. In general, students who score above fifty points on the diagnostic test are a good risk for assignment to student instructors and students who score below forty points are not.

Concrete aids are important in teaching arithmetic but can often be omitted in the instruction of ninth grade children. Much of the instruction was carried on at the abstract level and the students experienced no difficulty. It was necessary sometimes to make sketches of pies or to use the inch and fractional markings on a ruler to illustrate certain principles of common fractions. This was the extent of the use of concrete aids in this situation. It is possible that in rare cases, such as that of Betty S., described above, that the use of a pocket chart for illustrating the number system would have been helpful. In most instances this was not needed.

i. Evaluation of the clinical teaching phase of the Pilot Study:

Two items were summarized with regard to the students who attended the Mathematics Clinic during this phase of the study. These were the gain in the Cody High School Diagnostic Test score and a prognosis of success in future work. For purposes of this evaluation, three out of twenty-five cases were not considered. Two cases, continually truant, had attended so little, that it was impossible to consider that a learning relationship had been established. A third case, that of Tom E., concerned a boy so emotionally unstable that it was impossible to get any kind of reasonable measure of his achievement.

A control over the test score data was established by checking the author's classes in Remedial Mathematics II for students who had completed a year of instruction, but had not had clinical instruction.
Twenty-three cases were tabulated, for which complete test data was available. It should be noted that the members of this control group were part of the experimental group described in the classroom teaching part of the experiment. Table 14 repeats the data of Table 13, with the additional information acquired in the second phase of the study added.

Examination of the critical ratios listed in Table 14 reveals that the gain in the mean Diagnostic Test score is significant at the .001 level of significance, and indicates the most important gain in the entire table. The increment is significantly greater than that of the control group, and the gain for the entire year is approximately twice that of the control group.

In addition to the numerical gains described, the author was concerned with the possibility that some of the gains described might be temporary in character. An attempt was made to estimate the possibility that the gains were lasting. Six cases were listed, for whom the probability of lasting results were considered good, nine for whom it was considered as fair, and seven for whom it was considered doubtful. More objective evaluation of this problem must await the passage of time.

Some criticisms:

It was not originally intended that the data gathered in this pilot study were to be reported in this thesis. A study ended at this point would have obvious inadequacies, but it was felt that the lessons learned, and the information gained were too valuable not to be recorded. The most serious question which must be raised is the problem of where and when the gains took place. Since the students attended both the large classes and the clinic groups under the supervision of the same teacher, it is difficult to tell whether the gains took place in one situation or
Table 14

Summary of Cody High School Diagnostic Arithmetic Test Scores Before and After Remedial Instruction

<table>
<thead>
<tr>
<th></th>
<th>Mean Score</th>
<th>Mean Score</th>
<th>Increases</th>
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<tr>
<td></td>
<td>Sept. '53</td>
<td>Jan. '54</td>
<td>June '54</td>
<td>1st sem.</td>
</tr>
<tr>
<td>Overall Student Body</td>
<td>64.2</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>General Mathematics</td>
<td>69.7</td>
<td>70.9</td>
<td>x</td>
<td>1.2</td>
</tr>
<tr>
<td>Remedial Classes</td>
<td>42.7</td>
<td>56.6</td>
<td>x</td>
<td>13.9</td>
</tr>
<tr>
<td>Remedial Classes*</td>
<td>41.9</td>
<td>55.1</td>
<td>58.1</td>
<td>13.2</td>
</tr>
<tr>
<td>Clinical Group</td>
<td>33.3</td>
<td>51.0</td>
<td>65.5</td>
<td>17.7</td>
</tr>
<tr>
<td>Clinical Group**</td>
<td>x</td>
<td>50.1</td>
<td>67.6</td>
<td>17.4</td>
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</tbody>
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Mean Score: Mean Score: Mean Score: Increases

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<td>Clinical Group**</td>
<td>x</td>
<td>50.1</td>
<td>67.6</td>
<td>17.4</td>
</tr>
</tbody>
</table>

x = no data or incomplete data available
n = sample size
s = sample standard deviation
z = critical ratio using Student's t test

* Excerpted from previous group as control over clinical group.
** Includes the eleven cases in the previous group.

Underlined increments are significant at the .01 level, using Student's t test for the difference of sample means. All sample variances were tested for compatibility with that of the Overall Student Body by means of Snedecor's F test. No variance yielded a significant ratio.
the other. It must also be noted that a large number of the twenty-two cases had worked with the author for six to eight months before entering clinical instruction. In most of these cases, a close, friendly relationship had been developed. In several instances, the author had made contacts with the students concerning problems of school adjustment not specifically concerned with the subject matter of arithmetic. It must be asked, therefore, whether the gains were due in greater measure to the special teaching situation which was established, or to the pupil teacher relationships considered independently of the former. Both factors are important, but the author believes that the specialized teaching situation was much more important in this study. The final phase of the study, described in Chapter VI, was designed, in part, to aid in clarifying this question.

SUMMARY

Two teaching experiments were conducted during the school year 1953 to 1954, one a classroom teaching experiment based on common deficiencies found in the Diagnostic Test survey, and the other a clinical teaching experiment in which completely individualized instruction was carried on in groups of five or six students. The students instructed in both experiments showed significant gains in score on the Cody High School Diagnostic Test, while the students in the control group showed no significant gain. Using some of the students taught in the first (classroom) experiment as a control, the evidence shows that the clinical teaching experiment was more effective. Individualized teaching, after a semester of specialized classroom teaching, was about twice as effective as a year of classroom teaching alone.
It should not be concluded that individualized instruction must be conducted in small groups exclusively. Successful experiments in individualizing instruction in large classes were reported in Chapter III.

Important gains in self-confidence and other morale factors were observed. It is probable that the opportunity for greater informality in the clinical teaching situation with small groups, and the concurrent opportunity for friendly human relations makes the clinical situation vastly superior to the large class situation for dealing with the personality problems of individuals.

The attempt to utilize student assistants as clinical teachers met with limited success, and was referred to the next phase of the study.

The success of the instructional methods and of the diagnostic approach to the problem verifies, to some degree, the theories of causation proposed by Fernald, and expounded at greater length by the author in Chapters III and IV.

A major problem in the area of personality factors in low achievement is the behavior of certain pupils in examination situations. Teachers must learn to be sensitive to the factor of nervousness during examinations, so that they can distinguish spuriously low achievement from genuinely low achievement.

Seven cases were reported to illustrate both common and rare patterns found in clinical teaching. The patterns include the student who:

a. exhibits gaps in instruction, with no complicating personality factors.

b. has spuriously low achievement, symptomatic of personality conflicts.
c. has complex emotional problems which have a bearing on achievement, but no bearing on the teaching methods used.

d. gets too nervous on examinations to achieve up to his true knowledge of the subject.

e. is unable to bridge the gap from more concrete to more abstract number conceptions.

f. has subconscious conflicts which lead to low achievement.

g. has feelings of resentment toward a student teacher which retard his improvement.
Chapter VI
A LARGE SCALE CLINICAL STUDY

1. Arrangements for a larger clinical study:

The success of the pilot study led to the approval of the following phase of the work. The pilot study could not, of itself, have stood the test of careful scientific evaluation. The difficulty lay in the fact that the cases studied were those of students who had experienced several different learning situations, all related to the area under study. Most of them had attended Remedial Mathematics I during the classroom phase of the study, and were attending traditional size classes in Remedial Mathematics II during the clinical phase. The gains demonstrated in the clinical phase were an increment added to the first phase. In some cases, the increment could be questioned, as it was not clear whether the gains were made in the Mathematics Clinic or in the class in Remedial Mathematics II. In addition, all of the learning situations described were taught by the author, so that the factor of the pupil teacher relationship could be inferred to be much stronger than might be expected in a few weeks of clinical instruction alone.

With these considerations in mind, the author was assigned no large classes in Remedial Mathematics for the semester starting September, 1954. A schedule was arranged during which four periods daily were to be devoted to Mathematics Clinics, taught by the author, with six pupils in each clinic. The students selected for clinical instruction were
limited to entering ninth grade students in Remedial Mathematics and General Mathematics. The criterion used, at first, was a score on the Iowa Every Pupil Tests of Basic Skills, Part D, of 6.2 or less.

Later, other criteria, such as a score of less than 50 in the Diagnostic Test, or the request of the classroom teacher to examine a case of potential failure, were used. As in the previous phase of the study, the time of instruction was kept flexible. Students attended the Mathematics Clinic as little as one day (such cases usually had no deficiency on the Diagnostic Test, but a low score on the Iowa Every Pupil Test of Basic Skills), or as much as the entire semester. When a student had completed the planned instruction, he was returned to his originally scheduled class and another student was sent to the clinic. Students were told that they were not required to take two periods of mathematics a day, and could be excused from their large class instruction while attending the Mathematics Clinic. However, those who preferred to attend both classes were permitted to do so. Very few of the students exercised this option, but there appeared to be little difference in the results obtained either way. In general, the contact of the student with remedial instruction of a clinical character and the contact, in terms of human relations with the instructor, were limited to the period of attendance at the Mathematics Clinic. The results obtained can thus be fairly attributed to the special instructional situation. All together, fifty-nine cases were considered to be completed by the end of the semester. Ten cases were still listed as in process. It was the intention to program those cases, listed as still in process, for the beginning of the following semester. However, it was considered desirable to close the research study at this point, since little new
information was likely to be produced. The Mathematics Clinic became a permanent feature of the program of instruction at Cody High School. Two additional teachers took part in its operation and expansion during the semester following the close of the research part of the study.

A careful attendance record was maintained for each case, and the number of actual lessons attended was recorded. The mean value for attendance for the fifty-nine cases was 17.2 lessons. This figure is somewhat misleading, since one would judge that the 'average' student could be instructed in about three and one-half weeks. The fact is that at least one week must be added to this figure. This stems from the problem of attendance, which is often irregular with such children, and from the difficulty of arranging schedules. It was estimated, before the study entered this phase, that from seventy-five to eighty cases could be completed by the end of the semester. Simple arithmetic, based on five weeks per student, and six students at a time, would lead to such an expectation. The discrepancy between the expectation and the actual result of fifty-nine cases must be explained by the difficulties of the schedule. However, from the standpoint of evaluation, the sample size is satisfactory.

2. Some qualitative observations:

Before considering the overall statistical evaluations which were made, a few general impressions, comparing this part of the study to the pilot study, are noted. In general, the clinical principles observed in the pilot study were verified. The bulk of the cases were handled in a manner very similar to that described for the cases of Lela W. and Jean G. (See Chapter V, pp. 110-112) The major emotional problems encountered were lack of confidence and nervousness on tests. Another problem which
was more clearly understood as a result of dealing with many cases, was the problem of timing on tests. Some students made many random errors on arithmetic tests because of a tendency to hurry. Such a student is often relaxed in the examination situation, but is overconfident and finishes well ahead of time. Those who are nervous and lacking in confidence, usually do not finish, and their "hurrying" actually slows them down. The instructor must take pains to aid the former in slowing his pace for the sake of accuracy, and to help the latter to relax.

A fairly good rule for retesting was evolved. It had been the practice in the pilot study to retest the student after mastery of the items in the diagnosis. Some students, on the retest, would exhibit some of the behaviors in the diagnosis, although it was generally noted that there was an improvement on a majority of such items. This was more often true with a student with eight or more diagnostic items. In such a case, the procedure followed was reinstruction in the residual items, followed by a second retest. At this time, further improvement was generally noted. During the pilot study, the attempt was made to carry this process even further. The experience of this phase shows that this is generally a fruitless undertaking. A third and fourth retest will generally show little or no improvement, even some regression. This is often due to the fact that nervousness in the examination situation is the limiting factor.

A very important observation is the fact that some students could not meet the requirements of a test limited to a specific period of time. These students could show very satisfactory mastery of the arithmetic processes if they were told to take as much time as they wished to complete the test. The time they actually used never exceeded eighty
minutes, and generally was about fifty to fifty-five minutes for the forty minute test.

The cautious scientist will ask, at this point, "Doesn't a slow rate of work also indicate lack of mastery?" Indeed it does. The problem of distinguishing between the student who is slow from lack of mastery and the student who really has control over the subject matter, but is the victim of nervousness, is a difficult one, especially since the proposition is seldom one of an either/or distinction. Both factors appear at once in the same individual. In this situation, however, steps have been taken to improve mastery and control. After everything possible has been done, the teacher must make a judgment, based on previous knowledge concerning the student, regarding the importance of imposing an examination time limit upon that student. This, of course, is a subjective judgment, and the procedure is open to debate. However, the eleven students in question made power scores ranging from sixty-three to eighty-five points out of eighty-eight points on the Cody High School Diagnostic Test. While their slowness might demonstrate a lack of skill, at the level of automatic response, it would be difficult to argue a lack of understanding of the processes tested.

As a result of this observation, the statistical evaluation will show the growth of the students in the sample in both speed and power scores. It would be unreasonable to attempt to get speed scores from the type of student just described. The insistence on a time limit will result in two undesirable outcomes, namely an unduly large proportion of random errors, and a tendency to regress to earlier behavior responses. This conclusion has very important implications for the teaching profession. What is a desirable educational program for these students?
When the experiment was about half over, another attempt was made to utilize student assistants as teachers for the students attending the Mathematics Clinic. The principles formulated in Chapter V were carefully observed. Several orientation meetings were held, during which the nature of and need for meaningful instruction was emphasized. The author was present during the instruction carried on by these student teachers, and difficult problems in explanation were referred to him. Students were carefully chosen so that extreme difficulties would not be encountered. With these arrangements, excellent results were obtained. The student teachers gained in self confidence as they gained experience, and the growth exhibited by their cases was similar to that of other students in the study. The sample was too small to evaluate separately, and these observations must remain qualitative in character.

Another important conclusion concerns the degree of training needed by classroom teachers in order to carry on this kind of instruction. In all but three or four cases, instruction at the symptomatic level was successful and judged to be sufficient. It was enough to conclude that a student did not understand the nature of the number system, and explain it to him without searching for causal factors in his history which would explain the lack. It was usually enough to show a student that his sloppiness was the cause of many errors, without going into the kind of personality description and analysis which might explain the causes of the sloppiness. It follows that the teacher of remedial arithmetic does not need intensive training in clinical psychology, as far as a high ratio of cases are concerned. Any teacher, trained in mathematics and meaningful arithmetic methods, with some capacity to understand the implications of personality factors and good human relations in this
type of situation, can learn the basic techniques in a very short time. Cases, such as Marlene P., described in Chapter V, would have to be referred by most teachers to more highly trained personnel.

3. Five case studies reported to illustrate specific principles:

a. The over-meticulous student:

Another behavior type, which is inherent in the psychology of the test situation, is that of the over-meticulous student. Consider the case of Ken W.:

Ken's initial score on the diagnostic test was 46. However, careful observation of his paper revealed only one error pattern. The low score could be attributed solely to an unfinished test. It was further observed that Ken's work was extremely neat, indeed, quite feminine, and much more legible than that of the average 15 year old boy. Discussion with Ken revealed that he did everything twice, once on the examination paper and once on a piece of scrap paper to verify his results. We examined his paper carefully and showed him that he had erased and corrected only two problems in a total of twenty-eight completed. Both of these were wrong. His careful checking had actually done harm. Ken was asked to complete the unfinished portion of the test at his leisure. Four diagnostic items were actually found on this portion of the test, which explained why he had done the second page during the initial examination. The usual teaching procedures were employed for these items. (For details, see Appendix C.) After ten days, Ken was given a retest and was encouraged to do the work just once. He completed the test in forty minutes and scored eighty-two points out of eighty-seven. When he saw the results, he said, "Gee, am I smart." He was actually more accurate than most students, without rechecking.

In the following paragraphs four additional cases will be described to illustrate specific principles. While these will not occur with great frequency, the clinical teacher should be aware of their existence.

b. A case of blocking because of extreme hostility toward the subject: Before describing this case, it should be noted that the students do not hate arithmetic, at this stage of the game, as deeply as one might expect. Most of the students indicated that their dislike was due to poor achievement, and future achievement brought about a greater liking. A
few students made the statement that they liked the subject a great deal, even though they did poorly at it. Hostility toward the subject was a much simpler problem than was anticipated before the study began. However, an occasional individual would exhibit very strong feelings on the matter. Consider the case of Jean B.:

Jean was referred to the clinic by her counselor, after her mother had come to school for a consultation about her mathematics. At this time, 15 weeks of the semester had gone past and Jean was failing in General Mathematics I. Her teacher testified that she seemed utterly incapable of the simplest problem and generally made little or no attempt. Her mother told the counselor that she had always had trouble with arithmetic and didn't even know her tables of fundamentals. Jean's initial score on the diagnostic test was nineteen points, an unusually low score. However, large portions of the test were completely untouched.

At the first interview, the instructor sounded Jean out about her feelings regarding arithmetic. She gave vent to unusually violent feelings of hostility. "I hate this stuff like poison! I don't want anything to do with it." The instructor's acceptance of her feelings, without censure, caused her to relax somewhat. She admitted that possibly, she might like the subject much better, if she could do well at it. The instructor proceeded to assure her, with a great deal of confidence, that every student who entered the clinic showed a great deal of improvement, and that it was merely a matter of time and application. Because of the extremely low score, no attempt to make a complete diagnosis was made during the first lesson. It was felt that the number of items requiring instruction was too large and that Jean would be discouraged by seeing them all at once. The procedure was simply to take them up one at a time. The first items had to do with misunderstanding the placement of decimal numbers for addition and subtraction. After a brief explanation, during which she showed a surprisingly quick grasp of the situation, Jean proceeded to do an exercise rapidly and without mistakes. The next item taken up was subtraction. Jean had acquired the habit of borrowing twice when borrowing across a zero. She understood the explanation of her fault immediately, but her first attempt to do a corrective exercise was only one-half successful. She was encouraged to try again. The second attempt was better, but still full of error. At this point, the instructor sensed that Jean was getting annoyed with the problem, and suggested that she go on to something else and return to it later. This pleased her a great deal. The next item brought surprising results. As it was pointed out, Jean said, "Oh, I know how to do that!" and proceeded to show a perfect solution to the problem. We proceeded to the next item and the same thing happened. The instructor now told Jean that he really had no idea of what she could or could not do, but that it was obvious that she understood a great deal more than her first test had indicated. The interpretation that was made to her was the belief that she had become nervous and tense during the test, and that her experienced diffi-
culty with one or two problems, coupled with her intense hatred of the subject, had caused her to freeze up, with the consequences described. She was encouraged to write the test over, with no time limit, only attempting those items which she felt sure she understood. She did so and scored sixty-six out of eighty-eight, much to her own surprise. Only five points of this remarkable gain could fairly be attributed to the corrective exercises of the first two days. After this, instruction proceeded as in other cases and Jean's second re-test showed a score of sixty-five, with three residual items. Her final sixty-five was a power score, accomplished in sixty minutes. The major gain in this situation was the realization on Jean's part that her arithmetic was not nearly as bad as she had previously thought. Further interviews with Jean helped her to see that her own strong feelings were the basis of most of the trouble. When she was questioned about the course and asked to indicate some of the things that had been difficult for her, she found one set of problems, and with a little questioning and encouragement was able to explain how they should be done. This helped to verify, in Jean's mind, that her feeling about the subject had more to do with her attitude than her actual ability. The final score was lower than expected because of a larger number of random errors.

A later interview with Jean's counselor revealed that she was also having trouble in Business Science. The counselor also explained that Jean was taking a commercial course at her mother's insistence, and that her mother was very successful in business and mathematics. While this data was interpreted as symptomatic of hostility which Jean felt toward her mother, displaced to her mathematics and business teachers, both women, it was determined to let the treatment rest on the symptomatic level for the time being. Jean had indicated that she felt encouraged about her work. It was decided that it was better to see if her work in these areas would improve without going into the matter of family relations more deeply. For other details, see Appendix C.

c. A case of failure to achieve because of initial conflict related to authority figures. The case of George C.:

George was a tall boy for his age (15), about 70 inches, of medium build with dark hair. When he first reported to the Mathematics Clinic, he seemed rather tense and apprehensive of the situation. His initial score on the diagnostic test was twenty-two points, but the low score is deceiving since he had only completed eighteen problems out of a total of fifty-three. We told him that we really had no clear picture of what he could or could not do with basic arithmetic and asked him if he would complete the test so that a more accurate diagnosis could be made. He was told to take his time and not be concerned about racing the clock. He needed about forty-five minutes to finish the test, but he was observed working fairly rapidly, and then very slowly. The power score was sixty-three points, a fairly respectable total. A slightly higher ratio of random errors was noted in the part he had done during the forty
minute test. The diagnosis revealed only three items:

1. He placed whole numbers after the decimal point in two of the four problems concerned with addition of decimals.

2. There was some difficulty with zero place holders in division, a tendency to put in zeros which were not needed.

3. He made some errors in the use of the decimal rule in division.

Before proceeding with instruction, an attempt was made to sound out George about the apparent contradiction between his actual skill in arithmetic and his current failure in General Mathematics coupled with a low score on the initial test. George talked quite readily about the problem, explaining that he had had trouble for the past two years with at least one teacher every semester. He said that if he thought the teacher a bit easy going, he would try to get away with things. By this he meant not doing the regular classwork, talking excessively, and occasionally indulging in boyish pranks which would get him into trouble. It should be noted at this point, that George's teacher in General Mathematics was a young man in his first teaching position, who appeared to be very affable, and gave the misleading impression of being easy going. Without any comment from the teacher, George went on to say that this was his own fault, that he really did not know why he did it, but that he managed to stay out of serious trouble. He apparently had enough contact with reality to be able to gauge the seriousness of an offense and to avoid those which might lead to suspension. The fact that George's trouble was always with male teachers provided an obvious clue, for further conversation which revealed to George that his conflict was essentially related to feelings about his father. It is unnecessary to go into detail about the long interview, which followed procedures known to counselors and social workers for many years. The attempt to bring George a degree of insight concerning his conflict with his father was partly successful. He was helped to gain some fresh perspective towards school, to a limited degree.

Instruction proceeded, after the first interview, to take up the three points listed in the diagnosis above. In the case of number one and three, it was evident that George really understood them. In order to be sure, and also to provide a means for continuing the relationship, practice exercises were prescribed. There was a misunderstanding of item two, but some explanation quickly cleared it up. George took a retest about two weeks after the contact began and received a speed score of fifty, but a power score of seventy-eight out of eighty-seven. The only points he lost were due to random errors. His slowness on the test could be attributed to day-dreaming. After a brief discussion of the problem of day-dreaming with him, the relationship was terminated and he was sent back to class. The teacher later reported that, while he still was inconsistent, his behavior had improved sufficiently for him to pass the course, at the next card marking. The general prognosis for George is doubtful.
A case of specific blocking on a particular item. The case of John W.

John was a tall boy for his age (fifteen, about seventy-one inches, of medium build with straw colored hair. He had a very pleasant manner, and seemed pleased about the opportunity to receive clinical instruction. He said he was very interested in drafting and might like to make a career of it. He understood how important mathematics was to drafting, and wanted to clear up his difficulties. He was also happy about the assignment since it took him out of a study hall. He was also a member of the freshman basketball squad, and gave the general impression of a normal boy of average intelligence.

John's initial speed score on the diagnostic test was twenty-nine. The opportunity to finish the test raised the score to forty. The diagnosis indicated trouble with the tables of fundamentals and he was tested on these also. The complete diagnosis follows:

1. Slow on the subtraction tables. (four minutes)
2. Slow and inaccurate on the tables of multiplication and division, particularly on combinations involving 7, 8, and 9, particularly the latter two.
3. Does not understand the nature of the decimal point in addition and subtraction problems, such as 3.15 plus 54.3 plus 19.
4. Borrows 10 instead of the unit involved when subtracting denominate numbers.
5. Fails to reduce fractional answers to lowest terms.
6. Does not multiply correctly when numbers end in 0.
7. Fails to invert when dividing a fraction by whole numbers, and inverts whole numbers, when dividing them by a fraction.
8. Does not use decimal rule correctly in problems like .009/72 (adds one 0 instead of 3, counting from the left of the dividend).

Instruction proceeded along two lines. Each day, John did short practice exercises with the subtraction and multiplication tables, for ten to fifteen minutes. The time was limited on this to prevent boredom. The exercises consisted of flash cards or written drills of the following type:

---

1. This is a diagnostic item in the sense the omission may point to a misunderstanding of the process. An unsimplified answer is not, of itself, serious.
When the daily drill was over, John proceeded to work on the other items in the diagnosis. He showed a quick grasp of the principles involved in all of the items but number seven. After completing exercises for each item, he was given a retest, and scored fifty-seven, out of eighty-seven. Seven points of the loss could be attributed to random errors, the remainder to the multiplication and division tables and item seven in the diagnosis. Three days instruction and practice seemed to clear up the latter, but there seemed to be a block on the problem of the multiplication table.

The instructor made the error of having John continue with the same methods of practice, which did little real good. The same cues which had helped many other students did not work with him. For instance, he knew that the digits of all multiples of 9 added to 9, but that did not help. Many students, confronted with 8 X 7, would think "7 X 7 is 49 and one more 7 makes 56". This did not help John. After much fumbling, the following mental procedure was the one which solved the problem:

For 7 X 9 "7 X 3 is 21 and 3 X 21 is 63"
6 X 8 "3 X 8 is 24 and 2 X 24 is 48"
9 X 8 "9 X 4 is 36 and 2 X 36 is 72", etc.

This is admittedly slow, and not as desirable as an automatic response with the correct answer. However, the latter could not be achieved with John, and the procedure above gave him his first real control over the process. His control over the corresponding items in the division table also improved, which is customary in such cases.

2The student writes out the rows of nines, puts down the sums he knows, and fills the remaining answers in by addition or subtraction.
John took his second retest two weeks later and scored seventy-one out of eighty-eight. He made errors related to item eight in the diagnosis, and still lost three points attributable to the multiplication tables, but the remainder could be charged to random errors alone. He remained in the clinic a few more days to practice the tables further, then volunteered to remain the rest of the semester to assist in the work of the clinic. His motives in this were two-fold, since he had become very interested in a girl who was a student assistant that hour. He was very useful for the remainder of the term, spending a good deal of time helping another boy with the multiplication and subtraction tables, and, it seems, improving his own control at the same time.

e. A case of achievement in arithmetic which had little or no effect on over-all social adjustment. The case of Ray J.:

Ray was a short, blond, fifteen year old boy. His low score on the Iowa Every Pupil Test caused him to be taken from his General Mathematics class during the first week of the semester, for clinical instruction. He was to attend during the last hour of the day. Note that Cody High School was on double session, and the last hour of the day met from 4:00 to 4:40 p.m. However, Ray's schedule had not been lengthened to attend the clinic. It had really been substituted for his General Mathematics class. Ray was very quiet, relatively unresponsive and did not speak unless spoken to. This unresponsiveness carried over to the other students present, as well as the teacher.

Ray's initial score on the Cody High School Diagnostic Test was twenty-six points, unusually low, even for clinical students. Twenty-three problems on the test remained unanswered, even after he was given the opportunity to do so without time limit. He simply refused to answer any question when he was uncertain of the procedure. When there are large numbers of items in the diagnosis, it is unwise to discuss a complete diagnosis with the student at the outset. It is better to simply take them up one at a time, which was the procedure followed with Ray. When the diagnosis was eventually completed, nine items were listed. It is unnecessary for purposes of this explanation to go into detail about them. If interested, see Appendix C.

In the weeks that followed, Ray's attendance was very spotty. Most of his absence was truancy, including both truancy from an entire day of school and truancy from the last period of the day, which was the period to be spent in the Mathematics Clinic. Discussion with the boy's counselor indicated that he was a highly disturbed boy who tried to solve his problems by withdrawing from them, and by running away from the authority elements in society. The instructor did not attempt to enter into the counseling aspects of the situation. We simply dealt with Ray on the basis of subject matter instruction through the medium of a friendly personal relationship. When he returned after truancy, little or nothing was said about it. Ray was treated as if his last attendance had been the day before. He was a willing worker and caught on rapidly to everything explained to him. Every practice exercise was done to near perfection the first time.
About six weeks after the contact began, he was given a second test in which he received a speed score of forty-five and a power score of seventy-one. Three residual items in the diagnosis were found on this paper. For detail, see Appendix C. After brief instruction on each of these, he was tested again and made a speed score of fifty-nine and a power score of seventy-nine. His increase of fifty-three points in power score was the largest recorded in the entire study.

In spite of the friendly atmosphere, Ray remained sufficiently unresponsive, so that we never could tell whether he was really learning something clearly for the first time, or whether it was an experience he had mastered before but was unsure of. The teacher made a great point of complimenting him periodically about his improvement and taking his test papers to his counselor. Both the teacher and the counselor felt that something about which he could be complimented was badly needed.

After the second test, Ray was returned to his General Mathematics class. Subsequent reports from both his regular teacher and his counselor indicated that, from the standpoint of social and emotional adjustment, the special clinical instruction had had no observable effect. From the standpoint of other people observing him, it was as though the experience had not happened at all. The boy's inner feelings regarding the matter were impossible to judge, since he remained quite unresponsive.

It is very seldom that the process of clinical instruction, with the special attention and encouragement experienced by students, has so little effect in the area of attitudes. Even in cases where the over-all adjustment remains poor, some improvement is generally noted.

4. Some additional observations:

One of the greatest values for students and teachers alike, which comes out of this type of teaching, is the tremendous boost in morale. Most of the students, while expressing some slight discomfort at being singled out for special instruction, were really very happy about the experience. Any resistance about the situation usually disappeared after the first day. The friendly, informal atmosphere and the entry into a group which was obviously enjoying its work, were important factors in this regard. Of the sixty-nine cases dealt with, during the semester, only one expressed great feelings about being singled out for a special
class. This boy preferred the anonymity of a large class. While he was unhappy about the situation, it actually goaded him to faster progress, since he quickly learned that the faster he improved, the faster he would return to his regular class. Many of the other cases requested a longer period in the clinic. This was not necessary, but it was pleasant to realize that they were expressing their enjoyment of the situation and their sense of accomplishment. Several students half seriously threatened to fail the retest so that they would be retained in the clinic. A few expressed some fears about returning to class, but were encouraged to feel that the things they had learned would be helpful to them.

One important observation must be made, which came from comparing the results of the pilot study with those of this second phase. It is safe to say that the sooner the student enters clinical instruction, after coming to a new school, the better. There was a definite feeling that the best results were obtained during the first five weeks of the semester, when the students had had no large class contact as yet, and were still becoming adjusted to the school. The students in the pilot study had experienced over a semester of poor or at best fair achievement. They had already had the opportunity to learn that high school was just another school, the same old stuff all over again. The children helped in the second phase of the study were given assistance before they had acquired very great feelings of failure or frustration. While this observation is difficult to verify with objective evidence, it emerges nonetheless as a firm conviction of the author, based on the experience of the semester under discussion. Numerous students, meeting us in the hallways of the school, weeks after leaving the mathematics clinic, would smilingly say that they were getting along very well, and a few would testify that
mathematics was now their best and/or their favorite subject.

5. A three-fold statistical evaluation:

For purposes of analysis and evaluation, three tables of data concerning the students in this phase of the study have been prepared:

1. A group diagnosis of the fifty-nine cases, similar to that shown in Table 8, was prepared.

2. A statistical evaluation of the test-retest scores for the fifty-nine cases, and of the number of diagnostic items before and after instruction.

3. A table of classroom grades of those students discharged from the clinic early enough in the semester so that such analysis could have meaning.

The group diagnosis:

1. The composite group diagnosis was made for purposes of comparison with the overall student body diagnosed in Table 8. For convenience in analysis, two tables have been prepared. Table 15 shows the comparison of twenty-five items. These include the thirteen items studied in Table 9 as the most common, and nine others, with higher incidence in the clinical group than in the large group. In addition, three items are listed, which did not appear on the original list, but which appeared in sufficient quantity in the clinical group to warrant attention. It is to be expected that the incidence for any particular item ought to be greater in a group especially chosen for clinical instruction than in the student body at large. Table 15 shows very clearly that this is generally true. However, the most important purpose to be served by this comparison is the discovery of items for which this is not true. Table 16 reveals seventeen items for which no significant difference exists between the two groups.
Table 15
Comparison of Percents of Incidence of Selected Diagnostic Test Items for Two Groups

<table>
<thead>
<tr>
<th>Item No.*</th>
<th>Item</th>
<th>Overall Percent</th>
<th>Students Selected For Clinical Teaching*** Percent</th>
<th>Students Selected For Body** Percent</th>
<th>Before Percent</th>
<th>After Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Sloppiness leads to error.</td>
<td>11.2</td>
<td>3.3</td>
<td>3.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Works slowly and does not finish.</td>
<td>19.1</td>
<td>37.3</td>
<td>28.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>Misunderstands nature of decimal point in 7.1 + 3.15 + 16.</td>
<td>25.2</td>
<td>64.4</td>
<td>8.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>Fails to reduce correct answer.</td>
<td>9.5</td>
<td>27.1</td>
<td>8.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Does not borrow correctly.</td>
<td>7.8</td>
<td>3.3</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>Does not understand nature of decimal point in 42 - 6.7.</td>
<td>13.0</td>
<td>74.6</td>
<td>17.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>Brings down figures instead of borrowing.</td>
<td>16.8</td>
<td>44.1</td>
<td>7.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>Does not borrow correctly with denominate numbers.</td>
<td>18.9</td>
<td>39.0</td>
<td>1.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>Does not use 0 correctly with numbers ending in 0's (mult.).</td>
<td>11.7</td>
<td>27.1</td>
<td>3.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>36</td>
<td>Omits 0's needed as place holders (division).</td>
<td>6.4</td>
<td>18.6</td>
<td>5.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>Does not use decimal rule correctly (division).</td>
<td>27.0</td>
<td>50.9</td>
<td>23.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>41</td>
<td>Does not invert divisor fraction.</td>
<td>12.8</td>
<td>18.6</td>
<td>11.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>Inverts dividend fraction.</td>
<td>2.3</td>
<td>8.5</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Adds numerators without changing to a common denominator.</td>
<td>1.3</td>
<td>7.0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Cannot keep sums in head with long columns of numbers.</td>
<td>3.2</td>
<td>15.2</td>
<td>1.7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Numbers taken from Table 8.
** 529 Cases.
*** 59 Cases.
Table 15 (continued)

Comparison of Percents of Incidence of Selected Diagnostic Test Items for Two Groups

<table>
<thead>
<tr>
<th>Item No.*</th>
<th>Item Description</th>
<th>Overall Percent</th>
<th>Students Selected For Clinical Teaching***</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>Changes denominator incorrectly</td>
<td>7.6</td>
<td>13.6</td>
</tr>
<tr>
<td>16</td>
<td>Does not borrow correctly across 0's.</td>
<td>1.1</td>
<td>10.3</td>
</tr>
<tr>
<td>21</td>
<td>Does not change fractions to a common denominator (subt.).</td>
<td>1.5</td>
<td>10.3</td>
</tr>
<tr>
<td>22</td>
<td>Borrows 10 instead of the denominator of the fraction.</td>
<td>1.1</td>
<td>23.7</td>
</tr>
<tr>
<td>31</td>
<td>Places decimal points incorrectly (multiplication).</td>
<td>6.4</td>
<td>18.6</td>
</tr>
<tr>
<td>30</td>
<td>8 x 0 equals 8, 2 x 0 equals 2, etc.</td>
<td>0</td>
<td>5.2</td>
</tr>
<tr>
<td>35</td>
<td>Does not change mixed numbers to improper fractions but multiplies parts separately.</td>
<td>1.1</td>
<td>7.0</td>
</tr>
<tr>
<td></td>
<td>Has no idea what to do to borrow in subtracting.</td>
<td>x</td>
<td>18.6</td>
</tr>
<tr>
<td></td>
<td>Omits problems in dividing fractions (not due to lack of time).</td>
<td>x</td>
<td>7.0</td>
</tr>
<tr>
<td></td>
<td>Cannot divide 2 place numbers.</td>
<td>x</td>
<td>11.8</td>
</tr>
</tbody>
</table>

* Numbers taken from Table 8.
** 529 Cases.
*** 59 Cases.
**** Not diagnoses in original group. Diagnosis depends partly on interview, is very difficult to judge from test paper alone.

Other items exist in the diagnosis, but are too rare to afford comparison.
<table>
<thead>
<tr>
<th>Item No.*</th>
<th>Item</th>
<th>Overall Percent</th>
<th>Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Carrying numbers not there, or wrong number.</td>
<td>1.5</td>
<td>3.3</td>
</tr>
<tr>
<td>7</td>
<td>Not carrying (addition).</td>
<td>1.3</td>
<td>1.7</td>
</tr>
<tr>
<td>8</td>
<td>Leaving out decimal point (add.).</td>
<td>2.6</td>
<td>1.7</td>
</tr>
<tr>
<td>13</td>
<td>Forgets the whole number part of mixed numbers (add.).</td>
<td>2.5</td>
<td>1.7</td>
</tr>
<tr>
<td>17</td>
<td>Omits decimal point (subt.).</td>
<td>2.1</td>
<td>0</td>
</tr>
<tr>
<td>25</td>
<td>Misunderstands use of 0 as a place holder (mult.).</td>
<td>3.2</td>
<td>0</td>
</tr>
<tr>
<td>26</td>
<td>Mixes up 7x8, 7x9 and 6x9.</td>
<td>6.2</td>
<td>1.7</td>
</tr>
<tr>
<td>27</td>
<td>Forgets to carry (mult.).</td>
<td>1.1</td>
<td>0</td>
</tr>
<tr>
<td>28</td>
<td>Carries numbers not there (mult.)</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>29</td>
<td>Ignores the decimal point (mult.).</td>
<td>4.7</td>
<td>0</td>
</tr>
<tr>
<td>32</td>
<td>Inverts fractions when multiplying.</td>
<td>3.8</td>
<td>5.2</td>
</tr>
<tr>
<td>33</td>
<td>Multiplies fractions correctly, inverts answer.</td>
<td>0</td>
<td>0***</td>
</tr>
<tr>
<td>34</td>
<td>Cancels numerator with numerator, denominator with denominator.</td>
<td>0.2</td>
<td>1.7</td>
</tr>
<tr>
<td>37</td>
<td>Subtracts incorrectly (div.).</td>
<td>1.0</td>
<td>0</td>
</tr>
<tr>
<td>38</td>
<td>Mixes up $7/56$, $7/63$, $6/54$, etc.</td>
<td>1.7</td>
<td>0</td>
</tr>
<tr>
<td>39</td>
<td>Omits decimal point (div.).</td>
<td>3.2</td>
<td>0</td>
</tr>
<tr>
<td>43</td>
<td>Cancels before inverting fraction</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>44</td>
<td>Cancels before changing mixed numbers to improper fractions.</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>45</td>
<td>Seem all mixed up (div. fractions)</td>
<td>6.1</td>
<td>3.3</td>
</tr>
</tbody>
</table>

* Numbers taken from Table 8.
** 529 cases.
*** 59 cases.
**** Observed behavior in other pupils previous to study.
The large number of these items is surprising at first, but a careful examination of their character indicates a remarkable consistency. Fifteen of these items are errors of omission, or errors in which a process is reversed. For example, failure to carry in multiplication, or carrying a number not there, or carrying the wrong number, can all be characterized as errors of omission, or the reverse of inserting a process sometimes used but not required in the example. Subtracting the top number from the bottom one is another example of reversing a process. Two items deal with the basic multiplication and division tables. It is now clear that the student who responds incorrectly to 7x8, or 6x9 is often committing an error which can be called a reversal, similar to subtracting the top number from the bottom one, or inverting the wrong fraction in division. The most conclusive supporting evidence is the fact that students who do this never answer 7x8 is 55 or 59. The wrong answers are 63, 72, 64, 54, 49 and 81, all of which are answers memorized for some other problem. Item thirty-seven (Subtracts incorrectly in division) means little, since this type of diagnosis is difficult to make on this test. Item forty-five (Seems all mixed up in dividing fractions) is actually incorrect, since Table 16 shows that students deficient in this process often omit it, rather than attempt strange and garbled solutions.

Errors of omission, reversals of process or responses and errors due to sloppiness, (see Table 16) are clearly functions of the whole population rather than the group selected for clinical help alone. Perhaps teachers are dealing with a facet of human nature which is a limiting factor, which no amount of instruction will aid. Certainly the presence of calculating machines in offices and laboratories is eloquent testimony for such a point of view.
The preceding analysis indicates that the initial diagnostic check list which was used in the study bears a considerable amount of revision. Many items could be deleted, since their incidence is small. Other items could be grouped in different ways, rather than in the traditional manner of considering the four processes of addition, subtraction, multiplication and division as the natural means of classification. For example, Items 15, 19, 20, 16 and 22 in Table 16 are all concerned with the nature of borrowing in subtraction. Items 14, 21, and 12 are all concerned with changing to common denominators. Items 9, 24, 36, 40, 31 are concerned with an understanding of the tens number system. The manner of regrouping these items has been left open for further research, for several reasons. The most important reason is the multicausal nature of the phenomena described. Item 19, for instance, could be attributed to a misunderstanding of the borrowing process, but may, in some cases, be a product of hurrying or carelessness in a student who really understands borrowing. Another important reason is the future use of any diagnostic check list. Even if an item is rare, a knowledge of its existence and nature should be part of the training of any teacher doing this kind of work. Indeed, the problem of how to regroup the important diagnostic items is really a problem of teacher training.

One natural function of educational research in remedial areas is the problem of interpretation of the data in terms of a program of prevention. To aid in such a task, the commonest items in the diagnostic check list were compared with the total list and the percentage of incidence was calculated. The areas chosen can be classified as follows:

a. Understanding the nature of the decimal number system in all four basic processes. (Items 9, 18, 31 and 40)
b. Understanding the use of 0 in multiplication and division.  
(Items 24 and 36)  
c. Understanding borrowing in subtraction.  (Items 15, 19, 20, 16, 22)  

The total number of items under these headings was compared to the grand total of items in both groups listed in Table 15. For the group selected for remedial instruction, these items comprised 67.4 percent of the total. For the overall student body, they comprised 78.9 percent of the total. The lower ratio for the remedial group is explained by a higher incidence of items which reveal a complete breakdown of a process, such as a complete inability to divide two place numbers. The conclusion concerning the importance of these areas for elementary instruction is consistent with the review of articles on research in error diagnosis in Chapter III, with one exception. (See page 58 ) That exception is the problem of carrying in multiplication and addition. While these errors certainly exist, Table 16 indicates that they are not matters of remedial concern, or possibly not matters for general concern. The only issue which requires solution in this regard is whether students should, or should not be encouraged to write down the number carried. The author believes, on the basis of clinical experience, that they should.

2. The statistical evaluation:

Table 17 shows the test retest scores for the cases taught in this phase of the summary, with sufficient data reprinted from Table 14 to enable the reader to make quick comparison with the control group and the other phases of the study. It was necessary to show the comparison twice, once in terms of speed scores, that is, scores based on a forty minute time limit, and once based on power scores, based on no time limit. A word of explanation is in order. Students usually took the
Table 17

Diagnostic Test Score Summary of Various Groups
Before and After Remedial Instruction

<table>
<thead>
<tr>
<th>Group</th>
<th>Mean Score Before Instruction</th>
<th>Mean Score After Instruction</th>
<th>Increment</th>
<th>Critical Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebra Students (from Overall Student Survey)</td>
<td>73.8</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Control Group General Mathematics Students (classroom instruction)</td>
<td>69.7</td>
<td>70.9</td>
<td>1.2</td>
<td>.44</td>
</tr>
<tr>
<td>Pilot Study Remedial Classroom Group</td>
<td>42.7</td>
<td>56.6</td>
<td>13.9</td>
<td>4.04</td>
</tr>
<tr>
<td>Pilot Study Clinical Group (6 mos.)</td>
<td>50.1</td>
<td>67.6</td>
<td>17.5</td>
<td>4.37</td>
</tr>
<tr>
<td>Pilot Study Group (Classroom &amp; Clinical) (one year)</td>
<td>33.3</td>
<td>65.5</td>
<td>32.2</td>
<td>5.33</td>
</tr>
<tr>
<td>Experimental Clinical Group Speed Scores</td>
<td>48.4</td>
<td>71.7</td>
<td>23.3</td>
<td>9.17</td>
</tr>
<tr>
<td>Experimental Clinical Group Power Scores</td>
<td>54.0</td>
<td>73.1</td>
<td>19.1</td>
<td>8.36</td>
</tr>
</tbody>
</table>

\[ n = \text{number of cases} \]
\[ s = \text{sample standard deviation} \]
\[ F = \text{F ratio (see note below)} \]
\[ x = \text{No data} \]

Note: The sample standard deviations shown were compared to that of 17.2 for the Overall Student Body tested in the original survey, by the Snedecor F test. Since all are significantly different, the problem of which to use to test the critical ratio of the differences becomes important. To minimize the chance of erroneous conclusions, the larger figure of 17.2 was used. Critical ratios were figured for the null hypothesis for testing the difference of two sample means.
first test in their large classes, with a forty minute time limit. When they reported to the Mathematics Clinic, those who had not completed the test were asked to do so, and the papers were rescored, so that a power score was available. Most of the students finished the retest in the forty minute period. Some did not, and were asked to finish the following day, so that both a speed and a power score were available. In eleven cases, placing a time limit upon the student would produce undesirable behavior, such as regression to earlier patterns due to nervousness. It was decided to place no time limit upon these students, so that the only final score available for them is the power score. Two cases were not retested. They had initial scores of seventy-two and seventy-four, with one diagnostic item apiece. Each attended the mathematics clinic for two days to clear up the one item, and it was considered pointless to test them again. Thus the sample of speed scores contains forty-six cases, and the sample of power scores contains fifty-seven cases.

The increments shown for the sample are significant and consistent with the findings of the pilot study. The mean of 73.1 for the final power score for fifty-seven cases is comparable to the mean of 73.8 for the students entering ninth grade Algebra in the original Overall Student Body tested.

The variances of the four samples shown were compared to that of the original Overall Student Body (standard deviation 17.2) by means of the F test. Unlike the pilot study, each variance was found to differ significantly from the overall group. This does not affect the validity of the conclusions, since the selection of the group would lead to a smaller variance, as high scoring students are not included. To further illustrate this argument, the variances of each sample before and after instruction were also compared by the F test. The variance for the re-
test was significantly smaller. Note that the lowest retest score was fifty-six, with only two below sixty. (Power score) Another factor influencing the change in variance, and the rise in the means is the manner of closing cases. Of the ten cases not closed, two had shown very limited growth, and would probably have shown little more after another semester. The other eight were making satisfactory progress and would probably have shown change patterns like the closed cases. The variances of the data for the group in the pilot study exhibited the same tendency described in the previous paragraph (see Table 14), even though the F tests did not reveal significant differences. The F ratios were actually much like those calculated for the sample under discussion, but the critical region was differently located because of the smaller sample. In terms of subject matter control, there is little doubt that the unusual instructional situation is highly successful and more efficient than other types of organization or less individualized methods.

In addition to the test score, a tally was made of the number of diagnostic items inferred from the initial and final test papers. The mean number of initial diagnostic test items was 6.0 and the mean number of final diagnostic test items was 1.1. Table 18 below shows the number of cases closed according to the number of residual items on the final paper.
Table 18
Number of Diagnostic Items Per Test Paper for 59 Closed Cases After Clinical Instruction

<table>
<thead>
<tr>
<th>Number of Items</th>
<th>Number of Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>2*</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>

*One of these had been scheduled for further instruction, but was unable to attend.

These results simply corroborate the information from the scores on the same paper. Table 15, in addition to the comparison of the clinical group to the Overall Student Body for the previous year, also shows the breakdown of the residual diagnostic items for the clinical group. It is easily seen that the area which was most difficult to teach was the understanding of the decimal system, particularly for subtraction and division. A word about decimal division may help to clarify this part of the problem. The deficiency does not generally take the form of complete misunderstanding. Many students will do all problems correctly except this type:

\[ \frac{.009}{72} \quad \frac{.55}{2215} \]

The error in these is to count the places in the dividend from the left of the number instead of the right. A separate explanation must be made for this type of problem and a special exercise of such problems must be
prepared. The semester was half gone before the author realized the situation clearly, and the students taught during the latter part of the semester exhibited fewer deficiencies in this area than the first group. The fourteen residual items in this category can be fairly said to be unnecessarily high.

3. An examination of the classroom grades:

This part of the evaluation is less valid than the previous section, not simply because of the more subjective nature of classroom grades, but because the grades were a composite of the classroom teacher's marks for the time spent in the classroom and the author's grade for the time spent in the Mathematics Clinic. During clinical instruction the student was excused from the regular classwork. When he returned to the regular class, the author reported a grade for the improvement shown, which the classroom teacher averaged in with the other marks. While the information is clearly biased, it is worthy of some consideration. Since the students for this instruction were drawn from both classes in Remedial Mathematics and General Mathematics, they are considered separately. Table 19, below, shows the final semester grades for students who were discharged from the mathematics clinic up to eight weeks before the end of the semester.
Table 19
Grade Distribution of Students Who Completed Clinical Instruction Eight Weeks Before End of the Semester

<table>
<thead>
<tr>
<th>Grade</th>
<th>General Mathematics</th>
<th>Remedial Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>D</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>8*</td>
<td>0</td>
</tr>
</tbody>
</table>

*Classroom teachers reported that six of these cases failed because of personality problems rather than subject matter competence.

In spite of the bias which enters this data, it indicates a degree of future gain from clinical instruction. In addition, teachers have made many verbal reports about changes in attitude toward the work, increased self-confidence and improved social behavior.

SUMMARY

The final phase of the clinical study corrected the biases of the pilot study and established controls by taking students for clinical instruction from the classes of teachers in General and Remedial Mathematics other than the author. Fifty-nine cases were closed by the end of the semester, and ten were listed as still in process. The results of the pilot study were confirmed. Most cases followed the pattern of the cases of Lela Y. and Jean G. described in Chapter V, but important new patterns were observed. Five of these are described in detail. The patterns include the student who:
a. is over-meticulous and does not finish a test.
b. exhibits extreme hostility toward the subject.
c. has conflict with authority figures leading to low achievement.
d. has specific subject matter blocks.
e. showed no change in social adjustment as a result of successful clinical instruction.

The closed cases showed a mean gain on the diagnostic test of 23.3 points (speed score) and 19.1 points (power score). The group had a mean of 6.0 diagnostic items on the test before instruction and 1.1 after instruction. This information, with added evaluation of the classroom grades of the students, leads to the conclusion that the instruction was very effective.

A diagnostic comparison of the fifty-nine cases with the five hundred and twenty-nine in the testing survey phase of the study gave the expected result of higher incidence of diagnostic items among the fifty-nine cases. A very important exception was noted among items which can be classified as errors of omission or reversal, which are no different for the Overall Student Body than for the specially chosen clinical group.

The principles and qualitative observations of the clinical phase of the pilot study were verified.

Further qualitative observations included:

More than two retests per student are generally unprofitable, usually because of nervousness during tests.

Some students get nervous and regress to earlier wrong behavior if examination time limits are imposed.

Student assistants were used as teachers, under supervision, with good results.
Professionally trained teachers can do this kind of teaching, for most cases, without intensive training in clinical psychology.

The sooner a student is placed in remedial instruction, after entering the senior high school, the better.
Chapter VII
SUMMARY, CONCLUSIONS and IMPLICATIONS

Large numbers of students, deficient in basic arithmetic, have been entering the public high schools for some time. Although some students of the problem of low achievement in arithmetic have argued against meeting the problem with remedial rather than preventive devices, it is clear that some kind of instructional policy must be made in the high school. The research in remedial arithmetic conducted at Cody High School, Detroit, Michigan, can properly be described as a series of related studies which culminated in the successful and permanent establishment of daily Mathematics Clinics at the school, organized to give individualized instruction in arithmetic to groups of five, six or seven students.

The study was carried on from several points of view. One aspect sought to reveal information regarding the specific nature and causes of arithmetic deficiencies, and to examine the findings of other research in this area. Another objective was to formulate a specific teaching program, with methods for correcting deficiencies, whether causes are completely understood or not. A third viewpoint examined the nature and causes of such deficiencies from the standpoint of recommendations for preventive teaching in the elementary school, and for the teaching of general mathematics in the high school.
Each theme described, and each successive phase of the study appears to have implications in terms of (a) the meaningful understanding of the tens number system and (b) the basic personality structure of the student. The study had four principal phases:

1. A careful examination of the literature, not only for specific research items, but for overall themes which appear with consistency.

2. A testing survey, made in September of 1953 of a large number of students entering the ninth grade at Cody High School. The Iowa Every Pupil Test of Basic Skills, Form D (arithmetic), the Cody High School Diagnostic Arithmetic Test, and the Detroit Public Schools Tests of the Basic Subtraction, Multiplication, and Division Facts were given to large numbers of these students. The intercorrelations of the tests were studied from the standpoint of classifying students for fast (Algebra), average (General Mathematics) and slow (Remedial Mathematics) classes. Check lists of common diagnostic deficiencies in arithmetic were prepared to accompany the Cody High School Diagnostic Arithmetic Test, and one such list was checked off for each student tested. These items were coded on McBee Keysort cards, from which the percent of incidence of each item, and the interrelationships among the items were studied.

3. Special corrective exercises, based on the more common diagnostic items, were prepared for use in classroom or individual instruction. (See Appendix B) These were presented in two classes in Remedial Mathematics. The instruction was evaluated by retesting with the Cody High School Diagnostic Arithmetic Test at the end of the semester, and by other observations. The growth of two classes in General Mathematics on the same test was used as a control.
4. Clinical teaching in groups of five, six or seven students constituted the final phase of the study, first in a pilot study of twenty-five cases, and later in a larger study in which fifty-nine cases were concluded, and ten cases partially concluded. This took place during the spring semester of the school year 1953-1954 and the following semester. The technique was to work out a complete diagnosis of the deficiencies in arithmetic with each individual student, and teach him to correct each item in the diagnosis by appropriate explanation, followed by corrective practice with the special exercises mentioned in 3. Teachings procedures included interpretation to the student of pertinent personality factors. The number of lessons per student was kept flexible, so that students could be returned to their regularly scheduled classes as soon as possible, and others could take their places in the Mathematics Clinic. Student assistants were used as teaching personnel to some degree. This part of the study was also evaluated by retesting with the Cody High School Diagnostic Arithmetic Test. The classes in Remedial Mathematics in the previous phase of the study were used as a control.

1. Conclusions From Review of the Literature:

A. Remedial instruction in arithmetic has been carried on by many research workers, with pupils ranging from the third grade to the first year of college. Many variations in organization and technique have been tried and found workable to some degree. The principles summarized by Fernald\(^1\) concerning both causes and cures appear consistent with the

work of all the studies reported and with the work of this study.

B. Individualized instruction was more effective than group instruction, not only for remedial teaching, but for the general teaching of arithmetic as well.

C. The pupil teacher relationship is a major psychological factor in the learning of arithmetic.

1. The emotional attitude of the student toward the subject and the teacher is an important item for the teacher to examine. Also, successful remedial instruction may be a major factor in improving attitudes, such as self-confidence, and liking for the subject.

2. The emotional attitude of the teacher in training toward arithmetic must be worked through, if teacher training is to be fruitful.

D. All successful remedial teaching studies used a system of lesson plans based on a diagnostic review of errors, of individuals or groups of students.

E. A large ratio of the errors found on arithmetic tests, possibly eighty percent, are random errors, not representive of systematic error patterns. Diagnosis of error patterns must take this fact into account if such patterns are to be accurately described.

F. In spite of the large percentage of random errors, error diagnosis reveals consistent patterns from one study to another. The most frequently found deficiencies can be grouped in the following areas:

(1) Errors in the use of zero in multiplication and division.

(2) Errors in all kinds of borrowing in subtraction.
(3) Misunderstanding the nature and use of decimal points in all four processes.

(4) Errors in carrying in multiplication and addition.

(5) Lack of control over the basic tables of fundamental facts, particularly the 7, 8 and 9 facts (with the higher numbers) in multiplication and division.

2. Conclusions from the Testing Survey:

A. No single test, of the three examined, could serve as a satisfactory instrument for classifying students. The coefficient of correlation between the Iowa Every Pupil Test of Basic Skills, Part D (arithmetic) and the Cody High School Diagnostic Arithmetic Test was .586. This shows a significant relationship, but hardly of the order needed to predict one from the other in individual cases. The coefficient of correlation between Part II (computation) of the Iowa Every Pupil Test of Basic Skills, Part D, and the Cody High School Diagnostic Arithmetic Test was .638, not significantly higher than the correlation mentioned above. Since the latter test is limited to basic computation, one would expect the correlation to be higher. From this and various qualitative observations, the author makes the following important inference:

Certain pupil behaviors, having roots in the nature of the examination situation, rather than the particular test, operate as limiting factors in the validity of examination scores. These include nervousness, poor timing, hurrying the work in order to finish, with a consequent increase in error, poor work as a result of headaches, etc. These factors may be temporary or recurrent. These factors operate in
all tests. Teachers must be trained to look for such behaviors, and make appropriate adjustments in classification and teaching procedures.

B. The coefficient of reliability of scores on the Cody High School Diagnostic Arithmetic Test, used as a power test, is .92. The diagnostic items checked off on the same papers used to compute this coefficient, had a ratio of agreement of 68.7 percent. The disagreements were most often of the type in which the student did a problem wrong two or more times on one paper, leading to a diagnosis, and only once on the other paper, giving an indication, but not a diagnosis. The objectivity of the diagnoses made by student assistants was checked, and the ratio of agreement was 58.9 percent. This unsatisfactory result indicates the need for more careful training of student assistants. Although the test score, when used as a power test, is reliable, the test should be used to obtain an introductory diagnosis only, to be improved by observing the student as instruction progresses.

C. The statistical summary of the error diagnosis made of five hundred and twenty-nine test papers is consistent with those of other investigators. (See 1F) Within the limitations of human judgment, random errors were separated from systematic errors, and only systematic errors were recorded. These patterns are generally consistent with clinical interviews and subsequent work performed by the student.

D. The error patterns described were studied in pairs, for significant relationships, by the Chi-Square contingency test. Of seventy-eight paired relationships, forty-one were significant. The interrelationships suggest patterns leading to a two category causal theory, (a) misunderstanding the nature of the tens number system, and (b) general
personality factors. This theory is consistent with the work of Fernald and others. (See 1A)

E. Many students were deficient in their mastery of the basic tables of fundamentals, particularly in subtracting from numbers larger than twelve, and in the 7, 8 and 9 facts in multiplication and division with higher numbers.

3. Conclusions from the Classroom Teaching Experiment:

The two experimental classes gained 13.9 points on the Cody High School Diagnostic Arithmetic Test by the end of the semester, and gained .55 months in grade equivalent on the Iowa Every Pupil Test of Basic Skills, Part D. The control group gained 1.2 points on the Diagnostic Test, a non-significant gain.

4. Conclusions from the Clinical Teaching Experiment:

A. Twenty-two cases taught in the pilot study had a mean gain of 17.4 points on the Cody High School Diagnostic Arithmetic Test. Most of these pupils had taken part in the classroom teaching experiment and the gains were increments over the first semester's work. Eleven of these, chosen for comparison to a control group for which similar data were available, showed a mean gain of 14.5 points compared to a gain of 3.1 for the control group.

B. Twelve cases are described in Chapters V and VI (and Appendix C) in varying detail, indicating both common and rare patterns experienced in the conduct of the study. Four prominent patterns found in most of the cases were:
a. Deficiencies based solely on gaps in understanding the number system, with no complicating emotional factors.

b. Spuriously low achievement scores with large numbers of random errors resulting from adolescent conflicts.

c. Insecurity, coupled with many gaps in understanding, and poor control of the fundamental tables of facts. Emotional problems did not change the method of instruction, and success aided greatly in self-confidence.

d. Gains from remedial instruction, much of which could not be demonstrated on objective tests because of nervousness in the test situation.

C. Fifty-nine cases were closed in the final phase of the clinical teaching study, and ten were partially completed. The closed cases had a mean gain of 19.1 points on the Cody High School Diagnostic Test (power scores) with individual gains ranging from one to fifty-three points. Forty-six of these cases had a mean gain of 23.3 points on the same test, in speed scores. The closed cases had a mean number of 6.0 diagnostic items before instruction and 1.1 items after instruction. All of these figures are very significant. The mean power score of the retest was 73.1, comparable to the mean for the previous year's Algebra students' of 73.8. The classroom grade distribution of forty-three students discharged from the Mathematics Clinic eight weeks before the end of the semester reveals only eight failures, six of which were attributed to personality problems. The mean attendance of the closed cases was 17.2 forty minute lessons.

D. Individualized instruction, dealing with all types of student deficiencies, was much more effective than instruction of large classes
with corrective exercises. This was particularly noticeable in those aspects of behavior which can be roughly classified under morale. Gains in self-confidence, and increased liking for mathematics were evident in both situations, but much more so in the Mathematics Clinic.

E. The use of student assistants as clinical teachers in the pilot study was unsuccessful, largely because the students, although capable in computation, did not understand the meaningful aspects of the number system. They were used successfully in the final phase of the study, after a brief training period.

F. A group diagnosis of the fifty-nine closed cases in the final study was prepared for purposes of comparison with the group diagnosis of the testing survey. As expected, most items had a higher percent of incidence than in the Overall Student Body examined in the testing survey. However, seventeen items were noted as no different than the larger group. These could be classified as errors of omission, insertion of unnecessary processes, or reversals of association, such as 7x8 equals 63 instead of 56. Errors in carrying in addition or multiplication were revealed as functions of the entire student body, rather than of the remedial group. This aspect of arithmetic assumes quite different proportions than suggested by previous studies.

G. In view of the previous statement, the percent of the total of the following three categories was calculated for the Overall Student Body and the fifty-nine closed cases in the final phase of the study. The percentage was 79.8 for the large group and somewhat less for the remedial groups. This information has great implications for preventive instruction. The categories are:
1. The use of zero in division and multiplication.
2. The borrowing process in all kinds of subtraction.
3. Understanding and use of the decimal point in all four processes.

In addition to the specific observations reported, some important qualitative observations are listed:

A. The earlier clinical instruction is given, after admission to the senior high school, the better.

B. Increased self-confidence, changes in personal self appraisal, and increased liking for mathematics were evidenced by many students participating in all phases of the study, but particularly so in the Mathematics Clinic.

C. The observations made of students attending the Mathematics Clinic, and the inference made regarding the causes of the various deficiencies were generally consistent with the work of Fernald with younger children. However, it was not necessary to make as extensive use of concrete teaching aids as she describes. Also, many emotional reactions which appear to have their roots in adolescent conflicts are not described in literature concerning younger children.

D. Some students cannot be required to complete an examination in a specified time without undesirable outcomes. Such students get nervous, make unusually large numbers of random errors, and sometimes regress to earlier patterns of wrong response, even though they know better. The same students, without time pressure, often do very well on the same test.

E. Examination of the group diagnosis indicates that the program of instruction described in this study does not deal with the
arithmetic deficiencies of the students in the "fast" and "average" classifications. Most of these students exhibit from one to four diagnostic deficiencies. One of the major unanswered questions is that of a program for these students. The final phase of the study, through inaccuracies in testing, included three students who attended the Mathematics Clinic for one or two days, and eliminated one or two deficiencies in that time. It seems impractical to consider small group arrangements for individualized teaching for the bulk of the students in a public school.

On the basis of the numerous conclusions presented in these pages, the author wishes to add some inferences which seem quite reasonable, but must be subject to future testing:

A. For purposes of this study, group size was limited to six or seven students. It was the experience of the author that this was the largest number he could deal with effectively in a forty minute period. This should not be a fixed conclusion. It was merely the mode of operation of a particular teacher in a very limited type of operation. The literature suggests that a broader program of instruction, and varying methods of organization would make it possible to individualize instruction in larger classes.

B. Considering that the large majority of the cases taught in the Mathematics Clinic were dealt with at the symptomatic level, it appears that a teacher does not have to be highly trained in clinical psychology to do this kind of work. The essential requirements are some competence in the teaching of arithmetic, sufficient human understanding to be able to infer nervousness, hurrying, "freezing", or other natural, emotional responses from a student's work, and the ability to interpret these inferences to the student in everyday language.
C. The concept of the slow learning child must be greatly modified. A large number of the children who took part in clinical instruction were not mentally retarded at all. Diagnosis and remedial instruction made it possible for them to compete with "average" students. Many of the others were able to exhibit mastery of arithmetic, and their inability to compete had more to do with reaction to time pressure than to basic intelligence or control of the subject. The difference between the individual who is generally low in intelligence and the individual who is nervous, insecure, and unable to respond to social pressure, is difficult to tell and little understood. With our present knowledge, it is next to impossible to diagnose a ninth grade child as so retarded that he is unteachable, barring organic complications.

D. A program for elementary schools which would help to reduce the incidence of arithmetic deficiencies in students entering high school should have two aspects. One is the shift in emphasis from rote to meaningful teaching, with special emphasis on a meaningful understanding of the use of zero, borrowing in subtraction, and the use of the decimal point in all basic processes. A second point should be the use of remedial teaching from the third grade up, whenever appropriate.

E. The program for teaching General Mathematics at the ninth grade level could well omit many aspects of review arithmetic in current practice, and substitute corrective exercises for the commonest diagnostic deficiencies. If possible, a portion of the time, if not all of it, should be devoted to individualized instruction.
Implications for further research:

The most serious limitation of the instructional program described in these pages is the narrowness of an arithmetic program which limits itself primarily to computation. The Mathematics Clinic was limited to instruction in computation, although the general mathematics program of Cody High School is not. It is important to examine the possibility of diagnostic and remedial work in other aspects of arithmetic. Important areas for examination are the understanding of simple measurement, including the reading of various scales; the understanding and use of percentage, and the applications of arithmetic to various verbal problems. The latter is most important, and the problem with which teachers have expressed the greatest concern. Such research would not only formulate individual teaching procedures, but could have vast influence on classroom practice.

The test diagnoses of Chapter IV revealed that a problem exists concerning the deficiencies of students classified as "average" and "fast", who would be unaffected by the kind of clinical program undertaken in this study. Work is needed which would define an instructional program to cure these incorrect behavior patterns.

It is clear that a system of individual diagnosis, followed by individualized teaching in a small group is a workable system for promoting growth in arithmetic and in some related personality areas. It is likely that such a system is superior to other methods. It is also likely that the pupil teacher relationship is a major factor which operates in promoting the growth described. What needs clarification is
the precise nature of the psychological mechanisms operating in these situations. Numerous hypotheses could be tested by properly established experiments, of which a few are suggested:

a. The pupil is aided in growth by becoming aware of his behavior and of its consequences. For example, the sloppy pupil sees errors in his arithmetic directly traceable to sloppiness.

b. The pupil is aided in growth by a change in personal self-appraisal. Many pupils who regarded themselves as "dumb" changed their feelings to some extent.

c. Greater permissiveness in the instructional arrangements promotes growth. The pupil who sees that an activity is a means to improvement rather than a means of satisfying authoritative requirements is more likely to gain from the activity.

d. The opportunity for pupil teacher relationships in a small group gives the pupil a means for satisfying basic personality needs, such as the need for a parent substitute, the object of an adolescent crush, or an adult who encourages the need for independence from parental influence when appropriate. Also, individualized attention meets the need for a feeling of self importance.

It is likely that some, or all of the factors proposed operate in such situations and the problem of separating them out for study is a difficult one.

Much reference has been made in the text to the less tangible items, loosely classified under morale. In addition to increased control of the subject matter, easily demonstrated in statistical tables, there have been described remarkable changes in the feelings and attitudes of
students, expressed both verbally and in social behavior. Many students acquired a liking for a previously hated subject. Many acquired feelings of greater self-confidence and of liking for school as a whole. As the achievement and morale of students rose, so did the morale of the teacher increase. As the semester progressed, the same teacher who had been telling students that he understood their problems and thought he could help them, found himself telling students that he had helped many others and knew he could help them. This seemed to have the effect of the expanding cycle. The more confident the teacher, the more hopeful, more eager, and more confident the students. This is one more indication that the problems of education are only partly the problems of the intellect. They are much more the problems of the human spirit.
Appendix A

MANUAL FOR CODY HIGH SCHOOL DIAGNOSTIC ARITHMETIC TEST

This manual is written for the Form B (Revised) version of the test. A form A, for which the same instructions hold, has also been prepared. The test is shown on pp. 185-186, with solutions written in.

The test may be used as a 40 minute test for ninth grade students, but is more reliable as a power test. All the work should be done on the paper or diagnosis is seriously impaired, if not impossible. Pages 182-184 describe a diagnostic check list of the most common error patterns, and a few general comments about possible causes. Note that random errors are likely to increase toward the end of the test, especially when used as a timed test. Students tend to hurry for fear of not finishing, and fatigue factors enter in as well. This check list is by no means exhaustive, although it probably describes over 90% of the error patterns found in clinical teaching.

I. Scoring: The numbers 1 or 2 appear to the right of each problem, indicating the score for each problem. One point problems are right or wrong. Two point problems involve fractions or decimals. One point is for correct handling of decimal points, or for correct reduction of fractional answers. If a problem is done incorrectly, but these phases are handled correctly, one point is scored. For instance, in problem IA, the solution

\[
\begin{align*}
9.8 & \\
8.9 & \\
5.2 & \\
62.3 & \\
86.3 &
\end{align*}
\]

would score one point, since the correct answer is

170
An answer of 862 would also score one point. In problem ID a solution of
\[
\begin{array}{c}
8.6 \\
4.13 \\
16 \\
\hline
12.89
\end{array}
\]
would score one point for correct addition.

In problem IG, the solution
\[
\begin{array}{c}
1 = \frac{4}{12} \\
3 = \frac{9}{12} \\
\frac{3}{12} = \frac{3}{12} \\
- \frac{5}{6} = \frac{10}{12}
\end{array}
\]
would score one point, since the incorrect sum was correctly reduced.

These examples are sufficient to illustrate the method of scoring. The maximum score possible is 88 points. In general, a score of less than 50 indicates serious remedial problems, and a score of under 65 has remedial aspects.

The discussion which follows describes the various diagnostic inferences which can be made from the test paper. The reader is advised to place the test paper and check list alongside the discussion, else the text will be largely meaningless, or require the reader to do much unnecessary shuffling of pages.

Diagnosis: In general, an error pattern should not be diagnosed from a single problem. Two problems are enough for a tentative statement, except for the items under

I General Comments:

a. This may turn up on IC, ID, and the first eight problems under III, multiplication. It should appear on 3 or more items before
it is considered serious.

b. This does not turn up often and is generally transient behavior.

c. Many people do sloppy work and get correct answers.

The criterion for this item should be that wrong answers are directly traceable to the lack of neatness.

d. This will generally be seen in copying answers from the problem to the space marked answer. Other wrong answers may be due to this behavior, but it is difficult to trace.

e. If the student does not complete the test beyond IV J in 40 minutes, or omits over 5 questions anywhere, this diagnosis should be made.

II Adding whole or decimal numbers:

a. The correct answers to IA and IB are 86.2 and 115.932. Answers of 85.2 and 115.832 indicate carrying 1 instead of 2. An answer of 116.932 for IB and $45.95 for IC (correct answer $45.85) indicates a tendency to carry 1, when not needed. There are many variations of this behavior possible on the test. If tentatively diagnosed, the student should be given an exercise of at least 10 problems for further diagnosis.

b. Answers of 84.2 and 105.712 (or variations thereof) to IA and IB, respectively, indicate this behavior.

c. This should show up on IA, IB, IF. It should appear on all three before a definite diagnosis is made.

d. The following solutions to IC and ID are common, and indicate this behavior:  

<table>
<thead>
<tr>
<th></th>
<th>IC</th>
<th></th>
<th>ID</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7.80</td>
<td></td>
<td>8.6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.85</td>
<td></td>
<td>4.13</td>
<td></td>
</tr>
<tr>
<td></td>
<td>36</td>
<td></td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td>12.89</td>
<td></td>
</tr>
<tr>
<td>$10.21</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
If the student has the above solution for ID but has done IC correctly, check IIE and IIF before a diagnosis is made. Sometimes the following solutions appear: IC 7.80 ID 8.6 In this case, the diagnosis is the same, but the misunderstanding is more deep seated.

- This is a rarer item, and may be reflected in wrong answers to IE and IF which conform to no other pattern. More exercises must be done for a check.

III Adding fractions and mixed numbers:

a. Answers of $\frac{28}{12}$ to IG, $\frac{37}{20}$ to IH, $17\frac{23}{20}$ to II, and/or $17\frac{6}{14}$ to IJ, and $22\frac{18}{17}$ to IL, indicate this behavior. However, this is not as serious as incorrect reduction, such as: IG $\frac{26}{12} = \frac{21}{4}$, or $\frac{4}{12}$

$$\text{IH} \quad \frac{37}{20} = \frac{17}{17} \text{ or } \frac{17}{20} \text{ or } \frac{14}{5}$$

$$\text{II} \quad 17\frac{23}{20} = 17\frac{13}{20} \text{ or } 18\frac{1}{13} \text{ or } 1\frac{13}{20}$$

$$\text{IJ} \quad 16\frac{20}{14} = 17\frac{1}{6} \text{ or } 1\frac{6}{14} \text{ or } 17\frac{6}{14}$$

or III I, $\frac{15}{32} = \frac{2\cdot2}{32} \text{ or } \frac{2\cdot2}{15}$.

There are many variations of this type of response, each traceable to a pattern which is logical, even though incorrect.

b. This behavior must be spotted as part of a larger problem, and often appears sporadically. For example:
c. The following answers reveal this behavior:

\[ \begin{align*}
I & \quad \frac{13}{20} \\
I & \quad \frac{13}{20} \\
I & \quad \frac{13}{20} \\
I & \quad \frac{13}{20} \\
I & \quad \frac{13}{20} \\
\end{align*} \]

\[ \begin{align*}
J & \quad \frac{1}{7} \\
J & \quad \frac{1}{7} \\
J & \quad \frac{1}{7} \\
J & \quad \frac{1}{7} \\
J & \quad \frac{1}{7} \\
\end{align*} \]

\[ \begin{align*}
K & \quad \frac{1}{10} \\
K & \quad \frac{1}{10} \\
K & \quad \frac{1}{10} \\
K & \quad \frac{1}{10} \\
K & \quad \frac{1}{10} \\
\end{align*} \]

d. This behavior shows up in two ways:

\[ \begin{align*}
IG & \quad \frac{1}{3} \\
IG & \quad \frac{1}{3} \\
IG & \quad \frac{1}{3} \\
IG & \quad \frac{1}{3} \\
IG & \quad \frac{1}{3} \\
\end{align*} \]

\[ \begin{align*}
2 & \quad \frac{1}{4} \\
2 & \quad \frac{1}{4} \\
2 & \quad \frac{1}{4} \\
2 & \quad \frac{1}{4} \\
2 & \quad \frac{1}{4} \\
\end{align*} \]

\[ \begin{align*}
3 & \quad \frac{1}{12} \\
3 & \quad \frac{1}{12} \\
3 & \quad \frac{1}{12} \\
3 & \quad \frac{1}{12} \\
3 & \quad \frac{1}{12} \\
\end{align*} \]

\[ \begin{align*}
5 & \quad \frac{5}{6} \\
5 & \quad \frac{5}{6} \\
5 & \quad \frac{5}{6} \\
5 & \quad \frac{5}{6} \\
5 & \quad \frac{5}{6} \\
\end{align*} \]

\[ \begin{align*}
12 & \quad \frac{12}{25} \quad \text{or} \quad \frac{12}{12} \\
12 & \quad \frac{12}{25} \quad \text{or} \quad \frac{12}{12} \quad \text{(Sometimes the latter answer is made equal to 1.)} \\
\end{align*} \]
IV Subtracting whole or decimal numbers:

a. This behavior may show up as an omission.

For example:  

\[
\begin{array}{c}
78.904 \\
-18.635 \\
\hline
60.269
\end{array}
\]

b. Incorrect borrowing across 0's takes on two forms:

\[
\begin{array}{c}
7.81 \\
18.635 \\
\hline
60.179
\end{array}
\quad \text{or} 
\begin{array}{c}
78.9004 \\
18.635 \\
\hline
60.1929
\end{array}
\]

This latter is rare and does not appear on the test. It is usually matched with other borrowing difficulties and will show up in clinical teaching.

c. Same comments as IIC.

d. This will show up in:

\[
\begin{array}{c}
164.3 \\
-2.8 \\
\hline
161.5
\end{array}
\quad \text{or} 
\begin{array}{c}
164.3 \\
-28 \\
\hline
164.02
\end{array}
\]

and

\[
\begin{array}{c}
53 \\
-7.6 \\
\hline
45.4
\end{array}
\quad \text{or} 
\begin{array}{c}
53 \\
-5.3 \\
\hline
47.7
\end{array}
\]

53 or 2.7, 23 or 2.3, 52.24.

e. This behavior shows up in the following:

\[
\begin{array}{c}
7.4 \\
-0.68 \\
\hline
6.88
\end{array}
\quad \text{and} 
\begin{array}{c}
71.18 \\
-7.1225 \\
\hline
64.0625
\end{array}
\quad \text{IIF} 
\begin{array}{c}
53 \\
-7.6 \\
\hline
45.4
\end{array}
\]

and in

\[
\begin{array}{c}
37 \\
-4\frac{1}{3} \\
\hline
32\frac{1}{3}
\end{array}
\quad \text{IIJ} 
\begin{array}{c}
17 \\
-9\frac{1}{4} \\
\hline
8\frac{1}{4}
\end{array}
\]

The behavior described in IIC and IID is usually accompanied by that shown in IIH and IIJ, but the reverse is not necessarily true.

f. This behavior shows up in the following:

\[
\begin{array}{c}
6 \text{ ft.} \\
1 \text{ ft.} \\
5 \text{ ft.}
\end{array}
\quad \text{IIL} 
\begin{array}{c}
1 \text{ in.} \\
9 \text{ in.} \\
5 \text{ in.}
\end{array}
\quad \text{IIM} 
\begin{array}{c}
\frac{1}{2} \text{ hrs.} \\
2 \text{ hrs.} \\
3 \text{ hrs.}
\end{array}
\quad \text{or} 
\begin{array}{c}
\frac{1}{2} \text{ min.} \\
50 \text{ min.} \\
70 \text{ min.}
\end{array}
\quad \text{IIM} 
\begin{array}{c}
\frac{1}{2} \text{ min.} \\
50 \text{ min.} \\
70 \text{ min.}
\end{array}
\quad \text{or} 
\begin{array}{c}
4 \text{ hrs.} \\
10 \text{ min.}
\end{array}
\]
It is clear that the student borrowed 10 inches instead of 12 and 100 minutes instead of 60.

V Subtracting fractions or mixed numbers:

a. This behavior shows up as follows:

\[
\begin{align*}
\text{II G } & \frac{8^2}{3} \quad \text{II I } \frac{8^5}{3} \\
& \frac{2^5}{6} \\
& \frac{6^2}{6} \text{ (2 from 5)} \quad \frac{6^4}{6}
\end{align*}
\]

\[
\begin{align*}
\text{II K } & \frac{28^3}{8} = \frac{27^3}{8} \\
& \frac{-9^2}{4} \quad \text{OR} \quad \frac{-9^2}{4} = \frac{-9^2}{4} \\
& \frac{18^10}{8} = 19\frac{1}{4} \quad 18^8 = 19 \quad \text{or} \quad 18^8 = 20
\end{align*}
\]

b. This behavior will appear as follows:

\[
\begin{align*}
\text{II G } & \frac{8^2}{3} = \frac{7^1}{4} \quad \text{or} \quad \text{II K } \frac{7^1}{4} \\
& \frac{2^5}{6} = \frac{2^5}{6} \\
& \frac{5^9}{6} = 6\frac{1}{2} \quad \frac{18^7}{8}
\end{align*}
\]

c. This behavior will appear as follows:

\[
\begin{align*}
\text{II G } & \frac{8^2}{3} = \frac{8^4}{6} \quad \text{or} \quad \text{II K } \frac{28^3}{8} = \frac{28^3}{8} \\
& \frac{-2^5}{6} = \frac{-2^5}{6} \\
& \frac{6^1}{6} \quad \frac{19^3}{8}
\end{align*}
\]

VI Multiplying with whole numbers and decimal numbers:

a. This behavior will appear as follows:

\[
\begin{align*}
\text{III A } & 700 \\
& 30 \\
& 2100 \\
\text{III B } & 640 \\
& 320 \\
& 1280 \\
& 1920 \\
& 20480
\end{align*}
\]
b. This behavior will appear as:

\[
\begin{array}{ccc}
\text{III B} & 640 & \text{III C} & 80 \\
320 & 240 & 201 \\
12800 & 3200 & 402 \\
1920 & 160 & 402 \\
32000 & 4800 & 2412 \\
\end{array}
\]

or in an exercise, as

\[
\begin{array}{c}
320 \\
12800 \\
1920 \\
32000 \\
\end{array}
\]

or in an exercise, as

\[
\begin{array}{c}
201 \\
402 \\
2412 \\
\end{array}
\]

c. While difficult to trace this behavior on a single problem, note that III A, B, C, and D contain none of these items, and III E, F, G, and H do. If a student has the first four correct and the latter four (or three of them) wrong, (not including errors in place holders or decimal points), we may infer difficulty in this area. The detective work needed would be to examine a solution such as:

\[
\begin{array}{c}
\text{III F} & 6.71 \\
\text{18} \\
\end{array}
\]

If we compare this to the correct solution:

\[
\begin{array}{c}
6.71 \\
18 \\
\end{array}
\]

it is clear that the error is in the first line. We can now infer that the student must have figured \(8 \times 7 = 63\), since the rest of the solution is consistent with such an error.

d. and e. This also requires detective work. For instance:

\[
\begin{array}{c}
\text{III G} \\
\text{Wrong} \\
\text{Right} \\
76.5 \\
53.55 \\
688.5 \\
1520 \\
225.805 \\
\end{array}
\]

\[
\begin{array}{c}
76.5 \\
53.55 \\
688.5 \\
1530 \\
227.205 \\
\end{array}
\]

The numbers circled show two omissions in carrying.
OR IIIF  | Wrong | Right
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6.71</td>
<td>6.71</td>
<td>18</td>
</tr>
<tr>
<td>53</td>
<td>53 68</td>
<td>68</td>
</tr>
<tr>
<td>121.88</td>
<td>120.78</td>
<td></td>
</tr>
</tbody>
</table>

The numbers circled are one too many after a product which did not require carrying. It is easy to infer that the student "carried" one.

OR III G  | 76.5 |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2.97</td>
<td></td>
</tr>
<tr>
<td>68.5</td>
<td></td>
</tr>
<tr>
<td>1530</td>
<td></td>
</tr>
<tr>
<td>227.105</td>
<td></td>
</tr>
</tbody>
</table>

In this case, noting the circled number, we infer that the student carried 3 instead of 4.

This type of error usually occurs because the student tries to remember the number carried without writing it down. While this is a satisfactory procedure for most people, teachers are making a mistake if they insist that everyone carry mentally. Carrying errors often vanish when the student goes back to writing the numbers down.

g. This behavior shows up as follows:

III B  | 640 |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>320</td>
<td></td>
</tr>
<tr>
<td>1280</td>
<td></td>
</tr>
<tr>
<td>205440</td>
<td></td>
</tr>
</tbody>
</table>

III H  | 30.29 |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>69</td>
<td></td>
</tr>
<tr>
<td>272.61</td>
<td></td>
</tr>
<tr>
<td>2090.01</td>
<td></td>
</tr>
</tbody>
</table>

The circled numbers show that the student must have "said"
9 x 0 2 = 11
and 6 x 0 1 = 7

VII Multiplying fractions and mixed numbers:

a. This can be seen in:

III I  | \( \frac{5}{8} \times \frac{3}{4} = \frac{5}{8} \times \frac{3}{4} = \frac{5}{6} \)
|---|---|
| or \( \frac{2\frac{1}{3}}{3} = \frac{6}{4} = \frac{11}{5} \)
| or \( \frac{\frac{8}{3}}{5} = \frac{32}{15} = \frac{2}{15} \)
b. This can be observed as:

\[
\begin{align*}
\text{III I} & \quad \frac{5}{8} \times \frac{3}{4} = \frac{15}{32} = \frac{32}{15} = 2 \frac{2}{15} \\
& \quad \text{or} \quad \frac{15}{32} = 2 \frac{2}{15}
\end{align*}
\]

c. This type of behavior does not show up on the test, but will appear in classwork as follows:

\[
\frac{4 \times 8^2}{5 \times 9} = \frac{2}{45}
\]

This type of behavior is rare.

d. This pattern will appear as follows:

\[
\begin{align*}
\text{III K} & \quad \frac{43}{5} \times 4 = 16 \frac{3}{5} \\
\text{III N} & \quad \frac{75}{6} \times \frac{13}{5} = 7 \frac{5}{30} = 7 \frac{1}{6}
\end{align*}
\]

VIII Dividing whole or decimal numbers:

a. Omission of 0's turns up as follows:

\[
\begin{align*}
\text{IV A} & \quad \frac{0.4}{7} \text{ or } \frac{840}{648240} \text{ or } \frac{804}{648240} \\
\text{IV D} & \quad \frac{7.2}{54} \text{ or } \frac{67962}{67962}
\end{align*}
\]

b. Subtraction errors in division must be found by detective work with individual problems. These errors are random in character, unless related to a diagnosed subtraction deficiency.

c. The combinations in question show up in IV F, G and H, but not in the other division problems. They usually appear as follows:
d. and e. need no general comment. However, many students will answer IV A, B, C, D, E, F, and G correctly, and answer IV C and/or IV H as follows:

IV C \[ \frac{5.91}{.07} \text{ or } \frac{521}{4137} \text{ or } \frac{591.00}{4137.00} \]

IV H \[ \frac{80}{.009/720} \text{ or } \frac{8}{.009/72} \text{ or } \frac{8.000}{72.000} \]

IX Dividing fractions and mixed numbers:

a. Failure to invert yields the following solutions:

IV I \[ \frac{13}{16} \div \frac{1}{8} = \frac{13}{128} \]

or IV M \[ 8 \div \frac{2\frac{3}{8}}{8} = 8 \times \frac{19}{8} = 19 \]

Some students will do problems like IV I correctly, but fail to invert in problems like IV M. The error can be explained psychologically in terms of the reaction pattern of the student, who feels that he has done one operation to the number by changing it to an improper fraction, and that should be sufficient.

b. Inverting the dividend shows as follows:

IV I \[ \frac{13}{16} \div \frac{1}{8} = \frac{16}{28} \times \frac{1}{8} = \frac{16}{104} = \frac{2}{13} \]

c. Canceling before inverting appears as:

IV N \[ \frac{1\frac{1}{2}}{5\frac{1}{3}} = \frac{2}{2} = \frac{2\cdot3}{2} = \frac{1}{8} \]
d. This behavior appears as:

\[
\text{IV M } \frac{5}{2} \div \frac{2}{1} = 1 \div \frac{5}{1} = 1 \times \frac{1}{5} = \frac{1}{5}
\]

or

\[
\frac{5}{2} \div \frac{1}{3} = \frac{5}{2} \times \frac{3}{1} = \frac{5}{2}
\]

The latter problem type does not appear on the test.

e. Not a very helpful comment, but serves to highlight a situation where everything is wrong for no clear reason.

f. This type of behavior is significant of itself when the student does IV I, J, L and N correctly, but does IV K and/or M as follows:

\[
\text{IV K } \frac{3}{4} \div \frac{1}{4} = 3
\]

\[
\text{IV M } \frac{8}{2} \div \frac{2}{8} = \frac{1}{8} \times \frac{19}{8} = \frac{19}{64}
\]

or

\[
\frac{1}{8} \times \frac{8}{19} = \frac{1}{19}
\]
TO THE STUDENT:
Your test reveals the following types of error which are probably a habit of yours when you do arithmetic:

I General comments:
   a. Does not write numbers in straight columns, leading to error.
   b. Does the wrong operation (adds when it says subtract, etc.)
   c. Sloppiness leads to error.
   d. Copies own numbers incorrectly.
   e. Works slowly and does not finish.

II Adding whole or decimal numbers:
   a. Carrying numbers not there, or carrying wrong number.
   b. Not carrying.
   c. Leaving out decimal point.
   d. Misunderstands nature of decimal point in problems like $7.1 + 3.15 + 16$.
   e. Cannot keep sums in head with long columns of numbers.

III Adding fractions and mixed numbers:
   a. Fails to reduce correct answer to lowest terms.
   b. Changes denominator incorrectly, i.e., $\frac{3}{4} = \frac{3}{12}$.
   c. Forgets the whole number part of mixed numbers.
   d. Adds numerators without changing to a common denominator.

IV Subtracting whole or decimal numbers:
   a. Does not borrow correctly.
   b. Does not borrow correctly across 0's.
   c. Omits decimal point.
   d. Does not understand nature of decimal point.
IV (continued)

e. Brings down figures instead of adding 0's and subtracting:

\[
\begin{array}{c}
\phantom{-123} \\
\underline{- 1.23} \\
\phantom{50} 50.23
\end{array}
\]

\[
\begin{array}{c}
16 \\
\underline{9.3} \\
7.23
\end{array}
\]

f. Does not borrow correctly with denominate numbers.

V Subtracting fractions or mixed numbers:

a. Does not change fractions to a common denominator.

b. Borrows 10 instead of the denominator of the fraction.

c. Subtracts top from bottom instead of borrowing.

VI Multiplying whole numbers and decimals:

a. Does not use 0 correctly with numbers ending in one or more 0's.

b. Does not use 0 as a place holder or omits them.

c. Mixes up 7x8, 7x9 and 6x9.

d. Forgets to carry.

e. Carries numbers not there.

f. Ignores the decimal point.

g. 8x0 equals 8, 2x0 equals 2, etc.

h. Places decimals incorrectly.

VII Multiplying fractions and mixed numbers:

a. Inverts fractions when multiplying.

b. Multiplies fractions correctly, then inverts answer.

c. Cancels numerator with numerator, or denominator with denominator.

d. Does not change mixed numbers to improper fractions, but multiplies parts separately.
VIII Dividing whole and decimal numbers:
   a. Omits 0's needed as place holders.
   b. Subtracts incorrectly.
   c. Mixes up $\frac{7}{56}$, $\frac{7}{63}$, $\frac{6}{54}$ and/or $\frac{8}{64}$, $\frac{9}{72}$.
   d. Omits decimal point.
   e. Does not use decimal rule correctly.

IX Dividing fractions and mixed numbers:
   a. Does not invert.
   b. Inverts dividend fraction.
   c. Cancels before inverting.
   d. Cancels into fractions before changing mixed numbers to improper fractions.
   e. Seems all mixed up.
# CODY HIGH SCHOOL
## Diagnostic Arithmetic Test
### Form B (Revised)

### I. Addition

<table>
<thead>
<tr>
<th></th>
<th>A. 9.8</th>
<th>B. 44.389</th>
<th>C. $7.80 + $1.85 + $36 + 20 cents</th>
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<tbody>
<tr>
<td></td>
<td>8.9</td>
<td>.096</td>
<td></td>
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<tr>
<td></td>
<td>5.2</td>
<td>-8.205</td>
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</tr>
<tr>
<td></td>
<td>62.3</td>
<td>63.242</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>(2 pts.)</td>
</tr>
<tr>
<td>ans. 86.2</td>
<td>ans. 115.932</td>
<td>ans. $45.85</td>
<td></td>
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| D. 8.6 + 4.13 + 16 | E. 98 | F. 6.8 | G.  
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<tr>
<td></td>
<td>27</td>
<td>.6</td>
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<td>43</td>
<td>3.5</td>
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<td>72</td>
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<td>86</td>
<td>71.4</td>
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<td>38</td>
<td>1.5</td>
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<td></td>
<td>69</td>
<td>2.6</td>
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<td>62</td>
<td>8.9</td>
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<td></td>
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<tr>
<td>(2 pts.)</td>
<td>(2 pts.)</td>
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<tr>
<td>ans. 28.73</td>
<td>ans. 495</td>
<td>ans. 101.7</td>
<td>ans. 21/6</td>
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<td>6/5</td>
<td>61/2</td>
<td>2/5</td>
<td>12/18</td>
</tr>
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<td></td>
<td></td>
<td>8/2</td>
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<td></td>
<td></td>
<td>8/9</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td></td>
<td>(2 pts.)</td>
<td>(2 pts.)</td>
<td></td>
</tr>
<tr>
<td>ans. 11/20</td>
<td>ans. 18/20</td>
<td>ans. 17/7</td>
<td>ans. 4/10</td>
<td>ans. 15/18</td>
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### II. Subtraction

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<thead>
<tr>
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<th>A. 29.00</th>
<th>B. 78.904</th>
<th>C. 7.4</th>
<th>D. 71.18</th>
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<tr>
<td></td>
<td>-2.75</td>
<td>-18.635</td>
<td>-.68</td>
<td>-7.1225</td>
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<td></td>
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<tr>
<td>(2 pts.)</td>
<td>(2 pts.)</td>
<td>(2 pts.)</td>
<td>(2 pts.)</td>
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<tr>
<td>ans. 26.25</td>
<td>ans. 60.269</td>
<td>ans. 6.72</td>
<td>ans. 64.0575</td>
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<table>
<thead>
<tr>
<th>E. 164.3 - 28</th>
<th>F. 53 - 7.6</th>
<th>G. 65/3</th>
<th>H. 37</th>
<th>I. 85/8</th>
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<td>(1 pt.)</td>
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<tr>
<td>(2 pts.)</td>
<td>(2 pts.)</td>
<td></td>
<td>(1 pt.)</td>
<td></td>
</tr>
<tr>
<td>ans. 136.3</td>
<td>ans. 45.4</td>
<td>ans. 5/6</td>
<td>ans. 32/3</td>
<td>ans. 61/8</td>
</tr>
</tbody>
</table>
CODY HIGH SCHOOL
Diagnostic Arithmetic Test (cont.)
Form B (Revised)

II Subtraction (cont.)

J. 17 - 9
K. 28 - 9
L. 7 ft. 4 in. - 1 ft. 9 in.
M. 6 hrs. 20 min. - 2 hrs. 50 min.

(1 pt.) (1 pt.) (1 pt.)

ans. 7 3/4 ans. 18 5/8 ans. 5 ft. 7 in. ans. 3 hrs. 30 min.

III Multiplication

30 : 320 : 240 : 30.8 : .48

ans. 21,000 : ans. 204,800 : ans. 19,200 : ans. 1641.64 : ans. 4,4496

F. 6.81 : G. 76.5 : H. 30.29 : I. .5 \times \frac{3}{4} : J. \frac{3}{5} \times \frac{10}{12}
17 : 2.97 : 67 :

(2 pts.) (2 pts.) (2 pts.) (1 pt.) (2 pts.)


K. \frac{4}{5} \times 4 : L. 9 \times 4 \frac{5}{9} : M. \frac{1}{3} \times 2 \frac{1}{7} : N. 7 \frac{5}{6} \times 1 \frac{2}{5}

(2 pts.) (2 pts.) (2 pts.) (2 pts.)

ans. 18 \frac{2}{5} : ans. 41 : ans. 5 : ans. 9 \frac{2}{5}

IV Division

A. 7 \div 0.028 : B. 6 \div 0.056 : C. 0.04 \div 2244 : D. 54 \div 37962

(2 pts.) (1 pt.) (2 pts.) (1 pt.)

ans. .004 : ans. 8040 : ans. 56100 : ans. 703

E. 63 \div 12.01 : F. .8 \div 0.056 : G. 7 \div 63.953 : H. .009 \div 72 : I. \frac{13}{16} \div \frac{1}{8}

(2 pts.) (2 pts.) (2 pts.) (2 pts.) (2 pts.)

ans. .19 : ans. .07 : ans. 9.136 : ans. 8000 : ans. 6 \frac{1}{2}

J. \frac{5}{16} \div \frac{3}{4} : K. \frac{3}{7} \div \frac{4}{7} : L. \frac{4}{8} \div \frac{3}{5} : M. 8 \div 2 \frac{3}{8} : N. \frac{11}{2} \div \frac{5}{3}

(2 pts.) (1 pt.) (1 pt.) (2 pts.) (2 pts.)

ans. \frac{5}{12} : ans. \frac{3}{16} : ans. \frac{20}{21} : ans. \frac{7}{19} : ans. \frac{9}{32}
Appendix B
CORRECTIVE EXERCISES FOR ARITHMETIC DEFICIENCIES

Foreword: These exercises were designed to assist in remedial instruction for the behaviors described in Appendix A. Special exercises are not needed for many of the behaviors described, since appropriate practice is to be found in most standard arithmetic books. We have found that these pages, coupled with a 9th Grade General Mathematics Text, will take care of all but a few remedial problems. In these situations the instructor will usually have to make up something on the spot.

Pages 190-193 are designed to provide practice for deficiency in understanding the addition and subtraction of decimal numbers. The student's addition will appear on his test paper as: I D 8.6 + 4.13 + 16

\[
\begin{align*}
8.6 & \\
4.13 & \\
16 & \\
\hline
12.89 & \\
\end{align*}
\]

His subtraction will appear as: II F 53 - 7.6

\[
\begin{align*}
53 & - 7.6 \\
\hline
27 & \text{ or } 5.3
\end{align*}
\]

Page 194 is designed for practice in the following: The student subtracts thus:

II D

\[
\begin{align*}
7.4 & \\
-0.68 & \\
\hline
6.88 & \\
\end{align*}
\]
This page can also be used for students with difficulty in borrowing across 0's.

Page 195 is designed for practice in borrowing fractions in subtraction. The student usually does this:

\[
\begin{array}{ccc}
16 & 1\frac{6}{5} & 1\frac{1}{5} \\
-9\frac{3}{5} & -9 & -9\frac{3}{10} \\
\hline
7\frac{2}{5} & 6\frac{8}{5} = 7\frac{2}{5} & 6\frac{8}{10} = 6\frac{4}{5}
\end{array}
\]

or variations thereof. Note that each problem in II and III is a slight variation of the corresponding problem in I, adding a slight complication with each successive group. This has been found to be a definite aid in instruction.

Page 196 is designed for practice in borrowing denominate numbers in subtraction. The student usually does this:

\[
\begin{array}{c}
\frac{6}{7} \text{ ft.} \quad \frac{1}{4} \text{ in.} \\
\frac{1}{5} \text{ ft.} \quad \frac{9}{5} \text{ in.} \\
\frac{5}{5} \text{ ft.} \quad \frac{5}{5} \text{ in.}
\end{array}
\]

Page 197 is designed to aid in correcting a variety of behaviors, described at length in Appendix A, III.

Pages 198-199 are designed to aid in correcting the following:

\[
\begin{array}{c}
\frac{308}{405} \quad \text{or} \quad \frac{180}{40} \\
\frac{1540}{1232} \\
\frac{13860}{720}
\end{array}
\]

It is advisable to assign no more than 10 problems at a time. Mastery will show up in 10 problems, and a lack of it will be aggravated by doing more than 10.

Pages 200-202 are designed to aid in correcting many behaviors described at length in Appendix A, IX.
Page 203 is designed to aid in mastery of the decimal rule in division. Estimating is a useful tool to aid in the process. The instructor must be careful, however. Some students are confused rather than aided, by estimating.

Page 204 is designed to aid the following: The student omits the zero place holders and gets $\frac{6.4}{4228}$ instead of 604. Every problem on this page has an answer with zero place holders.

Page 205 is different than page 203 in that it is aimed at this specific behavior:

\[
.07 \div 630 \quad .07 \div \underline{63.0} \quad \text{instead of 9000.}
\]

Page 206 is designed for students who usually omit this step for lack of understanding, or who fail to realize that two steps are involved. The student will show:

\[
\frac{45}{6} = \frac{72}{6} \quad \text{or} \quad \frac{45}{6} = \frac{15}{2}.
\]

Also, some students reduce "upside down", as

\[
\frac{5}{\underline{6}} = \frac{14}{\underline{9}} \quad \text{or} \quad \frac{14}{\underline{9}}.
\]
CODY HIGH SCHOOL
Remedial Mathematics - Class Exercise

Adding Decimal Nos.

1. $5.01 + .0074 + 6.9 + 71.
2. $38.03 + .52 + .064 + 9.
3. $973.22 + .65 + .0015 + 25.
4. $49.17 + $.78 + $5 + $15.07.
5. 2744 + 73.27 + .02462 + 73.69.

6. $4.85 + .0026 + 9.5 + 56.
7. $53.22 + .58 + .074 + 8.
8. 314.46 + .82 + .0037 + 31.
9. $29.63 + $.67 + $9 + $67.65.
10. 5469 + 23.16 + .08623 + 48.29.

11. A teacher gave a class of math students a test of ten problems. Since the class was small they were timed. Joe Smith 13.5 minutes, Alice Jones 12.5 min., Charlie Green 7.8 min., Jim Branen 11.7 min., Sylvia Brown 9.8 min., Jill Mack 6.5 min., Margie Byrne 11.7 min. What was the average time for the class?

12. Four boys were planning a camping trip and needed some equipment, so they bought the following items: 1 tent $8.95, 4 sleeping bags $50, 1 flashlight $3.07, 1 jackknife $2.50, 4 jackets $20.00. What was the total cost? (Add Michigan sales tax.)

13. An interior decorator was buying material to be used in making slip covers for three couches and three chairs. He bought 10 yards of one kind of material at $15.61 a yard; 20 yards of another type of material at $17.81 a yard; 5 yards of another at $6.50 a yard, and 10 yards of still another type at $20.55 a yard. How many yards did he buy altogether? What was the total cost? (Add sales tax.)

Subtracting Decimal Nos.

1. $615.98 - 88.78.
2. 986 - .650.
3. 198 - 56.987.
4. 868.87 - 516.89.
5. 567 - 19.856.

6. 567.94 - 59.78.
7. $815.23 - $65.98.
8. 9578.26 - 657.84.
10. 156 - .068.
CODY HIGH SCHOOL
General Mathematics - Class Exercise

Adding Decimal Nos.

ADD
1. \[2.73 + .0589 + 4.6 + 25\].
2. \[43.71 + .97 + .005 + 40\].
3. \[468.73 + .653 + .0094 + 73\].
4. \[\$94.01 + \$0.72 + \$3 + \$725.25\].
5. \[6542 + 65.21 + .05601 + 42.98\].
6. \[5.89 + .273 + 6.4 + 52\].
7. \[71.43 + .79 + .007 + 50\].
8. \[687.34 + .356 + .0049 + 37\].
9. \[\$49.10 + \$0.27 + \$5 + \$7.25\].
10. \[5426 + 21.56 + .01065 + 94.82\].

11. A tourist souvenir hunter went into a shop and bought a vase for \$3, some jewelry for \$2.79, a fancy shirt for \$6.23, and pennant for 25 cents. How much did he spend? How much did it come to after the Michigan sales tax was added?

12. Mrs. Jones went to a nursery and bought 3 small shrubs for \$4.75 apiece. She also bought some rose bushes at \$5.32 altogether. How much did she spend? (Add sales tax.)

13. A teenage girl made out her budget for the week. She decided to spend \$4.78 on school books, \$2.50 on lunches, and \$1.07 for extras. By the end of the week she spent \$9.00 altogether. How much more did she spend than she allowed for?

14. A hardware merchant cut a piece of pipe 2.7 inches long, a piece 11.6 inches long, and a piece 4.3 inches long. How much did he cut altogether?

15. Five students worked a set of arithmetic drill problems in 49.6 seconds, 58.2 sec., 51.2 sec., 52 sec., and 60 seconds. What was the average time?

Subtracting Decimal Nos.

1. \[8.465 - 2.09\].
2. \[7.281 - 2.0597\].
3. \[4.0023 - 2.7\].
4. \[9.72 - .00637\].
5. \[7.0032 - 4.2\].
6. \[7.00063 - .792\].
7. \[253 - 7.65\].
8. \[9.0456 - 8.04\].
9. \[765 - 2.53\].
10. \[4.06 - 1.0065\].
CODY HIGH SCHOOL
General Mathematics - Class Exercise

Adding Decimal Nos.

ADD

1. \(9.65 + 0.0368 + 2.7 + 53\).
2. \(14.37 + 0.65 + 0.008 + 0.25\).
3. \(265.21 + 2.41 + 0.0083 + 21\).
4. \($36.17 + $0.99 + $7 + $8.14\).
5. \(7327 + 24.62 + 0.07369 + 24.77\).

11. A lady went into a shoe shop. She bought shoes for each of her 4 children. She was charged $6.71, $5.32, $5.75, and $6.32 for the shoes. What was the total amount spent? How much would it be after the Michigan sales tax had been added?

12. A girl spent $14.95 on a sweater, $5.95 on a shirt, $8.45 for shoes and $3.95 for miscellaneous items. How much did the bill come to? (Add sales tax.)

13. A man paid his monthly bills: $8.95 for the telephone bill, $20.04 for the grocery bill, $2.50 for club dues, and $60.57 for rent. How much did he spend?

14. A tree grew 4.7 feet its first year, 3.6 feet the second year and reached the height of 10.1 feet at the end of its third year. How much did it grow during the third year?

15. 5 boys decided to time their 50 yard dash. The first ran it in 7.3 seconds, the second in 20.2 seconds, the third in 8.4 seconds, the fourth in 9.7 seconds and the fifth in 21 seconds. What was the total time made?

Subtracting Decimal Nos.

SUBTRACT

1. \(923 - 1.06\).
2. \(21.643 - 9.056\).
3. \(8.0021 - .209\).
4. \(279 - 6.32\).
5. \(5.007 - 2.35\).

6. \(4.70 + .7453 + 8.8 + 21\).
7. \(.73 + .001 + 86 + 60.07\).
8. \(41 + 19.76 + .0024 + .34\).
9. \($17.36 + $0.61 + $3 + $2.23\).
10. \(8311 + 86.46 + .07144 + 91.62\).

6. \(421 - 4.55\).
7. \(51.729 - 5.293\).
8. \(9.0025 - .151\).
9. \(825 - 6.84\).
10. \(8.0092 - 7.16\).
CODY HIGH SCHOOL
General Mathematics - Class Exercise

Adding Decimal Nos.

ADD

1. \(6.71 + .0074 + 10.1 + 7.\)  
2. \(64.09 + .85 + .017 + 10.\)  
3. \(297.15 + .001 + .6115 + 18.\)  
4. \($555.61 + $.75 + $10 + $16.07.\)  
5. \(8151 + 65.52 + 0151 + 71.75.\)

11. A landscaper ordered the following material for a job he was about to do for a business firm. 500 lbs. of seed for $92.50, 50 loads of top soil for $700, 25 lbs. of fertilizer for $87.50, shrubbery for $75.81. How much did it cost him? How much did it cost with Michigan sales tax?

12. In a small town election eight men were running for council. Here were the results of the election: John Erie - 58 votes, Jacob Bent - 66 votes, Paul Green - 81 votes, Samuel Brown - 75 votes, J. B. Jones - 45 votes, Roland Stone - 50 votes, Joe Smith - 79 votes, and A. R. Quil - 90 votes. What was the average vote?

13. Six couples were playing at a local tennis court. Each couple was timed for the length of time it took them to play one game. It took the first couple 5 min. 37 seconds to finish. The second couple took 8 min. 55 sec., the third couple took 3 min. 7 sec., the fourth couple 10 min. 3 sec., the fifth couple 6 min. 59 sec. and the sixth couple 7 min. 6 sec. What was the total time it took all the couples to finish? What was the average time?

Subtracting Decimal Nos.

SUBTRACT

1. \(329 - 6.01.\)  
2. \(64.213 - 5.096.\)  
3. \(8.0712 - .902.\)  
4. \(632 - 2.79.\)  
5. \(7.015 - 3.25.\)

6. \(455 - 3.24.\)  
7. \(72.519 - 2.359.\)  
8. \(1.0052 - .911.\)  
9. \(684 - 8.25.\)  
10. \(7.0016 - 2.89.\)
### Subtraction Drill - Decimals

<p>| | | |</p>
<table>
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<tr>
<th></th>
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<td></td>
</tr>
<tr>
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CODY HIGH SCHOOL  
General Mathematics - Class Exercise

Subtracting Denominate Numbers

**SUBTRACT**

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CODY HIGH SCHOOL
General Mathematics - Class Exercise

Drill - Dividing Fractions I

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CODY HIGH SCHOOL
General Mathematics - Class Exercise

Drill - Dividing Fractions II

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4. $7 \div \frac{1}{8}$
5. $5 \div \frac{2}{12}$
6. $2 \div \frac{2}{5}$
7. $2 \div \frac{3}{4}$
8. $10 \div \frac{2}{5}$
9. $2 \div \frac{1}{6}$
10. $5 \div \frac{3}{4}$

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2. $\frac{2}{3} \div 2$
3. $\frac{3}{4} \div 3$
4. $\frac{1}{8} \div 7$
5. $\frac{1}{12} \div 5$
6. $\frac{2}{3} \div 2$
7. $\frac{3}{4} \div 2$
8. $\frac{2}{5} \div 10$
9. $\frac{1}{6} \div 2$
10. $\frac{3}{4} \div 5$

III
1. $\frac{3}{2} \div \frac{1}{2}$
2. $\frac{2}{3} \div \frac{1}{3}$
3. $\frac{3}{4} \div \frac{1}{4}$
4. $\frac{7}{8} \div \frac{1}{8}$
5. $\frac{5}{2} \div \frac{1}{12}$
6. $\frac{2}{3} \div \frac{2}{3}$
7. $\frac{2}{7} \div \frac{3}{4}$
8. $\frac{10}{8} \div \frac{2}{5}$
9. $\frac{1}{6} \div \frac{2}{3}$
10. $\frac{3}{4} \div \frac{5}{8}$
CODY HIGH SCHOOL
General Mathematics - Class Exercise

Short Division Drill

I Divide the following numbers by 4. Then divide by 5. Then by 6, 7, 8, and 9. Then write down the answer for division by 10. TIME YOUR WORK FOR EACH SET.

(1) 46042  (6) 251392
(2) 99580  (7) 226760
(3) 40682  (8) 995411
(4) 88828  (9) 969598
(5) 24538  (10) 769801

II Divide the first number in the following set by 4, 5, 6, 7, 8, and 9. Then go on to the next number and repeat. After you complete the set for all the numbers, write down the answers for division by ten. TIME YOUR WORK.

(1) 76786  (6) 256919
(2) 70737  (7) 844099
(3) 95778  (8) 659120
(4) 74040  (9) 647881
(5) 41718  (10) 710626

III Divide the first number in the following set by 7, 8, 9, 10, 11, and 12. TIME EACH SET.

(1) 367916  (6) 523466
(2) 317902  (7) 666675
(3) 642644  (8) 957889
(4) 401879  (9) 525198
(5) 848588  (10) 921370
FIRST: Round off each number to the nearest whole number, then divide and get an estimate of the answer.

SECOND: Divide the numbers exactly. Then compare your answer with the one you got from part one.

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CODY HIGH SCHOOL
General Mathematics - Class Exercise

Long Division Drill for Zero Placeholders

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CODY HIGH SCHOOL
General Mathematics - Class Exercise

Decimal Division III

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\[ .55 \div 22022 \]
\[ 1.08 \div 1300 \]

\[ .8 \div 6008 \]
\[ .17 \div 3468 \]
\[ .0037 \div 111 \]
\[ 3.14 \div 6280 \]
\[ 7.2 \div 144144 \]

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\[ .006 \div 444 \]
\[ .71 \div 3550 \]
\[ .64 \div 128128 \]
\[ 2.15 \div 2300 \]

\[ .6 \div 5112 \]
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Appendix C

CASE STUDIES

Foreword: It was not considered necessary to describe every case in the study in detail. The twelve cases described in the text are offered here in somewhat greater detail for the reader who wishes to acquire more familiarity with the procedures used.

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Lela W:

Lela was a rather mature girl for her age. She was poised, reserved, but not to the point where one would consider her excessively shy and withdrawn. She was neat, well-groomed, and considered a good citizen by all of her teachers. In our contacts with her outside of class, she seemed to have her normal quota of friendships. In short she appeared as a normal adolescent, with no apparent emotional problems which would complicate learning. Her initial score on a diagnostic test was 53 and she exhibited the following deficiencies:

a. Copies own numbers incorrectly.
b. Omits decimal point in addition.
c. Misunderstands nature of decimal point in problems like 7.1 plus 3.15 plus 16.
d. Fails to reduce fractional answers to lowest terms. (Lela told the teacher that she would know how to do this, but had misunderstood the directions on the test.)
e. In multiplication, does not use zero correctly with numbers ending in one or more zeros.
f. In dividing fractions, she inverted the dividend fraction instead of the divisor.
g. Does not borrow correctly when subtracting denominate numbers.

Initial interview with Lela revealed that items a, b and d above were neither subjects for instruction, nor symptoms of personal maladjustment. a and b were rather the kinds of error expected of normal people who are not conscious of what they are doing or of the consequences.

The first step in instruction was to explain the nature of decimal numbers and to assign a corrective exercise in addition and subtraction. All Lela had to do was to recognize that placing a number like 15 after the decimal point changed the basic nature of the number. She completed the entire exercise in twenty minutes without error. The next step was to explain borrowing in denominate number subtraction. By the end of the next class period, she completed an exercise of thirty problems with two errors.

The following day, the type error Lela had been making with 0's in multiplication were discussed in terms of the tens number system. She grasped the principles readily and completed a practice exercise of ten problems without error. There were a few minutes left in the class period and the instructor checked Lela on problems in the reduction of fractions to lowest terms. She showed competence in the operation and it was decided not to pursue instruction in this area.

The remaining item in the diagnosis concerned the division of fractions. Explaining this computation took a considerable amount of time, since Lela had difficulty in understanding the logic behind the rule. The
clue most helpful to her was the fact that the division symbol means divided by, and the number used to divide by is the one inverted. This is not the most desirable method of learning the concept from the standpoint of meaningful instruction, but it was most helpful in this case. While there was some doubt about the meaningful grasp of the concept, Lela computed a set of sixty problems of varying types including mixed and whole numbers as well as simple fractions. There were no errors in the entire exercise.

This completed the instruction and Lela took the Cody High School diagnostic arithmetic test and scored 81 points out of 87. Examination of her paper reveals the other six points were lost because of random errors. Total time in clinic was five lessons.

Lela requested some additional instruction in percentage, and was retained as a student in the Mathematics Clinic for two additional weeks. The additional instruction was successful, but is not reported in detail, since this phase of the subject matter is considered beyond the scope of the study, and objective materials for evaluation had not been worked out.
2. Leonard L:

Leonard was a slender boy of 15, who presented to the casual observer what could be described as a sloppy appearance. His hair was unkempt, his shirttail often out. He had begun his high school career by being involved in several mishaps which led to disciplinary action. Some of his teachers regarded him as rather difficult to deal with. At the same time the boy exhibited a rather friendly attitude, and on occasion showed a genuine desire to please, coupled with many likeable qualities. Past clinical experience has indicated to the profession that a sloppy appearance is often symptomatic of a lack of self-respect, and may in the case of adolescence be a sign of conflict with the problems of growing up. An examination of Leonard's diagnostic test, with a score of 40, tends to verify such a diagnosis. Consider the items checked off:

a. Misunderstood nature of decimal point in problems like 7.1 plus 3.15 plus 16.
b. Does not use decimal rule correctly in division.
c. Mixes up 7/56, 7/63, 6/54.
d. Fails to reduce fractions.
e. Changes denominator incorrectly.
f. Copies answer incorrectly.
g. Does not invert divisor in dividing fractions.
h. We note, in addition, that the student clerk wrote the following comment on Leonard's paper: "What a mess!"

Discussions with Leonard revealed that of the above items, only the first two were due to misunderstanding. Other errors showed up on his paper which could not be held to any definite pattern, and could be explained only on the basis of sloppiness. Leonard is left-handed, a partial explanation of the slanted columns of numbers on his papers.

We discussed the consequences of sloppy writing with him, and showed him how to use a 3x5 white card as a margin when writing down numbers, so that the vertical columns would be correct. Leonard is quite intelligent, and the first explanation of problems like 7.1 plus 3.15 plus 16 was easy for him to grasp. He proceeded to do a practice exercise of that type, which he completed neatly and with only two errors by the end of the following class period.

The decimal rule in division gave a little more trouble, with incorrect procedures cropping up, even though Leonard appeared to understand the procedure fairly well. His neatness improved greatly and he discarded the 3x5 white card after two days. Several days practice on division resulted in mastery of the deficiency and he was encouraged to take the test again. The score was 81 out of 87 points, with the loss of six points due to random error only. We wish to note that during the period of
instruction, Leonard asked the instructor the following question: "Mr. Bernstein, do you come in extra early to give us kids this special help?" Since we sensed what was on his mind, we lied and answered his question in the affirmative. "Well," he said, "if you can do that for us, I guess we ought to put out for you."
3. Jean G:

This girl had had a very unstable home situation for three years previous to her enrollment at Cody High School. Her parents had been divorced and she had been moved around from one situation to another, attending several different schools over a period of three years. The semester she enrolled in Cody High School was the first stable period in her whole life subsequent to the divorce. She was living with an aunt, who apparently showed genuine interest in the child. Her counselor reported to the instructor that Jean was extremely insecure, and had great fears about her ability. However, she was very anxious to succeed and her insecurity presented no obstacle to her teachers in either a classroom or a clinical situation. The procedure, therefore, was to analyze her weaknesses and faults in arithmetic and deal with these without discussing Jean's emotional problems with her. The relationship was primarily instructional, and assumed therapeutic aspects only in that the situation served to build confidence and self-respect.

It should be noted that Jean scored 30 points out of 87 on the diagnostic test in September, 1953, and 61 points out of 87 on another form at the end of the semester in January, 1954. This improvement was achieved without clinical instruction, in one of the Remedial Mathematics classes conducted in the study. The diagnosis for clinical instruction was taken from the second test and included:

1. Failed to reduce fractional answers to lowest terms.
2. Misunderstood borrowing in the subtraction of fractions.
3. Misunderstood borrowing in the subtraction of denominate numbers.
4. Doesn't place decimals correctly in multiplication.
5. Does not use decimal rule correctly in division.

In addition to the above items, the errors in multiplication suggested difficulty with the basic tables of multiplication and division facts. Jean was given the Detroit Public Schools Test of the basic Subtraction, Multiplication and Division facts, and the times of completion noted. Completion time on the subtraction facts was 4 min. 25 sec., on the multiplication, 5 min. 10 sec., and on the division, 6 min. 25 sec. Since the average student completes these in three minutes without error, her times indicated lack of mastery. Using a technique similar to that described by Fernald, the papers were then examined for patterns of wrong answers. For example, Jean made 19 errors or omissions in the multiplication table. Analysis reveals that 10 of these errors involved multiplying by 0 (2x0 = 2, etc.). Further analysis revealed that the other errors were paired, i.e., 7x8 and 5x7, or that they fitted into the table of multiplying by nine. This kind of analysis was conducted with Jean watching so that the instructor was able to convince her that she only had six facts to master instead of nineteen in order to gain complete control over the table. We were further able to show her that the facts she had wrong in the division table were generally the same ones, i.e., 7 /56, etc. This process of narrowing down the area for drill
and mastery was a boost to the girl's morale, a factor noted in many cases where the problem of the mastery of the tables was involved.

Two programs were used to help Jean with the mastery of fundamental facts. One was the use of flash cards, with a student assistant giving her the drill. The other was to write out the tables of 7, 8 and 9, in the following manner:

77777777 63  
77777777 56  
77777777 49  
77777777 42  
77777777 35  
77777777 28  
77777777 21  
77777777 14  
7 7

Jean would then add the sevens (eights or nines) across and subtract the results successively to be sure the difference was always seven. Jean was also taught to speed up subtraction by the process of bridging tens.

Concurrent with the program of drill in the fundamentals, Jean was taught the four items in the remainder of the diagnosis. Borrowing in the subtraction of mixed numbers was the only concept which gave her trouble, but two exercises were sufficient to give her control over the process.

The combined program took seven weeks, at the end of which Jean was given the diagnostic test and scored 74 out of 87, losing only two points on what could be described as errors of misunderstanding, on a particular kind of decimal division. At appropriate times, she was retested on the fundamental tables, and achieved complete mastery with completion times of better than 3 min. 30 sec. on all three.
4. Jack S:

His examination as of January, 1954, showed a score of 57 points. A diagnosis revealed only two items of misunderstanding:

1. He tended to omit zero as a place holder in long division.
2. He misunderstood the decimal rule in division.

One wonders why Jack should have such a score with only two misunderstandings. His test paper revealed several items where points were lost by omissions, or by such carelessness as copying his own handwriting incorrectly. Subsequent work with Jack revealed that, although he was slow spoken and slow moving, he could do accurate work in a daily class work situation, when not rushed. However, if not pressured somewhat, he would dally and take too long to finish the work. In addition to the two items above, he was found to have weaknesses in the multiplication and division tables. After practice, he showed improvement on these, but when retested, still took 4 or 4 1/2 minutes to complete them. Since he could respond to flash cards automatically and correctly, we concluded that the poor time was a product of the test situation. When Jack took the arithmetic test after ten weeks of work, he scored 68. This time his paper revealed no patterns, and we concluded that the errors were random in character. A loss of 19 points due to random errors is considerable and we encouraged Jack to practice a little further and try the test again. He later scored 66 points on another form, with a pattern of random errors similar to the previous test. Jack's inability to respond to the pressures of the examination must be deduced as the principle limiting factor in his achievement.

Jack testified that he always got nervous when he took tests, in all subjects. He also said that he did better on those items which he had practiced the day before. We observed that Jack tended to reject those parts of an explanation which made a process more meaningful, saying, "I'm only interested in how you do the problem." It has been observed that those students who do so may well have temporary, rather than permanent control over an arithmetic process.
Betty was a tall, stocky girl with a blemished complexion and a somewhat withdrawn personality. She accepted assignment to the Mathematics Clinic readily, and appeared quite willing to try any procedure which was suggested to her. She seemed very nervous and fearful. On one occasion, the instructor sat next to her in a double seat to discuss a process. During the explanation, questions which came naturally out of the problem were asked in an attempt to get Betty involved in the solution. She was either very slow in answering, or completely unable to answer, and was trembling visibly. Apparently, the experience of having a teacher so close was very frightening to her. It took a few weeks before she felt at home in the teacher's presence, and felt able to speak out freely and spontaneously, about subjects other than the work, as well as the day's problems. We were able to make use of a very capable student assistant during this time, Miss J.W., a very understanding girl who often accepted the burden of instruction for Betty, to help her feel at ease.

Betty's score on the diagnostic test was 47 points out of 87, an improvement of 17 points over the test of September, 1953. The improvement had taken place in one of the remedial mathematics classes taught in the previous phase of the study. She still showed the following diagnostic items:

1. Misunderstood nature of decimal point and decimal numbers in addition and subtraction.
2. Failed to reduce fractional items to lowest terms.
3. Did not borrow correctly in subtraction.
4. Borrowed incorrectly in subtraction of denominate numbers.
5. Did not use 0 correctly in multiplication, especially in numbers ending in 0.
6. Placed decimal points incorrectly in multiplication.
7. When multiplying mixed numbers, did not change to improper fractions, but multiplied parts separately.
8. Omitted 0's needed as place holders in division.
9. Omitted the decimal point in division.
10. Cancelled into fractions before changing mixed numbers into improper fractions when dividing.

With so many deficiencies, it was felt advisable to test Betty for control of the tables of fundamental facts. Surprisingly, she showed 100% mastery of the tables, and was slow only on the division tables (four minutes). Another step deemed suitable in such a case was to procure an analysis of her Detroit General Aptitude Test from the psychological clinic. She was listed as an E mental rating, with a retarded visual factor, indicating that motor and aural methods of instruction would be better approaches, when possible.

With these considerations in mind, instruction was begun on the first item in the list. When a dollar sign was placed on the numbers in such
problems, Betty was able to comprehend the nature of the problem and add correctly. However, when we used the language of arithmetic and talked about ones, tens and hundreds to the left of the decimal point, and tenths, hundredths, thousandths, etc. to the right of the decimal point, she had great difficulty in understanding the explanation. She practiced this kind of exercise many times in the course of the semester, leaving it for other work when it appeared that she was blocked and discouraged. Examination of her exercises revealed that she could master the problem if the numbers did not exceed two digits on either side of the decimal point. She was unable, even with considerable help, to comprehend a number like 7,325, whether written with or without a decimal point.

Betty had similar difficulty in comprehending the use of the decimal point in multiplication and division. The work she had done in addition simply confused the issue in multiplication, since she often used the addition rule in multiplication problems.

Betty was able to master the deficiencies in fractions, (items 2, 7 and 10 in the diagnosis) needing approximately two practice exercises for each one. The same was true of the use of 0 in both multiplication and division.

Since mastery of the decimal processes was not achieved, it was decided to keep Betty in attendance, and approach the problem from the standpoint of the applications currently being taught in the author's classes. She did problems on commissions, discount, payroll computation, alternated with attempts at the practices used for the basic decimal processes. She demonstrated some control over the problems, but could not avoid mistakes in decimals, and did not realize when such errors made answers in practical problems quite ridiculous. She remained in the Mathematics clinic for the entire semester, and took the diagnostic test as a final examination, scoring 61 points out of 87, a fourteen point improvement. She did problems in the addition of decimal numbers correctly on this test, but the permanance of the learning is in question, since she had done such problems the day before.

While Betty never achieved real control over abstract decimal processes, it is likely that she improved her control over concrete decimal processes, such as dollars and cents problems. Her chances of holding a job as a salesgirl, for instance, were probably improved. We can safely say, that if the gains in arithmetic are questionable, the gains in human relations, through the relaxation of her fears, are quite positive. Betty's capacity to deal with people was doubtless improved.

After an entire semester of close relationships with this girl, the author is incapable of judging whether her inability to achieve is a product of low ability or of emotional blocking due to fear. The low test score on the General Aptitude Test is not a verification, since it could be a result of blocking.
6. Marlene P:

Marlene's scores had shown an improvement from 17 points to 27 to 44 on the basis of the previous term's class work. The diagnosis for clinical instruction was based on the most recent paper. Examination of the paper revealed the following items:

1. Misunderstood the nature of decimal numbers in addition and subtraction.
2. Made too high a percentage of error in addition, including errors in carrying, and adding numbers in the wrong column.
3. Did the wrong process, usually adding when the problem called for subtraction.
4. Omitted the decimal point, or misplaced it in multiplication.
5. Misunderstood the decimal rule in division.
6. Inverted the dividend fraction instead of the divisor in division of fractions.

Marlene said that she got nervous on tests. Items 2 and 3 above were symptomatic of this problem.

Instruction was begun on the first item and Marlene understood the explanation readily. She went to work on the first corrective exercise, set up every problem correctly, but got the wrong answer on half of them. Examination of the work indicated carrying errors in addition, and failure to borrow in subtraction in problems like

\[
\begin{array}{c}
629 \\
- 8.23 \\
\hline
621.23 \\
\end{array}
\]

Strangely enough, the latter behavior had not turned up on the diagnostic test. Since we inferred that Marlene understood the nature of the decimal system, the next step was corrective work on the two behaviors just described. A practice page in subtraction was completed and was reasonably free of error.

The next step was some practice work in addition, after she was shown the specific errors in carrying column writing. Another exercise was chosen in a drill book, to eliminate the factor of writing the problem, so that she could concentrate on the factor of carrying. Five out of the next ten problems were incorrect because of carrying errors. We noted that she did not write down the numbers being carried, and encouraged her to do so on the next practice set. She still got a majority of the problems wrong. Marlene expressed discouragement at this point, and it was decided to leave this problem for the time and work on something else.
The next item discussed was the decimal rule in multiplication. Marlene again understood readily and proceeded to practice. The practice showed mastery of the decimal rule, but she made errors in the use of 0 in multiplication, an item which had not shown up on the diagnostic test. It was necessary to go over this and have her do some practice work, which she completed without error.

Marlene then returned to the problem of column addition, with results similar to the previous ones. After two days, she left addition again and did an exercise on the subtraction of denominate numbers. Her ability to do these was in doubt since she had added instead of subtracting on the test. She did, in fact, require a brief explanation, and was able to show control over the borrowing process by the end of the first practice.

A return to addition resulted in no gains for the third time. By this time, several weeks had gone by, and it appeared that the instructor was dealing with a pathological problem. Marlene gave the impression of a fairly intelligent girl, who had no more than average difficulty understanding the concepts explained to her, but she could not avoid a high percentage of what appeared to be careless errors, in spite of every effort, every warning, and encouragement.

We abandoned the instructional approach, and tried to examine the problem in terms of her basic emotional problems. On the basis of several interviews, including a long private interview with Marlene, in which she seemed to feel that our analysis of the situation was correct, we wish to hypothesize the following explanation:

Marlene was the youngest of several children, the others having finished school and entered adult life with reasonable success, including a brother who successfully finished engineering school. The parents teased her about being the baby, and she said she liked being the baby. The father wanted her to be good at mathematics, and both parents wanted her to be a nurse. Marlene was well aware that algebra and chemistry were essential to a nursing career. When asked whether she wanted to be a nurse, she responded in the affirmative, but seemed unsure of herself.

Marlene indulged in a good deal of bizarre behavior, including being outspoken at the wrong time (in class) with humorous remarks. She had the knack of imitating the mannerisms of Red Skelton, and her classmates found her quite funny (as indeed she was). When we discussed this, it turned out that her imitation of Skelton was quite unconscious, indeed a real facet of her personality.

On the basis of this and other material not described here, we concluded that the deficiency in arithmetic had something to do with unconscious hostility toward the choice of a profession for her, in which she had not been consulted. (Her parents also had discussed being an airline hostess - Marlene has pretty features, but is quite obese.) In
addition, she was going through a pronounced struggle for maturity, in which the advantages of remaining immature were very great. I pointed out to her that it would be difficult for her to give her bizarre behavior up, because she enjoyed the advantages. She accepted most of my interpretations.

For the next few weeks, instruction was put on a permissive basis. She did what she wished, including nothing if she so desired. We made it a practice not to shower attention on the basis of mistakes, and sent Marlene back to class after seven weeks without a retest. She took the retest in class as a semi-final examination and showed a score of 61, another 17 point improvement. It is difficult to assess what this means. Her class behavior showed some improvement, but we must be cautious in gauging whether any real changes have taken place.
7. Will C:

Will entered the Mathematics clinic at a time when the author was experimenting with student assistants used as teachers, on the basis of one student assistant per pupil. Will was "taken over" by Carol B, one of the student assistants. It should be noted that Carol is a very large girl and towered over Will by five or six inches. Will has normal height for his age. Carol was quite eager to give him a lot of assistance, and indeed did a rather excellent diagnosis of his difficulties in arithmetic, which included:

1. Misunderstanding of the nature of decimal numbers in addition and subtraction.
2. Borrowing difficulty in subtraction.
3. Decimal rule in division.
4. Inverting dividend in division of fractions.

Note that Will's initial score was 57 out of 87, comparatively high for students selected for this instruction. The score had not changed after a semester in a Remedial Mathematics class in the previous phase of the study, and the diagnosis had remained unchanged also. Tests of the tables of fundamental facts revealed satisfactory control over the multiplication and division tables, but some deficiency in subtraction, particularly in the time of response. The instructor worked out a plan for Carol to use, in which she gave Will some daily drill in subtraction for a short period, then proceeded to the other items in the diagnosis.

Carol quickly found that Will did not know how to read large numbers and decimal numbers, and made up a series of practices for him. She sat with him and had him read numbers aloud to her for several days. She also would make up homework assignments for him to memorize, including higher decade subtractions and decimal numbers to read. After a while, she complained to the instructor that Will was not trying. We suggested that she shift to another item of instruction to give him a chance of educational diet, since he seemed to be resisting. After another week on the new item, with little apparent improvement, Carol again complained that he was not trying.

Carol was not conscious of the fact that she was actually dominating him. It appeared to the teacher that some of Will's failure to try was a polite way of showing his resentment of the situation. We tried to discuss the matter privately with Carol and tell her that Will was a boy who appeared to resent teacher pressure and preferred to be left alone. Carol did not accept this interpretation of the situation, and insisted that she was merely trying to help him, not dominate him. She insisted that she had not displayed impatience at his slowness, when it was obvious to the instructor and the other student assistants that she had.

The case had a very interesting conclusion in that Will was returned to a large class before he was thought ready for it, due to an accident.
in the school schedule. He proceeded to show a greatly improved adjustment
to the class and began to do excellent work. He took the arithmetic test
as a semi-final examination and scored 71, a 14 point gain over the previous
score. Since the class work was not directly related to this phase of
instruction, we must attribute much of the gain to the clinical instruction.
It was evident that Will was greatly relieved to be rid of Carol's
domination, and we are inclined to interpret his improvement as a delayed
reaction which had not been in evidence while he was in the clinical group.
8. Ken W:

Ken is a short, slightly chubby, 'baby-faced' boy. He seemed rather quiet and unresponsive at first, but appeared more outgoing and sociable after a week in the Mathematics clinic.

Ken's initial score on the diagnostic test was 46. However, careful observation of his paper revealed only one error pattern, a misunderstanding of the use of 0 as a place holder in multiplication. It was further observed that Ken's work was extremely neat, indeed quite feminine, and much more legible than that of the average 15 year old boy. Discussion with Ken revealed that he did everything twice, once on the examination and once on a piece of scrap paper to verify his results. As a result, he finished only 35 problems out of 53.

We examined his paper carefully and showed him that he had erased and corrected only two problems in the total completed. Both of these were wrong. His careful checking had actually done harm. Ken was asked to complete the unfinished portion of the test at his leisure. Four diagnostic items were found on this portion of the test:

1. When adding mixed numbers, he changed them first to improper fractions, then to common denominators. This procedure is logically correct, but more difficult to manipulate mentally, and, in Ken's case, more conducive to error.

2. In subtracting decimals, he either brought down part of the subtrahend instead of writing a 0 and subtracting from 10, or omitted the problem.

3. In subtracting fractions, he brought down part of the subtrahend instead of borrowing:

\[
\frac{17}{84} - \frac{91}{84}
\]

4. In subtracting fractions, he changed mixed numbers to improper fractions, then to common denominators, with consequent errors.

The diagnosis of these items explains why he skipped that portion of the test and did a later section instead.

The first item for instruction was 2, above. Ken understood the difficulty quickly and did an exercise of ten problems without error.

Since borrowing had been the first item for instruction, we proceeded to borrowing in the subtraction of fractions. This again was quickly understood, and Ken worked out a practice set of thirty problems. He made three errors, one in borrowing, and two errors of omission, in that he changed the fraction correctly, but failed to reduce the whole number borrowed from by one.
Since it was felt that he had control over the process, Ken proceeded to an exercise in addition of mixed numbers, with the instruction designed to break him of the habit described in 1, above. He completed two lines of eight problems each, missing the last four. The errors on these were not patterned, and were interpreted as a sign of fatigue, or possibly boredom, not reflective of lack of control of the process.

The next day instruction was directed to the use of zero place holders in multiplication. Ken said he understood the procedure after it was explained, and did ten problems in the practice set. He missed five of them, all through zero errors. After careful analysis of the five errors, he did the next ten on the page and got four wrong, also because of zero place holders. We could sense that he was getting discouraged, and determined to leave the process for the time being.

There was some slight doubt about Ken's accuracy in addition, since he missed two problems on the diagnostic test, so he was assigned a set of five six-column problems to work out. He did these without error. We then explained that the problem of zero place holders in multiplication was the only one left to master, and he proceeded to the next set of ten on the practice page. He missed three of these, but only two because of 0 errors. He was encouraged to try again, and completed ten more, this time without error.

The next day Ken was given a retest, and was encouraged to do the work just once, without rechecking. He completed the test in forty minutes, and scored 82 points out of 87. When he saw the results, he said, "Gee, am I smart." He was actually more accurate than most students, without rechecking.
Jean B:

Jean was referred to the clinic by her counselor, after her mother had come to school for a consultation about her mathematics. At this time, 15 weeks of the semester had gone past and Jean was failing in General Mathematics I. Her teacher testified that she seemed utterly incapable of the simplest problem and generally made little or no attempt. Her mother told the counselor that she had always had trouble with arithmetic and didn't even know her tables of fundamentals. Jean's initial score on the diagnostic test was 19 points, an unusually low score. However, large portions of the test were completely untouched.

At the first interview, the instructor sounded Jean out about her feelings regarding arithmetic. She gave vent to unusually violent feelings of hostility. "I hate this stuff like poison! I don't want anything to do with it." The instructor's acceptance of her feelings, without censure, caused her to relax somewhat. She admitted that possibly, she might like the subject much better, if she could do well at it. The instructor proceeded to assure her, with a great deal of confidence, that every student who entered the clinic showed a great deal of improvement, and that it was merely a matter of time and application. Because of the extremely low score, no attempt to make a complete diagnosis was made during the first lesson. It was felt that the number of items requiring instruction was too large and that Jean would be discouraged by seeing them all at once. The procedure was simply to take them up one at a time. The first items had to do with misunderstanding the placement of decimal numbers for addition and subtraction. After a brief explanation, during which she showed a surprisingly quick grasp of the situation, Jean proceeded to do an exercise rapidly and without mistakes. The next item taken up was subtraction. Jean had acquired the habit of borrowing twice when borrowing across a zero. She understood the explanation of her fault immediately, but her first attempt to do a corrective exercise was only one-half successful. She was encouraged to try again. The second attempt was better, but still full of error. At this point, the instructor sensed that Jean was getting annoyed with the problem, and suggested that she go on to something else, and return to subtraction later. This pleased her a great deal. The next item discussed was the subtraction of denominate numbers. Showing Jean this brought surprising results.

She said, "Oh, I know how to do that!" and proceeded to show a perfect solution to the problem. We proceeded to the next items and the same thing happened. The instructor now told Jean that he really had no idea of what she could or could not do, but that it was obvious that she understood a great deal more than her first test had indicated. The interpretation that was made to her was the belief that she had become nervous and tense during the test, and that the experienced difficulty with one or two problems, coupled with her intense hatred of the subject, had caused her to freeze up, with the consequences described. She was encouraged to write the test over, with no time
limit, only attempting those items which she felt sure she understood. She did so and scored 66 out of 88, much to her own surprise. Only 5 points of this remarkable gain could fairly be attributed to the corrective exercises of the first two days. After this, instruction proceeded as in other cases.

Three additional items were revealed by this more complete test:

1. She borrowed 10 instead of the denominator of the fraction, in subtracting mixed numbers.
2. She did not know the decimal rule for division.
3. In problems like \( \frac{3}{4} \div 4 \), she failed to invert the divisor, although she did other division problems with fractions correctly.

Jean caught on to the first item very quickly, completing a set of ten problems without error. She was now encouraged to return to the problem of borrowing in subtraction, which she mastered after two more attempts. She then proceeded to the other two items, which she mastered on the first attempt. A second retest on the diagnostic arithmetic test resulted in a score of 65 (60 minute score) with two residual deficiencies:

1. She made errors in the decimal rule in division. While this would still be diagnosed as an error pattern, it was better than the previous test.
2. She still failed to invert, or inverted the wrong fraction in the division of fractions. (the latter behavior was new).

The items accounted for the loss of six points. The remaining sixteen points lost must be attributed to an unusually high percentage of random errors. Jean still felt nervous when taking tests. Instruction was not carried further because of the end of the semester.

The major gain in this situation, was the realization on Jean's part, that her arithmetic was not nearly as bad as she had previously thought. Further interviews with Jean helped her to see that her own strong feelings were the basis of most of the trouble. When she was questioned about the course and asked to indicate some of the things that had been difficult for her, she found one set of problems, and with a little questioning and encouragement was able to explain how they should be done. This helped to verify, in Jean's mind, that her feelings about the subject had more to do with her trouble than her actual ability.

A later interview with Jean's counselor revealed that she was also having trouble in Business Science. The counselor also explained
that Jean was taking a commercial course at her mother's insistence, and that her mother was very successful in business and mathematics. While this data was interpreted as symptomatic of feelings of hostility which Jean felt toward her mother, displaced to her mathematics and business teachers, both women, it was determined to let the treatment rest on the symptomatic level for the time being. Jean had indicated that she felt encouraged about her work. It was decided that it was better to see if her work in these areas would improve without going into the matter of family relations more deeply.
10. George C:

George was a tall boy for his age (15), about 70 inches, of medium build with dark hair. When he first reported to the mathematics clinic, he seemed rather tense and apprehensive of the situation. His initial score on the diagnostic test was 22 points, but the low score is deceiving since he had only completed 18 problems out of a total of 53. We told him that we really had no clear picture of what he could or could not do with basic arithmetic and asked him if he would complete the test so that a more accurate diagnosis could be made. He was told to take his time and not be concerned about racing the clock. He needed about 45 minutes to finish the test, but he was observed working fairly rapidly, and then very slowly. The power score was 63 points, a fairly respectable total. A slightly higher ratio of random errors was noted in the part he had done during the 40 minute test. The diagnosis revealed only three items:

1. He placed whole numbers after the decimal point in two of the four problems concerned with addition of decimals.
2. There was some difficulty with place holders in division, a tendency to put in zeros which were not needed.
3. He made some errors in the use of the decimal rule in division.

Before proceeding with instruction, an attempt was made to sound out George about the apparent contradiction between his actual skill in arithmetic and his current failure in General Mathematics coupled with a low score on the initial test. George talked quite readily about the problem, explaining that he had had trouble for the past two years with at least one teacher every semester. He said that if he thought the teacher a bit easy going, he would try to get away with things. By this he meant not doing the regular classwork, talking excessively, and occasionally indulging in boyish pranks which would get him into trouble. It should be noted at this point, that George's teacher in General Mathematics was a young man in his first teaching position, who appeared to be very affable, and gave the misleading impression of being easy going. Without any comment from the teacher, George went on to say that this was his own fault, that he really did not know why he did it, but that he managed to stay out of serious trouble. He apparently had enough contact with reality to be able to gauge the seriousness of an offense and to avoid those which might lead to suspension. The fact that George's trouble was always with male teachers provided an obvious clue, for further conversation which revealed to George that his conflict was essentially related to feelings about his father. It is unnecessary to go into great detail about the long interview, which followed procedures known to counselors and social workers for many years. The attempt to bring George a degree of insight concerning his conflict with his father was partly successful. He was helped to gain some fresh perspective towards school, to a limited degree.
Instruction proceeded, after the first interview, to take up the three points listed in the diagnosis above. In the case of number one and three, it was evident that George really understood them. In order to be sure, and also to provide a means for continuing the relationship, practice exercises were prescribed. There was a misunderstanding of item two, but some explanation quickly cleared it up. George took a re-test about two weeks after the contact began and received a speed score of 50, but a power score of 78 out of 87. The only points he lost were due to random errors. His slowness on the test could be attributed to day-dreaming. After a brief discussion of the problem of day-dreaming with him, the relationship was terminated and he was sent back to class. The teacher later reported that, while he still was inconsistent, his behavior had improved sufficiently for him to pass the course, at the next card marking.
11. John W:

John was a tall boy for his age (15), about 71 inches, of medium build with straw colored hair. He had a very pleasant manner, and seemed pleased about the opportunity to receive clinical instruction. He told us he was very interested in drafting and might like to make a career of it. He understood how important mathematics was to drafting, and wanted to clear up his difficulties. He was also happy about the assignment since it took him out of a study hall. He was also a member of the freshman basketball squad, and gave the general impression of a normal boy of average intelligence.

John's initial speed score on the diagnostic test was 29. The opportunity to finish the test raised the score to 40. The diagnosis indicated trouble with the tables of fundamentals and he was tested on these also. The complete diagnosis follows:

1. Slow on the subtraction tables. (4 minutes)
2. Slow and inaccurate on the tables of multiplication and division, particularly on combinations involving 7, 8, and 9, particularly the latter two.
3. Does not understand the nature of the decimal point in addition and subtraction problems, such as 3.15 plus 54.3 plus 19.
4. Borrows 10 instead of the unit involved when subtracting denominate numbers.
5. Fails to reduce fractional answers to lowest terms.
6. Does not multiply correctly when numbers end in 0.
7. Fails to invert when dividing a fraction by whole numbers, and inverts whole numbers, when dividing them by a fraction.
8. Does not use decimal rule correctly in problems like .009 /72 (adds one 0 instead of 3, counting from the left of the dividend.)

Instruction proceeded along two lines. Each day, John did short practice exercises with the subtraction and multiplication tables, for 10 to 15 minutes. The time was limited on this to prevent boredom. The exercises consisted of flash cards or written drills of the following type:

```
999999999 81
999999999 72
999999999 63
999999999 54
999999999 45
999999999 36
999999999 27
999999999 18
999999999 9
```
The student adds across, adds down and subtracts the results successively to see if he gets nine, or whatever the case may be. When the daily drill was over, John proceeded to work on the other items in the diagnosis.

The first of the items chosen was the subtraction of denominate numbers, a customary procedure with these students, since it is one of the easiest to teach, and the student gains confidence from a good start. John grasped the borrowing principle readily, and mastered the process with the first exercise. He next proceeded to the addition and subtraction of decimal numbers. This presented a little more difficulty, as he made five errors in placing numbers out of twenty problems. The errors were interpreted to him and another exercise resulted in mastery of the process. John had no difficulty in understanding the concepts of using 0 as a place holder in multiplication, and mastered the first group of ten problems without error in 0's, although he made several errors in the basic facts.

John then worked on division of fractions. Each type of problem had to be handled separately, since his errors were unrelated in some ways. For instance, he inverted a whole number dividend, although he did not invert a fractional dividend. The misunderstanding seemed to clear up, and he completed several exercises correctly, although some misgivings were expressed. We then showed John that the decimal rule was based on the theory of multiplying both dividend and divisor by the same number, a concept he readily understood and applied.

John then took a retest and scored 57, out of 87 points. Seven points of the loss could be attributed to random errors. The remainder to the multiplication and division tables and item seven in the diagnosis (division of fractions). Three days instruction and practice seemed to clear up the latter, but there seemed to be a block on the problem of the multiplication table.

The instructor made the error of having John continue with the same method of practice, which did little real good. The same cues which had helped many other students did not work with him. For instance, he knew that the digits of all multiples of 9 added to 9, but that did not help. Many students, confronted with 8x7, would think "7x7 is 49 and one more 7 makes 56." This did not help John. After much fumbling, the following mental procedure was the one which solved the problem:

\[
\begin{align*}
7 \times 9 &= 63 \\
6 \times 8 &= 48 \\
9 \times 8 &= 72
\end{align*}
\]

This is admittedly slow, and not as desirable as an automatic response with the correct answer. However, the latter could not be achieved with John, and the procedure above gave him his first real
control over the process. His control over the corresponding items in the division table also improved, which is customary in such cases.

John took his second retest two weeks later and scored 71 out of 88. He made errors related to item 8 in the diagnosis, and still lost three points attributable to the multiplication tables, but the remainder could be charged to random errors alone. He remained in the clinic a few more days to practice the tables further, then volunteered to remain the rest of the semester to assist in the work of the clinic. His motives in this were twofold, since he had become very interested in a girl who was a student assistant that hour. He was very useful for the remainder of the term, spending a good deal of time helping another boy with the multiplication and subtraction tables, and, we suppose, improving his own control at the same time.
Ray was a short, blond, 15 year old boy. His low score on the Iowa Every Pupil Test of Basic Skills caused him to be taken from his General Mathematics class during the first week of the semester, for clinical instruction. He was to attend during the last hour of the day. Note that Cody High School was on double session, and the last hour of the day met from 4:00 to 4:40 p.m. However, Ray's schedule had not been lengthened to attend the clinic. It had readily been substituted for his General Mathematics class. Ray was very quiet, relatively unresponsive and did not speak unless spoken to. This unresponsiveness carried over to the other students present, as well as the teacher.

Ray's initial score on the Diagnostic Test was 26 points, unusually low, even for clinical students. Twenty-three problems on the test remained unanswered, even after he was given the opportunity to do so without time limit. He simply refused to answer any question when he was uncertain of the procedure. When there are large numbers of items in the diagnosis, it is unwise to discuss a complete diagnosis with the student at the outset. It is better to simply take them up one at a time, which was the procedure followed with Ray. When the diagnosis was eventually completed, nine items were listed. It is unnecessary for purposes of this explanation to go into detail about them. If interested, see Appendix C.

In the weeks that followed, Ray's attendance was very spotty. Most of his absence was truancy, including both truancy from an entire day of school and truancy from the last period of the day, which was the period to be spent in the mathematics clinic. Discussion with the boy's counselor indicated that he was a highly disturbed boy who tried to solve his problems by withdrawing from them, and by running away from the authority elements in society. The instructor did not attempt to enter into the counseling aspects of the situation. We simply dealt with Ray on the basis of subject matter instruction through the medium of a friendly personal relationship. When he returned after truancy, little or nothing was said about it. Ray was treated as if his last attendance had been the day before.

The initial diagnosis included:

1. Did not borrow correctly in the subtraction of denominate numbers.
2. Did not use decimal rule correctly in multiplication.
3. Multiplied fractions and whole numbers separately in the multiplication of mixed numbers.
4. Did not borrow correctly in subtraction of mixed numbers.

In addition, Ray completely omitted the following:
1. All problems in addition of fractions.
2. All problems in subtraction of decimals.
3. All problems in division of decimals.
4. All problems in division of fractions and mixed numbers.

Ray caught on quickly to all explanations and readily did any assigned work. Most students who attended the Mathematics clinic looked for their own folders at the beginning of the period and finished up any assigned work without being told. Ray was not rebellious, but did nothing on his own initiative. Each day, it was necessary to prod him, in a friendly manner, and indicate what he had not finished from the day before. It was often as if he had no memory of what happened from day to day. However, he did any assigned work rapidly and accurately, and it was necessary only to go through a process with him once. After five and a half weeks, he took the test again and scored a speed score of 45 points and a power score of 71 points, with residual difficulty in subtraction of fractions and in division of decimals. He did one more practice in each one, and completed a second retest with a speed score of 59 points and a power score of 79 points, with six points lost from random errors and two lost because of decimal division, a process still giving some difficulty. It was deemed unprofitable to retain him further, and he was returned to his class.

Ray's gain of 53 points in power score was the largest recorded in the entire study.

In spite of the friendly atmosphere, Ray remained sufficiently unresponsive, that we never could tell whether he was really learning something clearly for the first time, or whether it was an experience he had mastered before but was unsure of. The teacher made a great point of complimenting him periodically about his improvement and taking his test papers to his counselor. Both the teacher and the counselor felt that something about which he could be complimented was badly needed.

After the second test, Ray was returned to his General Mathematics class. Subsequent reports from both his regular teacher and his counselor indicated that, from the standpoint of social and emotional adjustment, the special clinical instruction had had no observable effect. From the standpoint of other people observing him, it was as though the experience had not happened at all. The boy's inner feelings regarding the matter were impossible to judge, since he remained quite unresponsive.
Appendix D

OTHER TESTS USED IN THE STUDY

1. Detroit Public Schools Test of 100 Subtraction Facts.
2. Detroit Public Schools Test of 100 Multiplication Facts.
3. Detroit Public Schools Test of 90 Even Division Facts.
4. Iowa Every Pupil Test of Basic Skills, Form D.
DETROIT PUBLIC SCHOOLS

TEST ON THE 100 SUBTRACTION FACTS

DEPARTMENT OF INSTRUCTIONAL RESEARCH

To the Teacher: This is a diagnostic test. Each pupil may use it to find out which facts he still has to master. Allow only time enough for each pupil to write answers for the facts he knows. Counting and other immature methods of obtaining answers should not be used on this test. A suggested time for pupils of average ability in the different grades is given in the table at the right.

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*To the Teacher: Many teachers prefer to give all or parts of this test by using flash cards or by dictating the facts. Then pupils use lined paper, number down the side of the paper, and write only the answers for the facts dictated.

The values to be derived from this test depend upon careful reteaching and effective study of facts missed. Each pupil should make his own list of facts to be studied in his notebook. As the pupil proves to the teacher that he has mastered certain facts, these are crossed off his list.

Teachers may obtain from the Exact Science office a bulletin called, "A Program for Helping Pupils to Achieve Mastery of the Basic Facts," File No. 5499.*
To the Teacher: This is a diagnostic test. Each pupil may use it to find out which facts he still has to master. Allow only time enough for each pupil to write answers for the facts he knows. Counting and other immature methods of obtaining answers should not be used on this test. A suggested time for pupils of average ability in the different grades is given in the table at the right.

### Directions for Pupils:
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<td>×1</td>
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</table>

To the Teacher: Many teachers prefer to give all or parts of this test by using flash cards or by dictating the facts. Then pupils use lined paper, number down the side of the paper, and write only the answers for the facts dictated.

The values to be derived from this test depend upon careful reteaching and effective study of facts missed. Each pupil should make his own list of facts to be studied in his notebook. As the pupil proves to the teacher that he has mastered certain facts, these are crossed off his list.

Teachers may obtain from the Exact Science office a bulletin called, "A Program for Helping Pupils to Achieve Mastery of the Basic Facts," File No. 5499.
DETROIT PUBLIC SCHOOLS
TEST ON 90 EVEN DIVISION FACTS

To the Teacher: This is a diagnostic test. Each pupil may use it to find out which facts he still has to master. Allow only time enough for each pupil to write answers for the facts he knows. Counting and other immature methods of obtaining answers should not be used on this test. A suggested time for pupils of average ability in the different grades is given in the table at the right.

<table>
<thead>
<tr>
<th>Grade</th>
<th>Time for 100 Facts</th>
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<tbody>
<tr>
<td>4B-4A</td>
<td>6 MIN.</td>
</tr>
<tr>
<td>5B-5A</td>
<td>5 MIN.</td>
</tr>
<tr>
<td>6B-6A</td>
<td>4 MIN.</td>
</tr>
<tr>
<td>7-8</td>
<td>3 MIN.</td>
</tr>
</tbody>
</table>

Directions for Pupils: Write the answers as fast as you can. Skip those you cannot write quickly.

PART A (18 facts)

No. right on Part A

\[
\begin{array}{cccccc}
2/4 & 1/0 & 2/8 & 2/14 & 2/2 & 2/16 \\
2/12 & 2/6 & 1/1 & 2/10 & 2/18 & 3/0 \\
1/2 & 3/3 & 3/9 & 1/3 & 3/6 & 2/0 \\
\end{array}
\]

PART B (18 facts)

No. right on Part B

\[
\begin{array}{cccccc}
\end{array}
\]
### PART C (18 facts)

<table>
<thead>
<tr>
<th></th>
<th>No. right on Part C</th>
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<tbody>
<tr>
<td>5/10</td>
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<tr>
<td>1/6</td>
<td>5/25</td>
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<td>5/15</td>
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### PART D (18 facts)

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<tr>
<td>7/28</td>
<td>8/0</td>
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<tr>
<td>7/63</td>
<td>1/8</td>
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### PART E (18 facts)

<table>
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<tr>
<td>8/24</td>
<td>9/72</td>
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<td>9/36</td>
<td>9/9</td>
</tr>
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<td>8/48</td>
<td>9/81</td>
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</table>

*To the Teacher:* Many teachers prefer to give all or parts of this test by using flash cards or by dictating the facts. Then pupils use lined paper, number down the side of the paper, and write only the answers for the facts dictated.

The values to be derived from this test depend upon careful reteaching and effective study of facts missed. Each pupil should make his own list of facts to be studied in his notebook. As the pupil proves to the teacher that he has mastered certain facts, these are crossed off his list.

Teachers may obtain from the Exact Science office a bulletin called, “A Program for Helping Pupils to Achieve Mastery of the Basic Facts,” File No. 5499.
This test consists of three parts. Part I is vocabulary and fundamental knowledge including judgments of distance, time telling, and the understanding of basic concepts. Part II deals with whole numbers, fractions, and decimals. Part III consists of story problems. In general, the test is appropriate for grades 5 through 9.

The test may be administered in a single period of about eighty minutes or in two periods of about forty-five minutes each. The actual working time for the parts are as follows:

- **PART I - VOCABULARY AND FUNDAMENTAL KNOWLEDGE** - 15 minutes
- **PART IIA - WHOLE NUMBERS AND FRACTIONS** - 17 minutes
- **PART IIB - PERCENTAGE AND DECIMALS** - 8 minutes
- **PART III - PROBLEMS** - 28 minutes
- **Total Working Time** - 68 minutes

Each problem on the test is followed by four possible answers, only one of which is correct or definitely better than the others. Record the correct answer to a problem in the following way. First, solve the problem mentally or if necessary work the problem on a separate piece of scratch paper. Do not write on the test booklet. Second, compare your answer with the ones given below the problem and decide which is the best answer. Third, note the number of this answer. Fourth, find the number of the problem on your answer sheet and place an (X) in the parentheses after the number of the correct response. For some problems the correct answer is not given. For these problems you are to place an (X) in the parentheses marked "N" (meaning "No correct answer is given") on your answer sheet. Specific directions will be given at the beginning of each section of the test.

Answer problems on all parts of the test in the order given, but do not linger too long over difficult problems. Skip them and return to them later if time permits. Caution: If you do skip any problems, be sure to skip the corresponding problem on your answer sheet.

Reduce all fractions to their simplest form.

**SAMPLE PROBLEMS**

1. How many apples are 5 apples and 1 apple?
   - (1) 7
   - (2) 6
   - (3) 4
   - (4) 5

2. Add
   \[ \frac{3}{2} \]
   - (1) 5
   - (2) 4
   - (3) 6
   - (4) 7

3. James had 10 marbles. He gave 5 away. How many marbles did he have left?
   - (1) 6
   - (2) 4
   - (3) 7
   - (4) N

**ANSWER SHEET**

<table>
<thead>
<tr>
<th>Problem</th>
<th>Correct</th>
<th>Incorrect</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>3( )</td>
<td>2( X )</td>
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<tr>
<td>2.</td>
<td>3( )</td>
<td>4( )</td>
</tr>
<tr>
<td>3.</td>
<td>3( )</td>
<td>4( X )</td>
</tr>
</tbody>
</table>

DO NOT BEGIN WORK UNTIL YOU ARE TOLD TO DO SO. DO NOT MARK OR WRITE ON THE TEST BOOKLET.

Copyright, 1943, by State University of Iowa. This test is a special edition of the Iowa Every-Pupil Tests of Basic Skills, Test D, Advanced, Form O, by H. F. Spitzer and others, published by Houghton Mifflin Company, Boston, Mass.
PART I

VOCABULARY AND FUNDAMENTAL KNOWLEDGE

Directions. After each question in this part of the test there are four possible answers, only one of which is correct or definitely better than the others. Place an (X) in the proper parentheses on the separate answer sheet. DO NOT MARK OR WRITE ON THE TEST BOOKLET.

1. 75 minutes is how many hours?
   (1) $\frac{3}{4}$  (2) $1 \frac{1}{4}$  (3) $1 \frac{1}{2}$  (4) $2 \frac{1}{2}$

2. How should two hundred twenty-two and thirteen thousandths be written?
   (1) 20022.13  (2) 222 13000  (3) 222.13000  (4) 222.013

3. About how high is an average dining table?
   (1) $2 \frac{1}{2}$ feet  (2) 3 $\frac{1}{2}$ feet  (3) 4 feet  (4) 5 feet

4. Which of these represents the largest value?
   (1) .6  (2) .400  (3) .3841  (4) .0893

5. Which of these is used in measuring an angle?
   (1) Meters  (2) Cubic feet  (3) Degrees  (4) Centimeters

6. Two and a half hours after midnight would be what time?
   (1) 12 A.M.  (2) 2:30 P.M.  (3) 9:30 P.M.  (4) 2:30 A.M.

7. How should 5" be read?
   (1) Five feet  (2) Five degrees  (3) Five inches  (4) Five hours

8. Which of these fractions is the largest?
   (1) $\frac{5}{12}$  (2) $\frac{11}{18}$  (3) $\frac{12}{25}$  (4) $\frac{1}{3}$

9. How many square feet are there in a square yard?
   (1) 3  (2) 4  (3) 6  (4) 9

10. How many faces or sides does a cube have?
    (1) 4  (2) 6  (3) 8  (4) 12

11. How would you read 100.001?
    (1) One hundred and one  
    (2) One hundred and one-tenth  
    (3) One hundred and one-hundredth  
    (4) One hundred and one-thousandth

12. A ton of coal is about equal in weight to how many men?
    (1) 4  (2) 9  (3) 13  (4) 20

13. In which of these figures is there a horizontal line?
    \( \times \) \( \_ \) \( \_ \) \( \_ \) \( \_ \) \( \_ \) \( \_ \) \( \_ \) \( \_ \)
    (1)  (2)  (3)  (4)

14. In which of the figures above do the lines form a right angle?
    (1) 1  (2) 2  (3) 3  (4) 4

15. How many digits are used in writing the number four hundred twenty thousand seven?
    (1) 3  (2) 5  (3) 6  (4) 9

16. About how many acres are in the shaded area in this diagram?
    \( \frac{1}{4} \) Mile
    (1) About $\frac{1}{4}$  
    (2) About 16  
    (3) About 160  
    (4) About 320

17. How many $\frac{1}{8}$'s are in $\frac{2}{3}$?
    (1) Less than one  (2) 2  (3) 3  (4) 4

18. A tree 24 feet high is about how many times as high as a tall man?
    (1) 2  (2) 3  (3) 4  (4) 6

19. Which of the following represents the largest quantity?
    (1) M  (2) C  (3) XL  (4) XII

(Go on to the next page.)
20. Why do we write the zero in 3.05?
(1) Because arithmetic books say we should.
(2) Because it holds the tenths place and shows that the 5 means 5 one-hundredths.
(3) Because it shows that there are no fractions in the number.
(4) Because the tenths place is always a zero when there are hundredths in a number.

21. 1000 B.C. is about how many years ago?
(1) 1000 (2) 940 (3) 1940 (4) 2940

22. In looking at three groups of calves, one man said, "There are 6 in the first group, 6 in the second, and 8 in the third." A second man said, "There are 20 calves." If you only wanted to know how many calves there were, why was the second man's answer best?
(1) Because it is easier to think of one group of 20 than of three groups of 6, 6, and 8.
(2) Because 20 tells you how many calves there were.
(3) Because 20 does not leave out any of the calves.
(4) Because the first man did not tell how many calves there were.

23. Which is equal to 4\%?
(1) \(\frac{4}{10}\) (2) \(\frac{1}{4}\) (3) \(\frac{4}{100}\) (4) .40

24. In telling how long a certain bridge is, four children gave the following answers. Each answer is correct, but one is better than any other. Which is best?
(1) About three times the distance across the school lawn.
(2) About 40 times the length of this room.
(3) About 12 times as far as the distance around the school room.
(4) A person can run across it in about 2 minutes.

25. Which of these shows a diameter?
(1) (2) (3) (4)

26. The length of the air field runway is 1800 feet. How many miles is this?
(1) Less than half a mile (3) 1 mile
(2) \(\frac{1}{2}\) mile (4) 2 miles

27. Which line is the circumference of the circle?
(1) A (2) B (3) C (4) D

28. In which number does the 3 represent hundreds?
(1) 431 (2) 3826 (3) 5319 (4) 30000

29. In the number 565, how does the first 5 compare in value with the last 5?
(1) It is the same.
(2) It is twice as great.
(3) It is 10 times as great.
(4) It is 100 times as great.

30. Which of these figures shows what \(\frac{1}{4} \times \frac{1}{2}\) equals?
(1) (2) (3) (4)

31. In the word "eighty-one", what does the "ty" mean?
(1) It is used to make the word sound rhythmical.
(2) It means tens.
(3) It means to add 80 and 1 together.
(4) It means less than nine and more than eight.

32. What is the perimeter of a rectangle?
(1) The distance around it.
(2) Its area.
(3) The distance from one corner to the opposite corner.
(4) One-half the base times the altitude.

33. The population of city A is 161,832; that of city B is 45,126. What is the best way of expressing the relationship between the two populations?
(1) A has 118,706 more people than B.
(2) A is many times larger than B.
(3) A is about four times as large as B.
(4) A is about six times as large as B.

34. In what units would the volume of a box be given?
(1) In centimeters (3) In degrees
(2) In square inches (4) In cubic inches

(Go on to the next page.)
35. In which of these situations would \( \pi \) be used? \((\pi = 3.14)\)
   (1) In finding the thickness of a tree.
   (2) In finding the area of a triangle.
   (3) In finding the perimeter of a hexagon.
   (4) In finding the volume of a cube.

36. Which of these is a measure of area?
   (1) An acre
   (2) A rod
   (3) A peck
   (4) A cubic foot

37. In the last election, candidate A beat candidate B "two to one". If A received about 15,000 votes, approximately how many votes did B receive?
   (1) 7,500
   (2) 10,000
   (3) 30,000
   (4) 45,000

38. About how long would it take an eighth grade boy walking at a fast rate to walk a mile?
   (1) 5 minutes
   (2) 15 minutes
   (3) \( \frac{1}{2} \) hour
   (4) 1 hour

39. If a farmer asks for the capacity of a grain bin, what units of measurement should a salesman use in answering?
   (1) Gallons
   (2) Cubic feet
   (3) Bushels
   (4) Tons

40. About how many 850-pound steers can be hauled in a truck with a load limit of 5 tons?
   (1) 6
   (2) 11
   (3) 16
   (4) 20

---

PART II

SECTION A: WHOLE NUMBERS AND FRACTIONS

Directions. On this section, beginning on the next page, you are to solve a number of arithmetic problems. Below each problem you will find three possible answers and a fourth marked "N". The "N" means that No correct answer for the problem is given.

First, solve the problem mentally or if necessary work the problem on a separate piece of scratch paper.

Second, compare your answer with one of the four answers given and decide upon the number of the correct response.

Third, note the number of the correct response.

Fourth, turn to your answer sheet, find the problem number and place an (X) in the parentheses after the number of the correct response. Do not re-work a problem simply because your answer is not like any of those given below the problem. Instead place your (X) in the parentheses numbered "4", which is N (No answer given), on your answer sheet and go on to the next problem.

Reduce all fractions to their simplest form.

The first two problems are sample problems and the correct answers are marked on your answer sheet. Be sure you understand them before beginning this section of the test.

Do not mark or write on the test booklet.
<table>
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<tr>
<th>Exercise</th>
<th>Operation</th>
<th>Sample 0</th>
<th>Sample 00</th>
<th>Sample 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (1)</td>
<td>Add</td>
<td>3 (2) 4</td>
<td>9</td>
<td>130</td>
</tr>
<tr>
<td>2 (2) 4</td>
<td>5 (3) N</td>
<td>186</td>
<td>42400</td>
<td>129</td>
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<tr>
<td>3 (3)</td>
<td>Subtract</td>
<td>186</td>
<td>2000</td>
<td>2089</td>
</tr>
<tr>
<td>4 (4) N</td>
<td>Multiply</td>
<td>36000</td>
<td>1072</td>
<td>31</td>
</tr>
<tr>
<td>5 (5) N</td>
<td>Divide</td>
<td>340</td>
<td>2000</td>
<td>1000</td>
</tr>
<tr>
<td>6 (6) N</td>
<td>Add</td>
<td>36000</td>
<td>1072</td>
<td>31</td>
</tr>
<tr>
<td>7 (7) N</td>
<td>Subtract</td>
<td>340</td>
<td>2000</td>
<td>1000</td>
</tr>
<tr>
<td>8 (8) N</td>
<td>Multiply</td>
<td>36000</td>
<td>1072</td>
<td>31</td>
</tr>
<tr>
<td>9 (9) N</td>
<td>Divide</td>
<td>340</td>
<td>2000</td>
<td>1000</td>
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</table>

(Do not start section B until you are told to do so.)
SECTION B: PERCENTAGE AND DECIMALS

Directions: Do all your work on scratch paper. Select the number of the correct answer from the ones given below each problem. Record this answer by placing an (X) in the parentheses on your answer sheet after the number of the correct response. DO NOT MARK OR WRITE ON THE TEST BOOKLET.

64. Multiply .66 by .12
   (1) 7.32  (2) 7.82  (3) .792  (4) N

65. Divide 244 by .02
   (1) 1.22  (2) 122  (3) 12200  (4) N

66. What is 20% of 400?
   (1) 80  (2) 20  (3) 8000  (4) N

67. 16 is 50% of what number?
   (1) 80  (2) 800  (3) 25  (4) N

68. What is 200% of $250?
   (1) $500  (2) $50  (3) $100  (4) N

69. Change $\frac{3}{4}$ to per cent form.
   (1) $\frac{3}{4}$%  (2) 25%  (3) 75%  (4) N

70. Change .495 to per cent form.
    (1) $\frac{495}{1000}$  (2) $0.495$%  (3) $4.95$%  (4) N

71. Change 76% to decimal form.
    (1) 76  (2) .076  (3) .76  (4) N

72. Change 90% to a common fraction and reduce it to its lowest terms.
    (1) .9  (2) $\frac{9}{10}$  (3) $\frac{9}{100}$  (4) N

73. What is 5% of $892.60?
    (1) $44.63$  (2) $446.30$  (3) $4463.00$  (4) N

(Do not start Part III until you are told to do so.)

PART III - PROBLEMS

Directions: Read each problem carefully. Follow the same procedure in marking your answer sheet as you did on Part II. In some problems, you are asked to give only an approximate answer. For these particular problems, no "N" is given. You are to select the best approximate answer from the four given and record your choice in the usual manner on the answer sheet. Do all your work on scratch paper. DO NOT MARK OR WRITE ON THE TEST BOOKLET.

At the beginning of the year, there were 13 girls and 18 boys in the third grade, 15 girls and 12 boys in the fourth grade, 11 girls and 16 boys in the fifth grade, and 13 girls and 13 boys in the sixth grade of the Jackson School.

74. How many girls were in the four grades?
    (1) 41  (2) 52  (3) 111  (4) N

75. How many more boys than girls were there in all four grades?
    (1) 108  (2) 110  (3) 112  (4) N

76. At the end of the year, there were 34 children in the fifth grade. How many more children were in the fifth grade at the end of the year than at the beginning?
    (1) 6  (2) 7  (3) 8  (4) N

77. The absences in the fifth grade during one week were as follows: Monday 3, Tuesday 0, Wednesday 5, Thursday 2, Friday 5. What was the average number of absences for each day?
    (1) 2  (2) 4  (3) 5  (4) N

On an automobile trip with his father, Tom kept a record of the speedometer readings as they drove along. At home it read 9209, at Salem the reading was 9217; at Vale City, 9291 miles; and at Greenville, 9356 miles.

78. How far was it from Salem to Vale City?
    (1) 74  (2) 84  (3) 147  (4) N

79. If it took 3 hours to make the trip from home to Greenville, how many miles per hour did they travel?
    (1) 47  (2) 49  (3) 52  (4) N

80. Before he started, Tom's father bought 8 gallons of gasoline at 17¢ per gallon and a quart of oil at 36¢ per quart. What was his bill?
    (1) $1.36  (2) $1.62  (3) $1.72  (4) N

(Do on the next page.)
The Girls' Club sold Christmas cards at $1.00 per box. The cards cost them 60¢ per box.

81. How much profit did they make on each box of cards they sold?
   (1) $0.40  (2) $4.00  (3) $6.00  (4) N

82. Their profit was what per cent of the selling price?
   (1) 40%  (2) 50%  (3) 60%  (4) N

83. How much would it cost to send a letter weighing $2\frac{1}{2}$ ounces to Australia if postal rates are 5¢ for the first ounce and 3¢ for each additional ounce or fraction of an ounce?
   (1) 7\frac{1}{2}¢  (2) 11¢  (3) 12\frac{1}{2}¢  (4) N

84. How many tons of coal can be stored in a bin 4 feet wide, 10 feet long, and 3 feet deep? (Coal weighs about 50 pounds per cu. ft.)
   (1) 3  (2) 60  (3) 120  (4) N

85. The seventh grade planned to take a trip to an Indian reservation. The teacher said, "Mr. Brown is taking 5 of the children in his car, and I can take 3. That means we have rides for one-fourth of the class." How many children were in the seventh grade?
   (1) 12  (2) 20  (3) 32  (4) N

86. Ship A is rated as of 12,480 tons. If Ship B is about one-fourth as large, what is its tonnage? (Note that in this problem no exact relationship is stated. Therefore, your answer will be only an approximation.)
   (1) 2,500 tons  (3) 3,000 tons
   (2) 6,000 tons  (4) 50,000 tons

87. A certain airplane that has a top speed of 435 miles per hour is approximately how much faster than an automobile which has a top speed of 90 miles per hour? (Only an approximate answer is required.)
   (1) 3 times  (3) 4 times
   (2) 5 times  (4) 6 times

88. If a man plants 105 of his 160 acres in corn, about what part of his farm does he plant in corn? (Only an approximate answer is required.)
   (1) 105 per cent  (3) one-half
   (2) two-thirds  (4) three-fourths

89. A dress in a store window has these two prices marked on it: "Was $12.98--Now $10.25." About how much of a price reduction has been made? (Only an approximate answer is required.)
   (1) one-third  (3) one-fifth
   (2) one-tenth  (4) four-fourths

90. John is waiting for a train that is scheduled to arrive at 9:35 a.m. but has been marked 8 hours late. John looks at his watch and sees that it is 9:00 a.m. About how much longer must he wait for the train? (Only an approximate answer is required.)
   (1) 6 hours  (3) 7\frac{1}{2} hours
   (2) 8\frac{1}{2} hours  (4) 9 hours

This is a section of a road map. The numbers between points indicate the number of miles between those points. The solid line indicates paved road. The double line indicates gravel road.

91. What is the shortest road distance from A to G?
   (1) 40 miles  (3) 47 miles
   (2) 50 miles  (4) N

92. In going from D to A, how many miles farther is it to go the all paved road than to go over part that is gravel?
   (1) 3 miles  (3) 6 miles
   (2) 7 miles  (4) 10 miles

93. About what per cent of the most direct road from B to G is paved? (Only an approximate answer is required.)
   (1) 15%  (3) 55%
   (2) 40%  (4) 75%
94. If the cost of building a paved road is $55,000 per mile, what was the total cost of the road from C to D?

(1) $132,000 (2) $1,320,000
(3) $1,300,000 (4) N

95. If the cost of building a gravel road is only $6,000 per mile, how many miles of gravel road can be built for the same amount of money that one mile of paved road costs ($55,000)?

(1) 7 miles (2) 91 miles
(3) 20 miles (4) N

96. On an auto trip, Mr. Brown goes from C to F by way of G. He returns by way of A. How many miles did he drive on the trip?

(1) 96 (2) 97 (3) 99 (4) N

97. What is the approximate area in square miles of the region enclosed by the road from A to C to D and then back to A by way of B?

(1) 51 sq. mi. (2) 240 sq. mi.
(3) 120 sq. mi. (4) 480 sq. mi.

98. What is the total area of this floor plan?

(1) 108 sq. ft. (2) 720 sq. ft.
(3) 680 sq. ft. (4) N

99. The kitchen floor and cabinet top are to be covered with linoleum which comes only in 6-foot widths. How many feet of this 6-foot width material should be purchased?

(1) 9 feet (2) 20 feet
(3) 108 feet (4) N

100. The floor carpeting for the living room of this house costs $5.10 a square yard. What will be the cost of this carpeting?

(1) $ 129.20 (2) $1162.80
(3) $387.60 (4) N

101. About what fraction of the total area is used in closets? (Only an approximate answer is required.)

(1) \( \frac{1}{18} \) (2) \( \frac{1}{10} \)
(3) \( \frac{1}{8} \) (4) \( \frac{1}{5} \)

102. The builder of this house thought that the estimate on the cost of doors was too high. The contractor pointed out that outside doors were $15 each, standard interior doors were $4.50 each, and closet doors were $3.00. The estimate for doors was $70.00. How much too high was this estimate?

(1) None (2) $.50 (3) $1.50
(4) N

103. If the large living room and dining room windows together cost $85.00, and the other windows cost $25.00 per unit, what was the cost of windows in this house?

(1) $185 (2) $280 (3) $285
(4) N

104. The loan on this house is $5,000, on which the owner pays $30.00 per month. If the rate of interest is 5%, what is the approximate amount of the principal that is paid the first month? (Only an approximate answer is required.)

(1) $9 (2) $15 (3) $18
(4) $22
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