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ARITHMETIC PROBLEM SOLVING STRATEGIES OF FOURTH GRADE CHILDREN

by

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A DISSERTATION

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TABLE OF CONTENTS

TABLE OF CONTENTS--Continued

LIST OF TABLES

CHAPTER I

SYNOPTICAL STATEMENT OF THE PROBLEM

This study will attempt to identify strategies employed by fourth grade children to solve verbal arithmetic problems, and to determine which strategies are successful and which are unsuccessful. In order to discover if profiles of successful and unsuccessful problem solvers can be identified, certain characteristics of the children will be selected, and, through a new technique of Profile Analysis, profiles will be clustered according to similar elements. Attempts will also be made to explore relationships between strategies chosen and selected pupil characteristics. It is hoped this study will provide insight into successful strategies which can be taught to children and that a more rational base for grouping children for instruction will result.

Background and Importance

The term problem solving refers to a great diversity of situations. Many writers define a problem in terms of a goal, progress toward which is blocked. Others have noted that when a person faces a problem, he is in a situation for which he does not have immediately available an appropriate response or answer. To solve the problem, he manipulates variables in order to make the appearance of solution more likely. Although most writers agree that problem solving should result in learning, the issue of how generalizable the solution should be has not yet been fully explored. Some so-called routine problems, such as finding something which is lost, might simply be classed as performance.

However, it has been established that the most important problem solving skills to teach are those which result in broad transfer such as Anderson's[1] experiment with pendulums. The children's experiments resulted in their discovery of the principle that the length of the pendulum alone determines the period.

There is a wide diversity of events included in the term problem solving and, to date, no satisfactory taxonomy of problem-solving tasks has been developed. Some problems can be solved using an algorithm or formula, which leads to

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[1] R. C. Anderson, "Can First Graders Learn an Advanced Problem-solving Skill," Journal of Educational Psychology, LVI (1965), pp. 283-94.

the solution in a finite number of steps. Others can only be solved by the use of heuristics which improve the possibilities of reaching, but do not guarantee, a solution.

Gagne[1] states that problem solving depends upon rules, and especially upon a particular type of rule governing the individual's own thinking behavior called a strategy. He mentions eight types of learning from signal-learning to problem solving, each of which is a prerequisite of the other, with problem solving at the summit. He maintains that "problem solving results in the acquisition of new ideas that multiply the applicability of rules previously learned. As is the case with other forms of learning, its occurrence is founded on these previously learned capabilities; it does not take place in a vacuum, devoid of any prior knowledge. Learning by problem solving leads to new capabilities for further thinking. Included among these are not only the higher-order rules but also "sets" and "strategies" that serve to determine the direction of thinking . . . and therefore its productiveness."[2]

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[1]Robert M. Gagne, The Conditions of Learning (New York: Holt, Rinehart and Winston, 1969).

[2]Ibid.

Ausubel[1] refers to trial-and-error problem solving as inevitable where no meaningful pattern of relationships exists or is discernible, being used extensively in the solution of most mazes and complex puzzle box problems. He refers to insightful problem solving as a type of meaningful discovery in which problem conditions and desired objectives are nonarbitrarily and substantively related to existing cognitive structures.

Guilford's[2] conceptualization of the "structure of intellect" may be considered another type of approach to the study of cognitive mediating processes. His factorial studies led to an unprecedented amount of research in regard to creativity and problem solving, his distinction between divergent and convergent thinking being of particular value. Creative solutions to problems are unique and original. Because of the relationship of divergent thinking to creativity, some individuals, who possess this type of thinking, can find many different solutions to the same problem.

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[1]David P. Ausubel, Educational Psychology: A Cognitive View (New York: Holt, Rinehart and Winston, Inc., 1968).

[2]J. P. Guilford, "Three Faces of Intellect," American Psychologist, XIV (1959), pp. 469-79.

Each person must be his own problem solver and must learn his own strategies. Many factors influence the mental behavior of the problem solver in each situation such as experiential background, sense of achieving, interests, meaningful purpose and understanding of the concepts and principles involved. The values and skills that are productive in dealing with real and vicarious problems experienced in personal and community living are learned ways of thinking and acting. Competency in living and working in a society with advanced technology such as our own is dependent upon a person's understanding of, and skills in using, the scientific method of problem solving in personal, social, and economic situations.

Just as adults are faced with problems in their daily living, so young children are also confronted with their own special problems. For a young child, a problem may just be how to divide a bag of candy among his friends, or how to become more proficient at throwing a baseball, but to him these are real problems. How he solves those problems will depend upon what skills he has previously learned. There is little doubt that an accumulation of related knowledge has bearing on the solution of problems. The abilities of perceiving a situation, reasoning analytically, thinking logically, and making judgments are also generally recognized as significant factors in problem solving.

There is concern, which has been expressed both by teachers and parents, that the skill of verbal problem solving is not being well developed in the formal education program. It is a known fact that many people have difficulty in coping with problems involving numbers or mathematical concepts. As verbal problems in arithmetic provide the essential ingredients of problematic situations in real life, it is imperative that educators find some way of developing in every youngster a general approach toward the solution of arithmetic problems that is transferable to problems of every day living.

Review of Related Literature

As the area of problem solving encompases all aspects of the school curriculum and is probably one of the major, if not _the_ major objective of education, it would be an impossible task to review all the literature in this field. The purpose of this section is to present a review of the major theoretical and philosophical contributions pertinent to this study.

One of the theories of problem solving that has had the most influence in the area of education is that of John Dewey.[1] He believed that problem solving involved five stages and that any problem solver who experienced failure did so because of failure in one of these steps. One of the criticisms of Dewey's theory is that much of what is termed problem solving does not take place always according to this type of sequence of events. For instance, some scientists describe the problem-solving process in a manner diametrically opposed to that of Dewey. Einstein noted that when he had tried to solve a problem without success, he sometimes abandoned the problem in pursuit of some other matters. Suddenly, "out of the clear blue sky" the solution would occur to him. Although he recognized that this was the solution to the problem, he would not always understand why it was the solution and formal demonstration of its correctness often came at a much later date and after considerable struggle. This process of problem solving did not occur after testing successive solutions at the conscious level, as Dewey described, but was somehow generated by the operation of some sub-conscious process

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[1]John Dewey, How We Think (Boston: D. C. Heath and Co., 1933).

over which the problem solver had no control. The great French psychologist, Binet, arrived at a similar conclusion after experimental studies of the nature of intelligence and wrote that the essential elements of problem solving were not observable to the problem solver himself and could not be included among the inner mental processes which he could observe.

The major historical conflict of theories has been between associationistic theories of problem solving and Gestalt theories. The associationistic theories of problem solving assumed that problems consisted of component stimuli which led to responses through the operation of associations which, in the more advanced forms of the theory, were connections in the nervous system. The associationist attempted to break down problem-solving behavior into elements but, lack of knowledge of what the elements were, tended to make the theory of questionable merit. The Gestalt psychologists rejected this theory and postulated that problem solvers responded to systems of relationships. Here again, the difficulty lay in the inability to describe the problem-solving processes with the precision needed for testing whether they can account for the success in problem solving.

An entirely new approach to many scientific problems has been developed in recent years through the availability of high-speed computers. Psychologists have also attempted to study complex behaviors in this way, especially those behaviors designated as thinking or problem-solving behaviors. Methods of solving problems that have been built into modern electronic computers closely resemble those used by humans solving similar problems. Newell, Shaw, and Simon[1] had college students "think aloud" as they worked on problems in symbolic logic, and then compared their methods with detailed records of the processes employed by a computer program. The resemblances between the problem-solving processes of the computer program and the human mind are summarized by the writers as follows:

> First, and perhaps most important, it is in fact capable of finding proofs for theorems--hence incorporates a system of processes that is sufficient for a problem-solving mechanism. Second, its ability to solve a particular problem depends on the sequence in which problems are presented to it in much the same way a human subject's behavior depends on this sequence. Third, its behavior exhibits both preparatory and directional set. Fourth, it exhibits insight both in the sense of vicarious trial and error leading to 'sudden' problem solution, and in the sense of employing heuristics to keep the

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[1]A. Newell; T. C. Shaw; and H. A. Simon, "Elements of a Theory of Human Problem Solving," Psychological Review, 1958, 65, pp. 151-66.

> total trial and error within reasonable bounds. Fifth, it employs simple concepts to classify the expressions with which it deals. Sixth, its program exhibits a complex organized hierarchy of problems and subproblems.

Some research has been undertaken on problem solving which utilizes an intervening-variable theory. Intervening variables are sought which will make it possible to predict problem-solving behavior. The most important of these variables in terms of predictive power are those which have been commonly designated as aptitudes. At least two reasoning factors, or aptitudes have been identified. However, to what extent they are effective has not yet been fully determined.

Kleinmuntz[1] reports two major theoretical approaches for problem solving today - (a) neo-behaviorist and (b) information-processing. The neo-behaviorists attempt to understand problem-solving processes by using principles derived from laboratory studies of human learning. Those who adopt an information-processing orientation assume that computers offer a useful research model.

Research studies on problem solving in the classroom have been difficult to carry out. This may be due to the

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[1]B. Kleinmuntz, Problem Solving: Research, Method and Theory (New York: John Wiley and Sons, Inc., 1966).

fact that classroom problems are somewhat heterogeneous, and, therefore, it is difficult to describe with precision children's behavior in solving them. One other reason may be the school's emphasis on the product rather than the process. Observations of the process in problem solving have been attempted by some researchers. Those pertinent to this study are reviewed below.

Duncker and Krechevsky,[1] after working with both the problem solving of human subjects and the learning of rats, found that their work in the two fields yielded somewhat similar conclusions concerning problem solving. Duncker's method was to present the subject with a rational problem and ask him to "think aloud" about it. His conclusion was that problem solving has two principal aspects: (a) analysis of the situation and (b) analysis of the goal. The subject attempts to find the source of difficulty but also considers what must be done in order to reach a satisfactory conclusion.

Some studies have attempted to analyze the thought processes of children while they were solving problems by

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[1]K. Duncker, "On Problem-Solving," trans. by Lynne S. Lees, Psychological Monograph, LVIII (1945), No. 270.

having the subjects think aloud. Corle,[1] working with seventy-four sixth grade pupils, administered eight problems typical of those appearing in sixth grade arithmetic text books. After each problem had been solved on paper, the children were interviewed individually and asked to explain step-by-step exactly how they arrived at the solution. Corle found that 114 computational errors occurred, although 70 were solved by the correct process. He attributed the errors to unclear concepts and a lack of understanding of quantitative vocabulary.

The thought processes of second grade children as related to subtraction problems were studied by Gibb.[2] Each of the 36 children was given nine problems to solve, three which could be characterized as "take-away," three as "additive," and three as "comparative," all of which could be solved using the subtraction process. Problems used were one step, with a minuend of ten or less, and children responded to them orally. Interviews were recorded on tape and unusual observable behavior was noted. One of the

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[1]Clyde G. Corle, "Thought Processes in Grade Six Problems," Arithmetic Teacher, V (October, 1958), pp. 193-203.

[2]E. Glenadine Gibb, "Children's Thinking in the Process of Subtraction," Journal of Experimental Education, XXV (September, 1956), pp. 71-80.

problems of each type was clothed in a concrete situation, one in a semi-concrete situation, and one was abstract. It was found that types of subtraction problems do not appear mathematically or psychologically the same to children of this age. There was a tendency to solve problems in accordance with the reality of the problem situation. It may be that a teacher perceives a concept one way and a child another way and a method of solution may be taught without consideration of how children think about various types of problem situations. This study points out that there are many situations solved by subtraction where a child does not think in a subtractive way.

McGeoch[1] saw all problem solving as a form of trial and error while Boelter[2] outlined three methods of problem solving: (a) the experimental, (b) the method of models, and (c) the analytic method.

Durkin[3] identified three strategies for solving

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[1]J. A. McGeoch, The Psychology of Human Learning (New York: Longman's, 1942).

[2]L. M. K. Boelter, "Technique of Problem Solving," Journal of Philosophy, Vol. 40, 1943, pp. 127-32.

[3]Helen E. Durkin, "Trial-and-Error, Gradual Analysis, and Sudden Reorganization: An Experimental Study of Problem Solving," Arch. Psychology, 1937, Vol. 30, No. 210.

problems: (a) trial and error, (b) insight, and (c) gradual analysis. Baker and his associates found the same varieties of attack in a group of seven-year-olds who worked on problems similar to those used by Kohler, Maier, and Duncker.

Guest[1] found that three different types of thinking were distinguishable, (a) superficial-illogical, (b) concrete-specific, and (c) analytic-deductive.

In a study with second and third grade children, Keisler and Stern[2] taught half the subjects to use a special heuristic by which they could systematically find the answer. The other half were required to solve these problems on their own finding out whatever strategies might work. It was found on the posttest that there was very little difference in the overall performance of the two groups but, contrary to expectations, the group exposed to a systematic strategy showed greater generalization by performing significantly better on a problem solving transfer

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[1]M. Guest, "Process and Product in Generalizing" (Doctoral Thesis, University of California, 1953).

[2]E. R. Keisler and C. Stern, "Acquisition by Young Children of Strategies for Concept Identification Problems." Proceedings of the 73rd Annual Convention, American Psychological Association, Washington, D. C., 1965, pp. 321-22.

test and retained the skill better on a retention test given two weeks later. The children who were left to solve the problems without instruction apparently discovered something which only worked for that particular set of problems; it did not generalize and was not retained.

Gagne and Brown[1] in an attempt to find out if children do learn best by a process of "learning by discovery" used the task where the subjects were required to find the formula for the sum of n terms in a number series. They found that students given only slight guidance, the Discovery Group, did not learn as well as a group given far more specific direction, the guided Discovery Group.

It appears that at the moment we know too little about what to teach students to aid them in the problem-solving process. Perhaps if we were to follow Gagne's hierarchy and made certain that children had the necessary language and motor skills and had acquired the relevant concepts and understanding of the necessary prerequisite principles, this would lead to more efficient problem solving.

Most of the procedures currently available for the

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[1]R. M. Gagne and L. T. Brown, "Some Factors in the Programming of Conceptual Learning, 22 Journal of Experimental Psychology, 1961, pp. 313-21.

teaching of problem solving are quite informal, such as the heuristics proposed by Polya.[1]

Many characteristics of children have been measured in an attempt to learn more about problem-solving behavior. Included have been such factors as intelligence quotient, achievement, chronological age, reading ability, effort, attitude, socio-economic status and sex.

The most extensive study relating reasoning ability to other measures of problem solving was undertaken by McNemar.[2] A battery of four types of reasoning items, not correlated highly with intelligence, was used to select a group of high and a group of low reasoners from a large group of students. It was found that the highs were better in accuracy and speed on the induction problem, but better only in accuracy on the deduction problem. Varied results accrued when the subjects were questioned about their use of various methods of attack on problems. However, the results did reveal that highs were better at "selecting"

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[1] G. Polya, How to Solve It. A New Aspect of Mathematical Method (New York: Doubleday and Co., Inc., 1957).

[2] Olga W. McNemar, "An Attempt to Differentiate Between Individuals with High and Low Reasoning Ability," American Journal of Psychology, LXVIII (1955), pp. 20-36.

among relevant and irrelevant aspects of past experience and the over-all picture showed that highs solved more problems than lows. It would seem that reasoning, as measured by tests, has been found to be related to most measures of problem-solving performance.

Factor analysis of tests by Burt,[1] Davis,[2] and Guilford[3] seem to support McNemar's findings that there is a reasoning factor which correlates highly with problem-solving ability.

Klausmeier and Loughlin[4] used arithmetic problems in which 11-year olds were given actual coins and bills and were required to make up amounts of money using a given number of coins. In analyzing problem-solving behavior as a function of intelligence quotient, he found that subjects with high intelligence quotients were markedly superior in

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[1]Cyril L. Burt, Mental and Scholastic Tests (London: Staples Press, Ltd., 1947.

[2]F. B. Davis, "An Analytical Study of Reasoning Among High School Boys," American Psychologist, Vol. 3 (1948), p. 257.

[3]J. P. Guilford and J. I. Lacey (eds.). Printed Classification Tests. Army Air Forces Aviation Psychology Program Research Report, No. 5. Washington, D. C.: Government Printing Office, 1947.

[4]M. J. Klausmeier and L. J. Loughlin, "Behaviors During Problem Solving Among Children of Low, Average and High Intelligence," Journal of Educational Psychology, LXIX (1961), pp. 148-152.

verifying proposed solutions and used more logical approaches than the subjects with average intelligence quotients.

Koyanagi and Gorman[1] also compared problem solving techniques of high and low mental ability subjects and found that the bright children were superior.

Two factor analytic studies of problem solving by Werdelin[2] indicated that the five factors isolated in each study were virtually identical. Test of problem solving loaded most strongly on a General Reasoning Factor and to a lesser extent on a Deductive reasoning and a Numerical factor. The other factors, Space and Verbal Comprehension, were unrelated to problem solving.

Erickson[3] found a correlation of .58 between intelligence quotient and problem-solving scores. However, Erickson points out the misleading qualities of these correlations, and the serious dangers of over-generalizing. When he divided the children into the upper twenty-seven

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[1]B. R. Gorman, "The Effect of Varying Kinds and Amounts of Information as Guidance in Problem Solving," Psychological Monogram, LXXI (1957).

[2]I. A. Werdelin, "A Synthesis of Two Factor Analyses of Problem Solving in Mathematics," Didakometary, No. 8, Sweden, 1966.

[3]Leland H. Erickson, "Certain Ability Factors and Their Effect on Arithmetic Achievement," Arithmetic Teacher, V (December, 1958), pp. 287-93.

per cent, the middle forty-six per cent, and the lower twenty-seven per cent in terms of total arithmetic test scores, he found no significant correlation for any of the three groups between intelligence quotient and problem-solving scores.

Brownell and Stretch[1] studied the effects of unfamiliar settings on problem solving by such devices as substituting foreign terms for money, areas, etc., in otherwise standard problems. They felt that the study was inconclusive and stated, "It is impossible to give an absolute answer to the question regarding the influence of unfamiliar settings. Complicating factors must be considered. Children most affected are those least skilled in operations and computations in problem solving."

Doty[2] in a study interviewed 151 fourth-grade, and 31 sixth-grade pupils to find out what methods they used as they solved verbal problems in arithmetic. He found that procedures resulting in an incorrect solution included:

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[1]William A. Brownell and L. B. Stretch, Effects of Unfamiliar Settings as Problem Solving (Durham: Duke University Press, 1931), pp. 57, 71.

[2]R. A. Doty, "A Study of Children's Procedures in the Solution of Verbal Problems" (unpublished Doctoral dissertation, Duke University, 1940).

(1) determining the solution from the numbers in the problem and from verbal clues, (2) neglecting the questions asked, (3) disregarding the data given, (4) selecting a process which was appropriate to only one part of the problem, (5) working one step before reading aloud, and (6) estimating the size of the answer and then juggling the figures to get that answer. More positively, six procedures leading to the attainment of the correct solution were (1) rereading the problem before computing it as a check on comprehension, (2) checking the problem, (3) labeling answers, (4) evaluating work as to reasonableness, (5) visualizing the problem, (6) recalling analogous problems. Doty also mentioned that one element of difficulty was the unfamiliarity of the problem setting.

Some studies have concentrated on the merits of selecting problems which are closely related to the students' interests and experience. In 1967, Travers[1] presented to high school freshmen two problems that were identical in structure (numbers used, operations required, etc.) but different in setting. The students were required to choose

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[1]Kenneth J. Travers, "A Test of Pupil Preference for Problem-Solving Situations in Junior High School Mathematics," Journal of Experimental Education, 1967 (1967), pp. 9-18.

and solve one of two problems. It was found that students showed a strong preference for problems dealing with social-economic situations in comparison with scientific-mechanical situations and abstract situations (such as solving secret codes).

Klausmeier and Loughlin[1] studied the characteristics of children of different levels of intelligence in solving problems graded according to their level of ability. Three groups of fifth grade children of different levels of intelligence were used. Problems of equal difficulty, determined by an equal length of time necessary to reach a solution, were presented to each group. Differences in problem solving were noted, in spite of the gradation of problems. Lower ability groups showed less persistence, greater acceptance of incorrect solutions, and greater use of random attack than the other two groups. Conversely, high ability groups took greater note of mistakes and their correction, verification of solutions, and logical sequence in problem solving.

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[1]H. J. Klausmeier and L. J. Loughlin, "Behaviors During Problem-Solving Among Children of Low, Average, and High Intelligence," Journal of Educational Psychology, LXI (1961), pp. 442-48.

Herrick[1] has reported on studies in relation to problem solving and cultural background. The results showed that there is a significant relationship between problem-solving performance and socio-economic background. Children from high status groups consistently do better than lower class children.

In his dissertation, Day[2] studied the relationship between socio-economic status and problem solving. After administering tests of problem-solving ability, he divided his group into Group A (high problem-solving ability) and Group B (low problem-solving ability). He found that more children in Group A were in the high socio-economic status bracket than in Group B.

Summary

The review of the literature revealed that most of the investigations in the area of problem solving in arithmetic had been devoted to treatments and training procedures, or

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[1] Virgil E. Herrick, "What is Already Known About the Relation of the Intelligence Quotient to Cultural Background," Intelligence and Cultural Differences (Chicago: The University of Chicago Press, 1951), pp. 10-23.

[2] David E. Day, "Relationship Between Socio-Economic Status and Problem Solving" (unpublished Doctoral dissertation, Wayne State University, 1962).

the relationships between problem solving ability and teacher or children characteristics.

Some studies attempted to determine the effect of a knowledge of the four basic operations - adding, subtracting, multiplying, and dividing - upon problem solving ability while others concentrated on unfamiliar settings or order of presentation of numerical data.

Contradictory findings were reported in several studies in relation to which variables are significant in distinguishing successful from non-successful problem solvers. The variables on which there was most general accord were those of intelligence, the reasoning factor included in most standardized mathematical tests, and quantitative vocabulary.

By far the most frequently cited strategies were of trial-and-error and insight. Blind guessing was cited and, May, in her recent publication, differentiated between "blind guessing" and "educated guessing," the latter being equated with trial-and-error. Guilford's structure of the intellect model led to the investigation of creativity as a means of problem solving and was the subject of studies by Torrance.

It is clear from the foregoing review that studies designed to identify and typify problem-solving strategies have been very few. Fewer still have been studies which sought to establish the interrelationships between strategy types, success, and pupil characteristics.

Specific Objectives

The purpose of this study is to investigate strategies which children in the fourth grade use to solve verbal arithmetic problems, and to determine if some are more successful than others. The study will also attempt to explore relationships between strategies and some characteristics of the children and to discover, through the technique of profile analysis, profiles of children with similar or identical characteristics.

Answers to the following specific questions will be sought:

1. Are there different strategies which children use to solve verbal arithmetic problems?

2. What is the relationship between strategies employed and success in problem solving?

3. Is there a significant interaction effect between strategy and the following pupil characteristics?

   a. Intelligence
   b. Arithmetic Achievement

c. Socio-economic Status
d. Interest in Puzzles
e. Hobbies
f. Parent Education

4. Is success more closely related to strategy than to the selected pupil characteristics?

5. Can profiles of children be identified containing strategy and the above-named characteristics?

Limitations and Assumptions

This study is based on tests administered to fourth grade children in a suburban school district. It is reasonable to assume that conditions in other suburban school districts do not differ significantly. To the extent that this is true, the conclusions drawn from the present study are generalizable to other populations.

Methodology

In order to determine which strategies children employ to solve verbal arithmetic problems and which strategies were most successful, a test of verbal arithmetic problem solving was administered to all 455 fourth-grade children attending a suburban school district. Eighteen fourth-grade teachers were asked to assist in the study by administering the test.

Each child was asked to solve only two problems. If a child had one or both correct, his performance was adjudged successful. Otherwise his performance was adjudged unsuccessful. The children were also asked to state as clearly as possible on their work-papers exactly how they solved the problems. Their written responses not only determined how successful they were but also assisted in identifying strategies.

Twenty-five problems were selected from the most widely-used textbooks in order to ensure a wide variety of problem types. The following is a list of the twenty-five problems:

1. Aunt Mary had 160 pretty buttons. She divided them equally among her four daughters. How many buttons did each girl get?

2. John's father bought a new car. He drove it 500 miles the first week and 267 miles the second week. How many more miles did he drive the first week than the second week?

3. Jerry has read 90 pages in a book. He has 34 more pages to read in order to finish the book. How many pages are in the book?

4. Ralph and his father have a garden, which is a rectangle, 30 feet long and 25 feet wide. How many feet of fence do they need to build a fence around the garden?

5. Helen had 54 United States stamps and 39 foreign stamps in her book. She also had 114 loose stamps. She gave Sally 25 stamps. How many stamps in all did she have left?

6. Mary wants to buy two books for $1.25 and $1.65. She has only $2.38. How much more does she need?

7. At a school luncheon each student was served half a grapefruit. How much did the grapefruit cost if 16 were bought and each cost 10 cents?

8. John needs airplane glue. It costs 8 cents a tube. How many tubes of glue can he get for 32 cents?

9. John bought some thin wire so that he could hang his airplanes from the ceiling in his room. He first cut pieces 45 inches long, but he found that pieces 19 inches long would be better. Can he cut two pieces from each of the longer pieces (Yes or No)? How much wire will be left over from each 45 inch piece?

10. Ann saw three of her friends at the flower show, 15 other friends at the 4H Building, and 18 other friends at the Livestock Building. How many of her friends did she see in all?

11. Betty bought two big pieces of cardboard and pasted leaves on them. There were 26 leaves on one board and 45 leaves on the other. How many leaves were in her collection?

12. During one week Fred sold 68 papers at 5 cents each. How much did he get for all the papers?

13. Bob has a collection of 24 model cars. If he arranges them on some shelves with 4 cars on each shelf, how many shelves will he need?

14. Eight frankfurters (hot dogs) cost 48 cents. How much does one frankfurter cost?

15. Brand A frankfurters are sold at 8 for 48 cents. Brand B are sold at 6 for 42 cents. Which would you buy to save money - 3 packages of Brand A or 4 packages of Brand B?

16. Objects weigh 6 times as much on earth as they do on the moon. If Glen's earth weight is 56lbs., how much will his moon weight be?

17. Ted had 45 cents. What is the greatest amount of candy bars at 5 cents each could he buy for his money?

18. Tom had 8 bags with 6 marbles in each bag. He had 29 extra marbles on the ground. How many marbles did he have in all?

19. Ted had 35 rocks. Jim had 63 rocks. Each boy decided to store his rocks, 7 rocks in one can. Who used more cans, Jim or Ted?

20. Jack's breakfast consists of juice 85 calories, cereal 95 calories, strawberries 43 calories, ½ cup milk 85 calories, and sugar 15 calories. How many calories does Jack eat at breakfast?

21. Carol got $5 for her birthday. She decided to buy some candy. She bought 6 candy bars at 10 cents each. How much did she have left?

22. A sleeping bag cost $18.98 and a tent cost $98.50. How much more does the tent cost than the sleeping bag?

23. Helen gave the clerk a $10 bill to pay for $4.73 worth of groceries. Part of her change was a $5 bill, a $1 bill, and a dime. Explain how you know the clerk made a mistake. What should the correct change be?

24. If one phonograph record cost 89 cents, how much would six records cost?

25. Jeff helps his father at the grocery store after school. He put 32 cans of peaches on the shelf in 8 rows of equal size. How many cans did he put on each row?

The criteria for the selection of the problems were as follows:

1. Those which tested cognitively complex processes rather than mechanical processes of addition, subtraction, multiplication, and division.

2. Those which were representative of word problems found in the most widely-used textbooks in our schools.

3. Those that involve processes of addition, subtraction, multiplication and division.

4. Those with one and two steps.

5. Those which involve whole numbers only.

6. Those with familiar and unfamiliar settings.

It was decided that, in order to sample many children and many problems, each child would be asked to solve only two problems, thus creating a data bank containing considerable information. Small booklets were constructed, containing two problems each, one per page, for administration to the 455 children.

The writer met with the 18 teachers involved in the study to explain the purposes of the research and to solicit their help in the administration of the verbal arithmetic problem-solving test. The teachers were given instructions on how to administer the test and were asked to adhere as strictly as possible to these instructions in order to ensure validity. During the in-service training session, it was decided that the tests be administered during the first period in the morning, all on the same day, so that the children would be fresh and alert.

Following the administration of the test individual interviews were scheduled on two occasions. The purposes of the interviews were (1) to ascertain exactly what strategies the children used to solve the two verbal arithmetic problems; (2) to determine why some children were unsuccessful, and (3) to obtain information relating to pupil characteristics which would be used to determine the relationship, if any, between these characteristics and success in problem solving and strategies employed. The first interview, which lasted approximately fifteen minutes, was completely unstructured and simply required that the children state in their own words exactly how they solved the problems. This information, plus the written answers to the problems, was used by the writer to determine the strategies employed. The second interview, which lasted approximately the same time, was for the purpose of determining the interests and hobbies of the children.

To determine the relationship between selected pupil characteristics and success in solving verbal problems and strategies employed, data relating to intelligence, arithmetic achievement, parent education and socio-economic status were secured from Cumulative Records maintained for each child. Data relating to pupil interests and hobbies were collected during the individual interviews.

The data for intelligence and achievement were in the form of scores extracted from the California Mental Maturity Test and the California Achievement Test which were administered to all children prior to the present experiment. The data for parent education and socio-economic status was in narrative form but were converted to numerical form for the purpose of analysis.

Definition of Terms

Strategies for Solving Problems

1. Trial and Error - This strategy is equated with the Gestalt psychologists' definition whereby the subject will attempt one process after another until a satisfactory solution is achieved.

2. Insight - The Gestalt psychologists' definition of insight will be employed to determine this strategy. The subject will view the situation as a whole, rather than one isolated part at a time, and may by "a flash of genius" solve the problem mentally.

3. Logical Analysis - Subjects who use this strategy will most likely solve the problems by mathematical equations or algorithms. Thinking will proceed in logical, sequential steps.

4. Creative or Divergent Thinking - Subjects who use this strategy will evolve creative and unusual solutions to the problems. They may even suggest many different solutions.

5. Blind Guessing - This strategy will be opposed to "educated" guessing or Trial and Error. The subject might indiscriminately add where there are many figures or subtract where there are only two figures. They might also include irrelevant data.

6. Intuition - Subjects who use this strategy may arrive at the answer, which may be right or wrong, with little if any awareness of the process by which they reached it.

Successful Problem Solvers

Those subjects who solve one or two problems correctly.

Unsuccessful Problem Solvers

Those subjects who do not solve any problems correctly.

One-step Problems

Problems which can be solved by using one process (adding, subtracting, multiplying or dividing) one time.

Two-step Problems

Problems which cannot be solved without a combination of any two of the processes of adding, subtracting, multiplying and dividing.

Familiar and Unfamiliar Settings

Familiar settings are those with which the subjects will have experience. Unfamiliar settings are those with which the subjects have had no experience.

Quantitative Vocabulary

Words such as sum, difference, quotient, product, equation and symbols such as is greater than, is less than, and equals.

Analysis of Data

The analysis of the data was carried out as follows:

1. Analysis of Verbal Arithmetic Test - The test of verbal arithmetic problems scored by the writer was the criterion for determining success. The written solutions to the problems plus the written statements explaining exactly how the problems were solved, plus the oral statements obtained through the individual interviews were the criteria for determining strategies.

2. Profile Analysis - In order to identify profiles and determine whether profiles fell into similar clusters, the technique of Profile Analysis was applied. Each subject's profile contained the following elements - intelligence quotient, arithmetic achievement, socio-economic status, interest in puzzles, hobbies, parent education, number of problems correct (success), and strategy employed to solve the problems.

   The shape-type[1] technique of profile analysis, which was employed to analyze the data, compares favorably with traditional measures of elevation and shape used to describe profiles. When shape-type criteria are used either to cluster profiles when groups are unknown, or to classify profiles into groups when the groups are known, the criteria not only indicate what clusters are similar but also in what ways the clusters are similar, how other clusters differ, and then describe the shape and other characteristics of the profile in each cluster.

   Profile shape is based upon gamma measures[2] of skewness and kurtosis from Pearson's β_1 and β_2 curve-type criteria around the central ordinal entry field of the profile. To

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[1]John L. Lindsey, "Shape-Type Criteria of Profiles" (unpublished Doctoral dissertation, Wayne State University, 1969).

[2]Ibid.

describe general profile types, the four shape-type criteria of profile elements were computed using coded data for each profile element for each student. Details of coded data are contained in Appendix A.

The resultant values of the mean, standard deviation, skewness, and kurtosis of the elements for each student's profile became the criteria for clustering profiles and describing profile types. The measures of skewness and kurtosis, used to describe shape, are based on moments taken about the mean on the horizontal axis of the profile (ordinal number of elements), while the mean and standard deviation of the profile used to designate elevation and scatter are taken along the vertical axis (or scalar values).

3. Correlation Matrix - In order to fulfill another purpose of the study, which was to investigate relationships between strategy and problem-solving success and between strategy and other selected pupil characteristics, and also between problem-solving success and the same characteristics, a Pearson Product-Moment Correlation was obtained.

CHAPTER II

FINDINGS

The major objective of this study was to discover how the children in the fourth grade in a suburban school district solved verbal arithmetic problems and to ascertain whether some strategies were more successful than others. Another objective was to investigate the relationships between success and selected pupil characteristics such as intelligence, arithmetic achievement, socio-economic status, interest in solving puzzles, hobbies and parent education and between strategy and the same characteristics. Another objective was to ascertain whether, by the technique of Profile Analysis, profiles of children could be identified and whether profiles fell into clusters.

The following is a detailed description of each problem indicating (1) whether it was one or two-step, (2) familiar or unfamiliar setting, (3) the number of children who had correct or incorrect answers, (4) process, and (5) explanatory remarks regarding difficulties encountered.

Analysis of Problems to Determine Success

Problem 1 was a one-step familiar setting division problem. Twenty-six children solved it correctly and 13 incorrectly. The children who worked the problem correctly understood that it could be solved by division, either by using the algorithm or writing an open sentence. The 13 unsuccessful problem solvers were either not sure how to deal with the zero or merely solved the problem by Blind Guessing. They made no effort to check the answer or ask if the answer were sensible. Those who looked for clues decided that the word "divided" was the key to the solution.

Problem 2 was a one-step familiar setting subtraction problem. Twenty-one children solved it correctly and 19 incorrectly. As a result of the interviews, it was found that the difficulty lay in the inability to regroup the figures in the minuend so that the subtraction process could be carried out effectively. Most of the children admitted that the words "how many more" made them decide that the process was subtraction.

Problem 3 was a one-step familiar setting addition problem. Thirty-four children solved it correctly and nine incorrectly. Most of the children realized that if Jerry had read 90 pages and was not finished, the total number of pages must be more than 90, to be exact, 90 plus 34 would give the correct result.

Problem 4 was a two-step unfamiliar setting problem. Six children solved it correctly and 23 incorrectly. The concept of perimeter was confused with area. Many who solved the problem correctly made a drawing which led to the realization that the amount of wire fence would be the addition of the four sides. Some who did not make a sketch found the area, while others added only one width and one length.

Problem 5 was a two-step familiar setting problem. Sixteen children solved it correctly and 26 incorrectly. Although the problem was a simple matter of adding and subtracting, it proved to be one with which the children had difficulty. Even the clue "How much did she have left?" did not facilitate the process. Others had difficulty regrouping before the subtraction process could be successfully carried out.

Problem 6 was a two-step familiar setting problem. Twenty-one children solved it correctly and 17 incorrectly. The interviews confirmed that, if the problem had been stated differently, the children might have been more successful in finding the solution. Many said they felt the first statement should have come second.

<u>Problem 7</u> was a one-step familiar setting multiplication problem. Twenty children solved it correctly and 16 incorrectly. There was, however, a factor which proved to be an obstacle. The first sentence stated that each student was served half a grapefruit. The interviews revealed that many children were unable to grasp that this figure was irrelevant and should not have been used in their calculations.

<u>Problem 8</u> was a one-step familiar setting division problem. Twenty-six children solved it correctly and 9 incorrectly. The diversity of processes used in the solution was interesting; for example, the algorithm, the open sentence or equation, the addition of eights until the total of 32 was attained and the subtraction of eights from 32 until it was reduced to zero. The interviews indicated that this problem was checked for accuracy by most of the students who were successful.

<u>Problem 9</u> was a one-step, unfamiliar setting subtraction problem. Seventeen children solved it correctly and 20 incorrectly. Interviews confirmed that some found

the problem too long to read. By the time they finished reading they had forgotten what was asked. Most of the children realized that they could cut 19" pieces of wire from pieces 45" long but did not write down the problem and subtract to find out how much was left over.

Problem 10 was a one-step familiar setting addition problem. Twenty-nine children solved it correctly and 9 incorrectly. Most children said they recognized the clue "How many friends did she see in all?" and responded to the clue. Most of the children chose the correct process but made mechanical errors probably because of the three addends.

Problem 11 was a one-step familiar setting addition problem. Twenty-two children solved it correctly and 10 incorrectly. Even with two addends there was considerable inaccuracy.

Problem 12 was a one-step familiar setting multiplication problem. Nineteen children solved it correctly and 14 incorrectly. Some children wrote an equation, some used an algorithm and others added 68 five times.

Problem 13 was a one-step familiar setting division problem. Twenty-two children solved it correctly and 12 incorrectly. Some children solved the problem by writing an equation, others by an algorithm. A few counted by fours until they reached 24, some subtracted four from 24 until they arrived at a zero remainder and one child made a pictorial illustration, drawing four cars on six shelves.

Problem 14 was a one-step familiar setting division problem. Twenty-four children solved it correctly and 17 incorrectly. The same strategies were attempted here as in Problem 13 which was basically the same type.

Problem 15 was a two-step familiar setting problem. Seventeen children solved it correctly and 17 incorrectly. The procedure necessary to obtain the correct answer was to multiply and then subtract. A few children found the solution easily by figuring out the cost of one hot dog of each brand and were able to state the answer without further calculation. Those who did not perceive that there was a quick way to solve it, worked it out step by step to arrive at the answer.

Problem 16 was a one-step unfamiliar setting division problem. Twelve children solved it correctly and 24 incorrectly. This proved to be one of the most difficult problems for the children. This was surprising considering the exposure children have to radio and television media and their apparent interest in space travel, astronauts and moon landings. The problem asked that they express an earth weight as a moon weight which could have been done by dividing. Most of the children used Blind Guessing tactics and tried the other three processes. At the interview, some admitted that they were confused by the expression that a person's earth weight is six times their moon weight. The two words six times caused them to multiply instead of divide. Those who were unsuccessful admitted that they had not asked themselves if their answers were sensible.

Problem 17 was a one-step familiar setting division problem. Twenty-eight children solved it correctly and 9 incorrectly. Like other division problems in the set, this one produced a plethora of solutions similar to Problem 13.

Problem 18 was a two-step familiar setting problem. Fourteen children solved it correctly and 17 incorrectly. Most of the children who had the wrong solution failed to see that they had to multiply 8 x 8 to find out how many marbles were in the bags and then add 29, the number of marbles lying on the ground. Some eliminated the difficulty by drawing eight bags with eight marbles in each and added 29 to the sum.

<u>Problem 19</u> was a two-step familiar setting problem. Sixteen children solved it correctly and 17 incorrectly. The two processes necessary for solution were division and subtraction. However, during the interviews, it was evident that some of the more astute children came to the early conclusion that Jim, who had more rocks, would require more cans and, therefore, were able to state the answer without more calculation.

<u>Problem 20</u> was a one-step unfamiliar setting problem. Thirteen children solved it correctly and 20 incorrectly. All that was required to find the answer was to add a column of figures. There was, however, an irrelevant figure in the problem which puzzled some of the students. The problem required that they find the total calories a child ate for breakfast and one item was stated - ½ cup milk - 85 calories. Many of them tried to include the one half in the answer but, as most of them were not conversant with fractions, some strange results accrued. There were five two-digit figures to add and perhaps this made the problem difficult. Most of the errors in this

problem, apart from the inclusion of the one-half, were computational in nature.

Problem 21 was a two-step familiar setting problem. Thirteen children solved it correctly and 25 incorrectly. The problem could have been solved by multiplying 6 x 10¢ to find out the cost of candy bars and then subtracting the 60¢ from $5 to find out how much money was left. During the interviews, the reason for the lack of success was revealed. Most of the children did not know how to write $5 in order to subtract 60¢. It was quite a revelation to discover that the children were unable to write $5 thus - "$5.00," and then proceed to subtract. Regrouping the values for easy subtraction of $5 also posed a major obstacle for some.

Problem 22 was a one-step familiar setting subtraction problem. Eighteen children solved it correctly and 18 incorrectly. The clue "How much more?" triggered the subtraction process for the majority but again regrouping presented for some an insurmountable barrier. When interviewed, those who employed the correct procedure but still failed to find the correct answer admitted that they had not estimated nor had they checked the result by adding the two bottom lines of the subtraction problem.

<u>Problem 23</u> was a two-step unfamiliar setting problem. Twelve children solved it correctly and 23 incorrectly. Children obviously need a considerable amount of experience with live situations where goods are purchased and money rendered in payment. During the interviews, it was found that their knowledge of how much change they should have received was very limited. This may be due to the fact that parents pay by check very often or charge the goods. Children do not seem to be sent on errands which involve cash transactions. As in some previous problems, it was discovered that the children were unable to rewrite $10 as $10.00 and regroup before subtraction.

<u>Problem 24</u> was a one-step familiar setting multiplication problem. Thirty-one children solved it correctly and 9 incorrectly. The children who multiplied 89¢ by 6 met with greater success than those who tried to add 89¢ six times.

<u>Problem 25</u> was a one-step familiar setting division problem. Seventeen children solved it correctly and 15 incorrectly. Again, as in other divisions, the procedures used to find the solution were varied.

The details in the preceding twenty-five paragraphs have been developed into a table for ease of comparison and for summarization. The table reads as follows:

TABLE I

LIST OF PROBLEMS SHOWING NUMBER OF CHILDREN WHO SOLVED THEM CORRECTLY AND INCORRECTLY, NUMBER OF STEPS, TYPE OF SETTING AND PROCESS NECESSARY FOR SOLUTION

No.	No. of Steps	Type of Setting	Process	No. of Children Correct	No. of Children Incorrect
1	1	Familiar	Division	26	13
2	1	Familiar	Subtraction	21	19
3	1	Familiar	Addition	34	9
4	2	Unfamiliar	Addition and Multiplication	6	23
5	2	Familiar	Addition and Subtraction	16	26
6	2	Familiar	Addition and Subtraction	21	17
7	1	Familiar	Multiplication	20	16
8	1	Familiar	Division	26	9
9	1	Unfamiliar	Subtraction	17	20
10	1	Familiar	Addition	29	9
11	1	Familiar	Addition	22	14
12	1	Familiar	Multiplication	19	14
13	1	Familiar	Division	22	12
14	1	Familiar	Division	24	17
15	2	Familiar	Multiplication & Subtraction	17	17
16	1	Unfamiliar	Division	12	24
17	1	Familiar	Division	28	9
18	2	Familiar	Multiplication and Addition	14	17
19	2	Familiar	Division and Subtraction	16	17
20	1	Unfamiliar	Addition	13	24
21	2	Familiar	Multiplication & Subtraction	13	25
22	1	Familiar	Subtraction	18	18
23	2	Unfamiliar	Addition and Subtraction	12	23
24	1	Familiar	Multiplication	31	9
25	1	Familiar	Division	17	15
Totals				494	416

Strategies Used by Children to Solve Verbal Arithmetic Problems

Strategies were identified (1) by analyzing the written solutions of the two problems, (2) by analyzing the written statements whereby the subjects attempted to explain exactly how they solved the problems and (3) by analyzing the oral statements obtained at the interviews. Strategies were then classified according to the definitions on page 15.

Of the 455 subjects, 304 employed the strategy of Logical Analysis, 20 used Creative or Divergent Thinking, 23 Trial and Error, 1 Insightful Thinking, 3 Intuitive Thinking, and 104 Blind Guessing.

In the pages which follow, the 25 problems have been classified according to number of steps, familiar or unfamiliar setting, and process. Examples of the strategies employed are also stated.

Strategies Employed to Solve Problems

Problems 1, 8, 13, 14, 17 and 25 were all one-step problems with familiar settings and the solution process was division.

Some of the strategies used to solve Problem 1 were as follows:

Problem 1. Aunt Mary had 160 pretty buttons. She divided them equally among her four daughters. How many buttons did each girl get?

Logical Analysis

1. $160 \div 4 = 40$

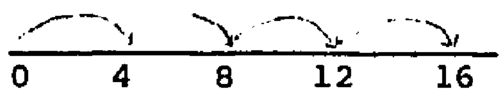
2. $4\overline{)24}$ with quotient 6

   24 6 x 4

   0

Creative

1. Drawing number line



   There are four four's in sixteen, therefore there are 40 four's in 160.

2. Subtracting 4 from 16 until 0 remained and stating, since there were four four's in 16, there would be 40 in 160.

3. Stating three or four combinations of the above.

Blind Guessing

1. Multiplying 4 x 160
2. Adding 160 + 4
3. Subtracting 160 - 4

Trial and Error

1. 160 x 4 = 640

2. Decided that the answer was too big as there were more buttons than we started off with.

3. 160 - 4 = 40

Problems 2 and 22 were one-step problems with familiar settings and the solution process was subtraction.

Some of the strategies used to solve Problem 22 were as follows:

Problem 22. A sleeping bag cost \$18.98 and a tent cost \$98.50. How much more does the tent cost than the sleeping bag?

Logical Analysis

1. \$98.50 - \$18.98 = n, therefore n = \$79.52.

2. \$18.98 plus n = \$98.50, therefore n = \$79.52.

3. $$\begin{array}{r} \$98.50 \\ \underline{-18.98} \\ \$79.52 \end{array}$$

Creative

1. One child said she worked the problem in her head by estimating \$18.98 as \$19 and \$98.50 as \$99. Her answer was correct - \$79.52. When asked at the oral interview if there were other ways of doing the problem she showed the algorithm.

Blind Guessing

1. $$\begin{array}{r} \$\ 98.50 \\ \underline{18.98} \\ \$117.48 \end{array}$$

2. \$98.50 plus \$18.98 = \$106.48

Problems 3, 10, 11 and 20 were one-step, familiar setting problems and the solution process was addition.

Some of the strategies used to solve Problem 3 were as follows:

Problem 3. Jerry has read 90 pages in a book. He has 34 more pages to read in order to finish the book. How many pages are in the book?

Logical Analysis

1. $$\begin{array}{r} 90 \\ \underline{+34} \\ 124 \end{array}$$

2. 90 plus 10 equals 100 plus 24 equals 124.

3. 90 plus 30 equals 120 plus 4 equals 124.

Blind Guessing

1. $$\begin{array}{r} 90 \\ \underline{-34} \\ 64 \end{array}$$

Trial and Error

1. 90 - 34 = 56

   This must be wrong because the boy had already read 90 pages so there must be more than 90 pages in the book.

2. 90 + 34 = 124

Problems 7, 12 and 24 were one-step, familiar setting problems and the solution process was multiplication.

Some of the strategies used to solve Problem 12 were as follows:

Problem 12. During one week Fred sold 68 papers at 5 cents each. How much did he get for all the papers.

Logical Analysis

1. $$\begin{array}{r} \$\ .05 \\ \underline{\times\ \ 68} \\ \$3.40 \end{array}$$

2. 68 x 5¢ = $3.40

3. 68 x 5¢ = 340¢ = $3.40

4. 68 plus 68 plus 68 plus 68 plus 68 = 340 - Answer $3.40

Creative

5. I said 20 nickels make a dollar so there are $3 and 8 nickels which is $3.40

Blind Guessing

6. 68 plus 5, 68 - 5

Some of the strategies used to solve Problem 24 were as follows:

Problem 24. If one phonograph record cost 89 cents, how much would six records cost?

Logical Analysis

1. $ .89 x 6 = n, n = $5.34

2. $ .89
   x 6
$5.34

3. Adding .89 six times - Answer $5.34

Creative

4. Estimating $.90 for $89 and taking away 6¢

Blind Guessing

5. $.89 plus 6

Problem 9 was a one-step problem with an unfamiliar setting and the solution process was subtraction.

Some of the strategies used to solve the problem were as follows:

> <u>Problem 9</u>. John bought some thin wire so that he could hang his airplanes from the ceiling in his room. He first cut pieces 45 inches long, but he found that pieces 19 inches long would be better. Can he cut two pieces from each of the longer pieces (Yes or No)? How much wire will be left over from each 45 inch piece?

<u>Logical Analysis</u>

1. $45 - 19 = 26$

2. $\begin{array}{r} 45 \\ \underline{-19} \\ 26 \end{array}$

3. 19 plus 1 equals 20 and 25 more equals 45

<u>Blind Guessing</u>

4. 45 plus 19

Problem 20 was a one-step unfamiliar setting problem whose solution could be found by addition.

Some strategies used to solve Problem 20 were as follows:

> <u>Problem 20</u>. Jack's breakfast consists of juice 85 calories, cereal 95 calories, strawberries 43 calories, ½ cup milk 85 calories, and sugar 15 calories. How many calories does Jack eat at breakfast?

<u>Logical Analysis</u>

1. $85 + 95 + 43 + 85 + 15 = 323$

2.
```
 85
 95
 43
 85
 15
___
323
```

3. $(85 + 95) + (85 + 15) + 43 = 323$

Problem 16 was a one-step unfamiliar setting problem and the solution process was division.

Some strategies used to solve the problem were as follows:

> <u>Problem 16</u>. Objects weigh 6 times as much on earth as they do on the moon. Glen's earth weight is 56 lbs how much will his moon weight be?

<u>Logical Analysis</u>

1.
```
   9R2
 6/56
  -54   9 x 6
    2
```

2. $6 \times n = 56$

<u>Blind Guessing</u>

3. $56 \times 9 = 504$

<u>Trial and Error</u>

4. $56 \times 9 = 504$

I know this is wrong because people are lighter on the moon so I divided.

$56 \div 9 = 6$ remainder 2.

Problems 5 and 6 were two-step unfamiliar setting problems and the solution processes were addition and subtraction.

Some strategies used to solve Problem 5 were as follows:

<u>Problem 5</u>. Helen had 54 United States stamps and 39 foreign stamps in her book. She also had 114 loose stamps. She gave Sally 25 stamps. How many stamps in all did she have left?

<u>Logical Analysis</u>

1. $54 + 39 + 114 - 25 = 182$

2.
$$\begin{array}{r} 54 \\ 39 \\ \underline{+114} \\ 207 \\ \underline{-25} \\ \underline{\underline{182}} \end{array}$$

<u>Blind Guessing</u>

3.
$$\begin{array}{r} 54 \\ 39 \\ 114 \\ \underline{25} \\ \underline{\underline{232}} \end{array}$$

Problems 15 and 21 were two-step unfamiliar setting problems and the solution processes were multiplication and subtraction.

Some of the strategies used to solve Problem 15 were as follows:

<u>Problem 15</u>. Brand A frankfurters are sold at 8 for 48 cents. Brand B are sold at 6 for 42 cents. Which would you buy to save money - 3 packages of Brand A or 4 packages of Brand B?

Logical Analysis

1. $\begin{array}{r} \$\ .48 \\ \underline{\times\ 3} \\ \underline{\underline{\$1.44}} \end{array}$ $\quad$ $\begin{array}{r} \$\ .42 \\ \underline{\times\ 4} \\ \underline{\underline{\$1.68}} \end{array}$ Answer - Brand A is cheaper

2. $\begin{array}{r} \$\ .48 \\ .48 \\ \underline{.48} \\ \underline{\underline{\$1.44}} \end{array}$ $\quad$ $\begin{array}{r} \$\ .42 \\ .42 \\ .42 \\ \underline{.42} \\ \underline{\underline{\$1.68}} \end{array}$ Brand A

Creative

3. Brand A cost 6¢ each and Brand B cost 7¢ each so Brand A would be cheaper.

Blind Guessing

4. 48¢ x 8 = \$3.84 $\quad$ \$.42 x 6 = \$2.52 $\quad$ - Brand B

Problem 19 was a two-step familiar setting problem and the solution process was multiplication and division.

Some of the strategies used to solve the problem were as follows:

Problem 19. Ted had 35 rocks. Jim had 63 rocks. Each boy decided to store his rocks, 7 rocks in one can. Who used more cans, Jim or Ted?

Logical Analysis

1. $35 \div 7 = 5$, $63 \div 7 = 9$ Jim needed 4 more cans than Ted.

2. $\begin{array}{r} 5 \\ 7\overline{)35} \\ \underline{-35} \end{array}$ $\quad 5 \times 7 \quad$ $\begin{array}{r} 9 \\ 7\overline{)63} \\ \underline{-63} \end{array}$ $\quad 9 \times 7 \quad$ Jim needed 4 more.

3. Counting by sevens until 35 and 63 were reached.

<u>Creative</u>

4. Using the number line.

5. Subtracting 7 from 35 and 63 until zero was reached.

6. Combination of above methods.

<u>Intuitive</u>

7. Jim had more marbles to start with so Jim needed more cans.

Problem 4 was a two-step problem with an unfamiliar setting which could have been solved by addition or multiplication and addition.

Some of the strategies used to solve the problem were as follows:

<u>Problem 4</u>. Ralph and his father have a garden, which is a rectangle, 30 feet long and 25 feet wide. How many feet of fence do they need to build a fence around the garden?

<u>Logical Analysis</u>

1. $30 + 30 + 25 + 25 = 110$
2. $(2 \times 30) + (2 \times 25) = 110$
3. $(30 + 30) + (25 + 25) = 110$

4. $\begin{array}{r} 30 \\ \underline{\times\ 2} \\ \underline{\underline{60}} \end{array} \quad \begin{array}{r} 25 \\ \underline{\times\ 2} \\ \underline{\underline{50}} \end{array} \qquad 60 + 50 = 110$

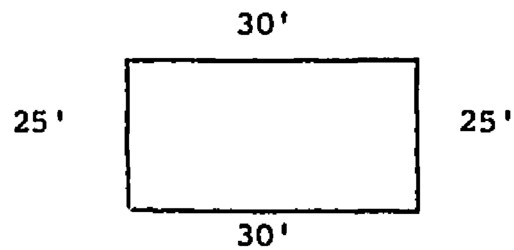
5. $\begin{array}{r} 30 \\ 30 \\ 25 \\ \underline{25} \\ \underline{\underline{110}} \end{array}$

Blind Guessing

6. 30 x 25 = 750

Creative

7. The fence should be the same as the perimeter.



I did it in my head - Answer 110'

Problem 23 was a two-step problem with an unfamiliar setting, and the solution processes were addition and subtraction.

Some of the strategies used to solve the problem were as follows:

> Problem 23. Helen gave the clerk a $10 bill to pay for $4.73 worth of groceries. Part of her change was a $5 bill, a $1 bill, and a dime. Explain how you know the clerk made a mistake. What should the correct change be?

Logical Analysis

1. $10 - $4.73 = n, therefore n = $5.27. Change could not be $1 bill and $5 bill.

2. $10.00
   -4.73
   $ 5.27   Change was more than $6. This is wrong.

3. $4.73 + n = $10, n = $5.27 Change could be $5 bill and 27¢

Creative

4. Estimated the groceries as \$5 and realized change must be less than \$6. Said there was \$1 bill too much. Found exact answer.

5. $\$4.73 + 2¢ = \$4.75$ and one quarter more makes \$5 and \$5 more makes \$10. Change should be \$5.27. Could be \$5 bill, one quarter and 2¢.

Blind Guessing

6.
```
$10
  4.73
  5.00
  1.00
   .10
$20.83
```

Intuitive

7. I know the change could not be more than \$6 as the groceries were more than \$5. Found correct answer \$5.27.

Summary of Analysis of Strategies

The main objective of this research was to determine if children use different strategies to solve verbal arithmetic problems. Another question was posed as to whether some strategies were more successful than others.

Analysis of the problems demonstrated that children do solve problems in different ways. Of the 455 children who took part in this study, 304 employed Logical Analysis. There were many variations of this strategy including the use of algorithms and equations.

The children who employed the Creative Thinking strategy numbered 20. Those children displayed great diversity and ingenuity in solving the problems.

Only one child chose the Insightful strategy and three the Intuitive.

The Trial and Error strategy was attempted by 23 children but most of them did not use intelligent trial and error.

Blind Guessing was chosen by 104 children but results clearly showed that this strategy did not produce accuracy.

In regard to whether some strategies were more successful than others, the analysis proved that by far the most successful strategy was Creative or Divergent Thinking. Unfortunately only 20 children employed this strategy. This may have been due to the fact that the problems did not stimulate the children to find novel and interesting solutions. This is the fault of many text book problems and it would be worthwhile to experiment with more exciting problems in order to find out if the challenge resulted in diverse ways of solving them. In view of the high degree of accuracy, this strategy should be developed by classroom theachers and different approaches to problem-solving taught.

The second most successful strategy was Logical Analysis. Arithmetic text books advocate this strategy and the influence of classroom training was evident in the children's work. Classroom teachers should be encouraged to teach this strategy.

Although the Insightful and Intuitive strategies produced accurate results, the small number of students attempting them indicated that further analysis would not be profitable.

Blind Guessing and Trial and Error proved to be the most unsuccessful strategies. Out of the 104 children who attempted Blind Guessing, 100 had no problems correct. This strategy should definitely be discouraged. Trial and Error could have resulted in accurate results if the children had used systematic Trial and Error instead of random Trial and Error. Teachers should not eliminate this strategy but should give children more guidance in order to achieve success.

Relationship Between Strategies and Pupil Characteristics

In order to determine the relationship between strategies and pupil characteristics Pearson Product Moment correlation coefficients were computed. Table 2 shows the correlation between children's characteristics and strategies.

The correlation between Intelligence and Blind Guessing is -.33, which is significant at the .001 level. The correlations between Intelligence and Trial and Error, Insight and Intuitive are negative and not significant. The correlation between Intelligence and Logical Analysis is .25, which is significant at the .001 level. The correlation between Intelligence and Creative Thinking is .14, which is significant at the .002 level.

The correlation between Arithmetic Achievement and Blind Guessing is -.32, which is significant at the .001 level. The correlation between Arithmetic Achievement and Trial and Error is -.08, which is significant at the .05 level. Correlations between Arithmetic Achievement and Insight and Intuitive are negative and not significant. The correlation between Arithmetic Achievement and Logical Analysis is .29, which is significant at the .001 level. The correlation between Arithmetic Achievement and Creative Thinking is not significant.

The correlation between Socio-economic Status and Blind Guessing is negative and not significant. The

TABLE 2

RELATIONSHIP BETWEEN CHILDREN'S CHARACTERISTICS AND STRATEGIES
USED TO SOLVE VERBAL ARITHMETIC PROBLEMS

Characteristic	STRATEGY					
	Blind Guessing	Trial & Error	Insight	Intuitive	Logical Analysis	Creative
Intelligence Quotient	-.33**	-.03	-.01	-.01	.25**	.14**
Arithmetic Achievement	-.32**	-.08	-.01	-.01	.29**	.06
Socio-economic Status	-.07	-.15**	-.01	-.01	.11**	.05
Interest in Puzzles	-.10**	-.11**	-.02	-.09	.10**	.11**
Hobbies	-.04	-.01	-.09	-.05	.07	-.04
Parent Education	-.12**	-.08	-.01	-.11**	.10**	.16**

**Level of Significance $\leq$ .01

correlation between Socio-economic Status and Trial and Error is -.15, which is significant at the .001 level. The correlations between Socio-economic Status and Insight and Intuitive are negative and not significant. The correlation between Socio-economic Status and Logical Analysis is .11, which is significant at the .01 level. The correlation between Socio-economic Status and Creative Thinking is .05, which is not significant.

The correlations between Interest in Puzzles and the strategies Blind Guessing and Trial and Error are negative, -.10 and -.12 respectively, which are significant at the .01 level. The correlations between Interest in Puzzles and Insight and Intuitive are not significant. The correlation between Interest in Puzzles and Logical Analysis is .10, which is significant at the .01 level. The correlation between Interest in Puzzles and Creative Thinking is .11, which is significant at the .01 level.

None of the correlations between Hobbies and strategies is significant.

The correlation between Parent Education and Blind Guessing is negative, -.12, which is significant at the .004 level. The correlations between Parent Education and Trial and Error and Insight are negative and not significant. The correlation between Parent Education and Intuitive is negative, -.11, and significant at the .008 level. The correlation between Parent Education and Logical Analysis is .10, which is significant at the .02 level. The correlation between Parent Education and Creative is .16, which is significant at the .001 level.

The correlations which seem to have the most bearing on strategy chosen are Intelligence Quotient, Arithmetic Achievement and Parent Education. Arithmetic Achievement has been employed by teachers to group children for arithmetic instruction. It might prove an interesting experiment for teachers to group them according to the other two characteristics.

The correlation between Success in Problem Solving and Strategy Employed is .76, which is significant at the .001 level. It would seem that strategies are more closely related to success than to pupil characteristics.

Relationship Between Success in Problem Solving and Strategies Employed

One of the objectives of this study was to discover what relationship existed between success in solving verbal arithmetic problems and strategy employed. The following table shows the correlations between strategies employed and success in problem solving:

TABLE 3

CORRELATIONS SHOWING RELATIONSHIP BETWEEN SUCCESS IN PROBLEM SOLVING AND STRATEGIES EMPLOYED

Strategy	Correlation
Blind Guessing	-.71**
Trial and Error	-.20**
Insight	-.01
Intuitive	-.01
Logical Analysis	.67**
Creative Thinking	.14**

**Level of Significance $\leq$ .01

The high negative correlation between Blind Guessing and Success in Problem Solving indicated that this strategy should not be used to solve verbal arithmetic problems. Teachers could, however, profitably substitute "educated guessing" which resulted in success for the few children who attempted it.

The same conclusions apply to Trial and Error. Experts in the field of arithmetic problem solving differentiate between systematic Trial and Error and random Trial and Error. The children who adhered to the latter type of Trial and Error did achieve at least partial success.

Logical Analysis was highly correlated with success and should be encouraged as a strategy for teaching verbal problems to fourth grade children.

The correlation between Creative Thinking and Success in Problem Solving was also significant but the number of children employing this strategy was small in comparison to those using Logical Analysis. This strategy, which is closely related to divergent thinking, should be taught as it was the most highly successful of all the strategies.

Relationship Between Success in Problem Solving and Children Characteristics

In order to answer the questions as to the relationship between Success in Problem Solving and Children Characteristics, correlations were obtained and appear in Table 4.

The two most significant correlations between Children Characteristics and Success in Problem Solving were Intelligence Quotient and Arithmetic Achievement. There was also a significant correlation between Success in Problem Solving and Interest in Puzzles and Success in Problem Solving and Parent Education.

TABLE 4

CORRELATIONS SHOWING RELATIONSHIP BETWEEN SUCCESS IN PROBLEM SOLVING AND CHILDREN CHARACTERISTICS

Characteristic	Correlation
Intelligence Quotient	.42**
Arithmetic Achievement	.41**
Interest in Puzzles	.12**
Hobbies	.01
Parent Education	.20**

**Level of Significance $\leq$ .01

The area of teaching children how to solve various types of puzzles might be explored further in order to find out if this type training was conducive to accuracy in solving arithmetic problems.

Profiles of Children With Similar Characteristics

In order to fulfill another purpose of the study which was to determine whether individual profiles of the subjects could be identified and whether profiles fell into similar clusters, the technique of Profile Analysis was employed to analyze the data. This was accomplished by use of the computer facilities at Wayne State University.

The elements contained in each profile were as follows - intelligence, arithmetic achievement, socio-economic status, interests in puzzles, hobbies, parent education, number of problems correct and strategy employed to solve the problems.

Shape-Type Criteria[1] of Profile Analysis

The technique of shape-type criteria of profile analysis was applied to the data. This is an innovative technique of profile analysis. The following is a description of the process:

The data were coded (details of coded information appear in Appendix A) and, with the aid of an IBM 360 Program, written specifically for this analysis, the four criterion measures were computed for each subject, the measures being mean (of whole data), standard deviation, skewness and kurtosis. (See Table 9, Appendix C)

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[1]John L. Lindsey, "Shape-Type Criteria of Profiles" (unpublished Doctoral dissertation, Wayne State University, 1969).

The measures for mean, standard deviation, skewness, and kurtosis were recorded on cards one for each child. The cards were numbered 1 through 455 in the exact order of the original coded data which had been prepared for the computer. A second number was written on each card which identified each child by name and school exactly as the names had appeared on the coding sheets.

The cards were then sorted and rank ordered, highest to lowest, according to the mean, and the appropriate number written above the mean. The same procedure was followed for the standard deviation, skewness and kurtosis.

The cards were then divided into quintiles and rank ordered 5, 4, 3, 2, and 1 (highest to lowest) for each of the four measures. The single digit number was written under each measure and the cards arranged according to similar one digit numbers for the four criterion measures. For example, one set of cards may have looked like this:

Mean	Standard Deviation	Skewness	Kurtosis
1	2	5	4

This would read Mean - Low (low elevation), Standard Deviation - Low Average (low-average scatter), Skewness -

Skewed Right, and Kurtosis - Cap shaped.[1]

The writer then consulted the original data to identify profiles according to the characteristics of the children. As mentioned before, each child was identified by a special number and letter representing his name and school. Profiles were then recorded according to characteristics. It was found that profiles fell into seventy-one clusters. Descriptions of the profiles are listed in Table 5.

In order to discover whether the shape-type criteria of profile analysis technique identified children with similar characteristics, the coded data of the students in each cluster were compared. Comparisons showed that the clusters of profiles were identical. The writer then examined the original data and arranged the profile types according to the eight profile elements.

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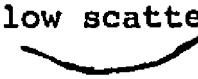
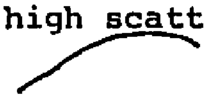
[1]Profiles which have average elevation, low scatter, no skewness, cup shape, would look like this 

Profiles which have average elevation, low scatter, no skewness, cap shape, would look like this 

Profiles which have average elevation, high scatter, skewed right, cap shape, would look like this 

Profiles which have average elevation, high scatter, skewed left, cap shape, would look like this 

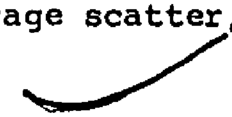
Profiles which have low elevation, average scatter, skewed right, cup shape, would look like this 

TABLE 5

DESCRIPTION OF PROFILES

Type	Elevation	Scatter	Skewness	Kurtosis
I	Low	Low Average	Slightly Right	Cup
II	Low	Low Average	None	Flat
III	High Average	Low	None	Cup
IV	High	Low	Slightly Right	Cup
V	High	Low	None	Cup
VI	High	Low	Slightly Left	Flat
VII	Average	Low	None	Cup
VIII	Average	Low	Right	Cup
IX	High Average	Low	Slightly Right	Cup
X	Low Average	Low	Slightly Right	Cup
XI	High Average	Low Average	Slightly Right	Cup
XII	High	Low Average	Slightly Right	Cup
XIII	High	Average	None	Cup
XIV	High Average	Low Average	None	Cup
XV	High Average	Low Average	Slightly Left	Flat
XVI	High Average	Low Average	Slightly Left	Very cupped
XVII	High Average	Low Average	None	Flat
XVIII	Low	Average	Slightly Left	Flat
XIX	Low Average	Average	Slightly Right	Flat
XX	Low	Average	Slightly Right	Very cupped
XXI	Low	Average	Slightly Right	Cup
XXII	Low Average	Low Average	None	Cup
XXIII	Low	Average	None	Flat
XXIV	Low	Average	Slightly Right	Very cupped
XXV	Low	Average	None	Cup
XXVI	Low	Average	None	Very cupped
XXVII	Average	Average	None	Cup
XXVIII	Average	Average	None	Flat
XXIX	Average	Average	Slightly Right	Flat
XXX	Average	Average	Slightly Left	Flat
XXXI	Average	Average	Slightly Left	Cup
XXXII	Low Average	Average	Slightly Right	Flat
XXXIII	Low	High	Left	Very cupped
XXXIV	Low	High	None	Flat
XXXV	Low	High	None	Cup
XXXVI	Low	High	Slightly Left	Cap

TABLE 5--Continued

Type	Elevation	Scatter	Skewness	Kurtosis
XXXVII	Low	High	Slightly Right	Cap
XXXVIII	Low	High	Left	Cap
XXXIX	Low	High	Slightly Left	Flat
XL	Low Average	High	Slightly Right	Flat
XLI	Low Average	High	Slightly Left	Very cupped
XLII	Low Average	High	Slightly Left	Cup
XLIII	High Average	High Average	None	Very cupped
XLIV	High Average	High Average	None	Very cupped
XLV	High Average	High Average	Slightly Right	Very cupped
XLVI	High Average	High Average	Slightly Left	Flat
XLVII	High Average	High Average	Slightly Left	Cap
XLVIII	High Average	High Average	Slightly Right	Cup
XLIX	High Average	High Average	None	Cup
L	High Average	High Average	Right	Cup
LI	Low Average	High Average	Slightly Right	Flat
LII	Low Average	High Average	None	Cup
LIII	Low Average	High Average	None	Flat
LIV	Low	High Average	None	Cap
LV	Low	High Average	None	Flat
LVI	Low	High Average	Slightly Left	Cup
LVII	Low	High Average	Slightly Right	Cap
LVIII	High	High	None	Very cupped
LIX	Average	High	Slightly Right	Cup
LX	Average	High	Slightly Left	Cap
LXI	Average	High	Slightly Left	Flat
LXII	Low Average	High	None	Cup
LXIII	Low Average	High	Right	Cup
LXIV	Low Average	High	None	Flat
LXV	Low Average	High	Slightly Right	Flat
LXVI	Low Average	High	Slightly Left	Cap
LXVII	Low Average	High	Slightly Left	Flat
LXVIII	Average	High Average	None	Cup
LXIX	Average	High Average	None	Very cupped
LXX	Average	High Average	Slightly Left	Cup
LXXI	Low Average	High	None	Cap

Table 6 shows those clusters of profile types who had two correct problems.

It will be observed that twenty of the twenty-two groups used Logical Analysis to solve the problems, one cluster used Trial and Error and the other Creative or Divergent Thinking.

In the cluster which employed Creative Thinking, Intelligence Quotient, Arithmetic Achievement, and Socio-economic Status were all average but the parents were college graduates. The subjects were interested in solving puzzles and chose sports as their hobbies.

In the group which used Trial and Error, Intelligence Quotient, Arithmetic Achievement, and Socio-economic Status were all average, the subjects were not interested in solving puzzles, the parents were high school graduates, and the children's hobbies were sports.

In the twenty clusters which used Logical Analysis, in almost every instance Intelligence Quotient, Arithmetic Achievement, and Socio-economic Status were either average or high. Profile Type LXIV was the exception. In this instance, Intelligence Quotient, Arithmetic Achievement, and Socio-economic Status were all low.

TABLE 6

PROFILES OF SUCCESSFUL PROBLEM SOLVERS
(Two problems correct)

Profile Type	I.Q.	A.A.	S.E.S.	Int. in Puzzles	Hobbies	Parent Education	Strategy
IV	High	High	High	Yes	Sports	H.S.	L.A.
V	High	High	High	Yes	Sports	Coll.	L.A.
VI	Av	Av	Av	Yes	Sports	Coll.	L.A.
XVII	High	High	Av	Yes	Acad.	H.S.	L.A.
XV	Av	Av	Av	Yes	Sports	Coll.	Creative
XIV	Av	Av	Av	Yes	Sports	H.S.	L.A.
XXXI	Av	Av	Av	No	Acad.	H.S.	L.A.
XLI	Av	Av	Av	No	A & C	H.S.	L.A.
XLII	Av	Low	Av	Yes	A & C	H.S.	L.A.
XLIII	High	High	Av	No	Sports	H.S.	L.A.
XLIV	High	High	High	Yes	A & C	Coll.	L.A.
XLV	High	High	Av	Yes	A & C	H.S.	L.A.
XLVI	Low	Av	Av	Yes	Sports	H.S.	L.A.
XLVII	Av	Av	Av	No	Sports	H.S.	L.A.
XLIX	High	Av	Av	No	Sports	H.S.	L.A.
LII	Av	Av	Av	No	Sports	H.S.	T&E
LVIII	High	High	Av	No	Sports	H.S.	L.A.
LXI	Av	Low	Av	Yes	Sports	L.H.S.	L.A.
LXIV	Low	Low	Low	Yes	Sports	H.S.	L.A.
LXVIII	Av	Av	Av	Yes	Sports	L.H.S.	L.A.
LXIX	Av	High	Av	Yes	A & C	H.S.	L.A.
LXX	Av	Av	Av	No	Sports	H.S.	L.A.

Code

I.Q. = Intelligence Quotient
A.A. = Arithmetic Achievement
S.E.S. = Socio-economic Status
L.A. = Logical Analysis
A & C = Arts & Crafts
T & E = Trial and Error
L.H.S. = Less than High School

The profile analysis demonstrated that successful types used predominantly Logical Analysis to solve the arithmetic problems. The elements in these successful profiles indicated that average or high Intelligence Quotients, Arithmetic Achievement, and Socio-economic Status were characteristics that contributed to this success. Almost identical profile elements contributed to success in the subjects who used Creative or Divergent Thinking.

Type LXIV was unique in that Intelligence Quotient, Arithmetic Achievement, and Socio-economic Status, were all low, but the subjects had both problems correct.

Almost twice as many in the successful groups indicated an interest in solving puzzles than did those who apparently had no interest.

The most popular hobby was sports, second was arts and crafts, and third, academic subjects.

The analysis left no doubt that the two strategies which should be taught in order to ensure success are Logical Analysis and Creative or Divergent Thinking. Type LXIV indicated that even when Intelligence, Arithmetic Achievement, and Socio-economic Status were low, children could be successful if they used Logical Analysis.

Undoubtedly, irrespective of the characteristics, success in problem solving was closely related to the strategy employed.

Table 7 shows the profile types who had one problem correct.

The clusters of similar Profile Types which had partial success (one problem correct) all employed Logical Analysis to solve the problems. Most of the groups had high or average Intelligence Quotients, high or average Arithmetic Achievement, and high or average Socio-economic Status. Only two out of the twenty-two groups were not interested in solving puzzles. Most of the parents were either high school or college graduates.

This analysis was conclusive in demonstrating that Logical Analysis is a very successful strategy.

As all the groups except two intimated that they were interested in solving different kind of puzzles, teachers might experiment with this activity in an effort to improve problem-solving ability.

It was noted that children whose hobby was Academic Subjects (for example Astronomy, Collecting Rocks, Space and Reading) were in the high Intelligence Quotient and Arithmetic Achievement bracket. It might be an interesting

TABLE 7

PROFILES OF SUCCESSFUL PROBLEM SOLVERS
(One problem correct)

Profile Type	I.Q.	A.A.	S.E.S.	Int. in Puzzles	Hobbies	Parent Education	Strategy
III	Av	Av	Av	Yes	Sports	H.S.	L.A.
VIII	Av	Av	Av	Yes	Acad.	H.S.	L.A.
IX	Av	Av	High	Yes	Acad.	H.S.	L.A.
XI	High	High	Av	Yes	Sports	H.S.	L.A.
XII	High	High	Av	Yes	Acad.	Coll.	L.A.
XIV	High	Av	Av	Yes	Sports	H.S.	L.A.
XVII	Av	Av	Av	Yes	Sports	Coll.	L.A.
XXI	Av	Av	Av	Yes	A & C	H.S.	L.A.
XXII	Av	Av	Av	Yes	A & C	H.S.	L.A.
XXV	Av	Low	Av	Yes	A & C	H.S.	L.A.
XXVII	Av	Av	Av	Yes	Sports	H.S.	L.A.
XXX	Av	Low	Av	Yes	Sports	H.S.	L.A.
XXXIII	Low	Low	Low	Yes	Sports	H.S.	L.A.
XXXVIII	Low	Low	Low	Yes	A & C	H.S.	L.A.
XLVIII	High	High	Av	No	Sports	H.S.	L.A.
L	High	High	High	Yes	Acad.	L.H.S.	L.A.
LVI	Av	Low	Av	No	A & C	H.S.	L.A.
LIX	High	High	Low	Yes	A & C	H.S.	L.A.
LX	Low	Low	Av	Yes	Sports	Coll.	L.A.
LXII	Low	Av	Av	Yes	Sports	L.H.S.	L.A.
LXVI	Av	Low	Low	Yes	Sports	H.S.	L.A.

Code

I.Q. = Intelligence Quotient
A.A. = Arithmetic Achievement
S.E.S. = Socio-economic Status
L.A. = Logical Analysis
A & C = Arts & Crafts
L.H.S. = Less than High School

experience to group children for arithmetic instruction according to hobbies rather than arithmetic achievement or mental capacity.

Table 8 illustrates clusters of profile types which were completely unsuccessful (no problems correct).

The analysis showed that, in most cases, the strategy employed was Blind Guessing. In three profile clusters, Logical Analysis was used, and in another three, Trial and Error was attempted. Arithmetic Achievement, Intelligence Quotients, and Socio-economic Status were mostly in the low and average range - slightly over fifty per cent of the Arithmetic Achievement and Intelligence scores being in the low bracket. Only one group cited academic subjects as a hobby, sports having pre-eminence over the other two. Parents of six of the groups had less than a high school education compared with two each in the other groups.

This analysis reaffirmed what had been stated in previous analyses - that Blind Guessing was completely unsuccessful and should be discontinued as a way of solving arithmetic word problems.

TABLE 8

PROFILES OF UNSUCCESSFUL PROBLEM SOLVERS
(No problems correct)

Profile Type	I.Q.	A.A.	S.E.S.	Int. in Puzzles	Hobbies	Parent Education	Strategy
I	Av	Av	Av	Yes	A & C	H.S.	B.G.
II	Av	Low	Av	Yes	Sports	H.S.	B.G.
X	Av	Av	Av	Yes	Sports	H.S.	B.G.
XVIII	Low	Low	Low	Yes	A & C	H.S.	B.G.
XIX	Av	Av	Av	Yes	Sports	H.S.	B.G.
XX	Av	Low	Av	No	A & C	L.H.S.	B.G.
XXIII	Low	Low	Av	Yes	A & C	H.S.	B.G.
XXIV	Low	Av	Av	No	A & C	H.S.	B.G.
XXVI	Low	Low	Low	No	A & C	L.H.S.	B.G.
XXVIII	Av	Av	Av	Yes	Sports	H.S.	L.A.
XXIX	Av	Av	Av	Yes	Sports	Coll.	L.A.
XXXII	Av	Av	Av	Yes	Sports	L.H.S.	B.G.
XXXIV	Low	Low	Low	No	Sports	L.H.S.	B.G.
XXXV	Low	Low	Av	No	Sports	H.S.	B.G.
XXXVI	Low	Low	Av	Yes	Acad.	Coll.	L.A.
XXXVII	Low	Low	Av	Yes	Sports	L.H.S.	B.G.
XXXIX	Av	Low	Low	No	Sports	H.S.	B.G.
XL	Av	High	Av	No	Sports	H.S.	T & E
LI	Low	Av	Av	Yes	Sports	H.S.	B.G.
LIII	Av	Low	Av	Yes	Sports	H.S.	B.G.
LIV	Low	Low	Av	Yes	Sports	H.S.	B.G.
LV	Low	Av	Av	No	Sports	H.S.	B.G.
LVII	Av	Low	Av	Yes	Sports	L.H.S.	T & E
LXIII	High	High	Av	Yes	A & C	H.S.	B.G.
LXV	High	High	Low	Yes	Sports	H.S.	T & E
LXVII	Av	Low	Av	No	Sports	Coll.	B.G.
LXXI	Low	Low	Av	Yes	Sports	Coll.	B.G.

Code

I.Q. = Intelligence Quotient
A.A. = Arithmetic Achievement
S.E.S. = Socio-economic Status
A & C = Arts & Crafts
B.G. = Blind Guessing
L.A. = Logical Analysis
T & E = Trial and Error
L.H.S. = Less than High School

Interviews

In order to fulfill the major purpose of this study which was to determine which strategies the subjects employed to solve the verbal arithmetic problems, 455 children were interviewed individually. This first interview was completely unstructured - the children simply being asked to state in their own words exactly how they solved the problems, step by step. Sample interviews are contained in Appendix A.

The second interview was for the purpose of ascertaining if the children were interested in solving puzzles and to determine their hobbies. This information was necessary in order to complete the profile analysis. Interest in Puzzles and Hobbies were two of the elements in each child's profile.

The children were also asked specific questions relating to the set of verbal arithmetic problems which were recorded later as percentages. Although the answers are recorded as "Yes" and "No," much more dialogue transpired between questioner and child. For instance, when the child was asked to state the problem in his own words, the interviewer only recorded an affirmative answer if he actually did so. When he was asked if he had checked or estimated the answer, he was then asked to tell exactly how he proceeded and an affirmative or negative recorded if the child demonstrated that he really knew how to estimate or check.

Percentages of affirmative and negative answers are as listed in Table 9. The interviews were conducted by the writer and each child was allocated about fifteen minutes.

The preliminary remarks are not included as they do not appear relevant, but the examiner tried to put each child at ease by introductory statements such as telling her name and asking theirs. Also mention was made as to whether they remembered the written problem test. Each child was permitted to look at the typed questions again while the interview was being conducted and was also allowed to refer to his written answers when the interviewer posed the questions.

The Interviews took place in a separate room with only the interviewer and one child present and in an atmosphere which had the least possible distractions and noise. As one child returned to his classroom, he was asked to send another according to a pre-arranged sequence with the classroom teachers. As many of the children were already familiar to the examiner and vice versa, it was felt that rapport was quickly established and most of the children appeared relaxed and frank in their responses.

TABLE 9

QUESTIONS POSED AT INDIVIDUAL INTERVIEWS

Questions	Per Cent of Children with Correct Answers	Per Cent of Children with Incorrect Answers
1. Can you tell in your own words what the problem says?	81	19
2. What figures in the problem are necessary in order to solve the problem?	76	24
3. What process is necessary to solve the problem?	66	34
4. Is your answer sensible?	54	46
5. Did you check your answer to prove that it was correct?	50	50
6. Did you estimate the answer before you found the exact solution?	28	72
7. What steps would you take to solve the problem?	67	33
8. Is there any other way of doing the problem than the one you chose?	13	87

It was noted that in very few instances were the children unable to solve the problems because of reading disability. Many children who could not rephrase the problem still understood the meaning of each word. As stated previously, the students were informed before the administration of the written test that, if they did not understand any of the words, they could ask questions.

Twenty-four per cent of the children were unable to state what figures in the problem were necessary for solution. In questions 7, 10 and 20, irrelevant data were included and this proved a stumbling block to the successful solution of the problems. Many were unable to determine that some of the figures were not necessary to solve the problem and were unable to visualize the problem as a mathematical sentence.

In question number 3, many of the responses could be categorized as wild guessing. However, a few of the children who were not successful in solving the problems during the written test were able to identify the correct method. Their lack of success resulted from mechanical errors and careless manipulation.

It was apparent that the children were not in the habit of asking themselves if their answers were sensible.

It did not occur to many that the size of the numbers in their answers was ridiculous in view of the numbers with which they started. For example, in the problem in which they had to find a boy's moon weight compared with his earth weight, the earth weight was stated as 56 pounds and those who multiplied arrived at the figure of 336 pounds. When asked if they had observed the astronauts when they landed on the moon and how they almost floated they realized the moon weight should have been much less than the earth weight. This particular problem was very badly done which was surprising when one considers the amount of exposure children have to space movies and even authentic real-life space accomplishments such as the moon landing.

Only 50 per cent of the children admitted that they had checked their answers. Many of the careless computational errors could have been eliminated by the process of checking. The successful problem solvers on the whole were the ones who had checked their answers. There were many children who did not know how to check a subtraction, division or multiplication problem but, even some who were cognizant of the process, failed to do so. Time was not a significant factor in the problem test and every child was afforded the opportunity to finish. Many utilized this time by checking the problems and showing the checking on their papers.

Many children did not know the meaning of the word "estimate" and 72 per cent answered negatively when asked if they had estimated the answer before commencing to solve the problem. This is an area which needs greater emphasis. Most of our mathematics in everyday life is based on estimation rather than exact answers.

Where two-step problems were solved incorrectly, in most cases the children were unable to proceed in orderly, sequential steps. Thirty-three per cent of the students were unable to articulate in regard to what steps should be performed in order to arrive at the solution.

The last question proved conclusively that children need to be taught, if it is possible, to practice divergent thinking in the solution of problems. When children were asked if there was any other way of solving the problem, 87 per cent answered they did not think so. However, the children who solved the problems in a creative way did use divergent thinking. Some of them cited two or three different ways to solve the problems and checked their answers.

CHAPTER III

SYNOPSIS OF PROBLEM AND METHODOLOGY

This study attempted to discover different strategies employed by children to solve verbal arithmetic problems and to find out if some strategies were more successful than others. Relationships were investigated between strategies employed to solve verbal arithmetic problems and selected characteristics of the children and between success in solving these problems and the same characteristics. Attempts were also made to identify profiles of successful and unsuccessful problem solvers and to see if profiles fell into similar clusters.

In order to attain these objectives, a group of 455 fourth grade children from a suburban school district were administered a test composed of two arithmetic word problems, chosen at random from a set of twenty-five problems. After the problems were scored, individual interviews were arranged with each subject to determine the strategy used to solve the problems. A second interview was arranged to discover the children's hobbies and interests and to obtain answers to some questions relating to the solutions of the problems. The techniques chosen to analyze the data were Pearson Product Moment Correlation Coefficients and Profile Analysis.

General Findings

The findings were as follows:

1. Children do solve problems in different ways. The following six strategies were identified:

   (1) Logical Analysis

   (2) Creative or Divergent Thinking

   (3) Intuitive Thinking

   (4) Insightful Thinking

   (5) Trial and Error

   (6) Blind Guessing

   However, as Intuitive and Insightful Thinking were employed by so few students, these strategies were not investigated further.

2. The most widely-used strategy was Logical Analysis. This strategy proved successful with children irrespective of Intelligence Quotient, Arithmetic Achievement, Socio-economic Status or Parent Education.

3. *Creative* or *Divergent Thinking* was a very accurate strategy and was mainly employed by students with average or high intelligence, arithmetic achievement, socio-economic status, and parents who were either high school or college graduates.
4. Subjects who used *Trial and Error* could have achieved more success if they had used systematic *Trial and Error*.
5. *Blind Guessing* was a completely unsuccessful strategy and teachers should be urged to discourage children from solving problems in this way.
6. There was a high correlation between strategy chosen and the problem-solving success of the subjects. *Logical Analysis* correlated more highly with problem-solving success than any other strategy. A very low negative correlation was obtained between problem-solving success and the strategy *Blind Guessing*.
7. The characteristics of the children which correlated most highly with strategy selected and the successful solution of the problems were intelligence, arithmetic achievement (as measured by the California Mental Maturity and California Achievement Test), and socio-economic status.

8. There was a significant correlation between children's interest in solving puzzles and the strategies Logical Analysis and Creative Thinking. There was also a significant but negative correlation between children's interest in solving puzzles and the strategies Blind Guessing and Trial and Error.
9. There was a significant correlation between Parent Education and the strategies Intuitive Thinking, Logical Analysis, and Creative Thinking. There was significant but negative correlation between Parent Education and the strategy Blind Guessing.
10. There was a significant correlation between children's hobbies and the strategy Logical Analysis.
11. The most significant correlation obtained was between success in problem solving and strategy employed. It would appear that success in problem solving depends more on the strategy used than on any of the characteristics of the children.
12. Examination of certain children's profiles (Types XXXVII, XXXIV, LI, LIV and LXXI) demonstrate that the students employing Blind Guessing are also students of low intelligence and arithmetic achievement. It would appear that Blind Guessing and to some extent

<u>Trial and Error</u>, are employed by students who do not know what else to do.

13. Examination of other profiles (IV, V, and VI) demonstrate that intelligence, arithmetic achievement, and socio-economic status are either average or high and that the strategy employed by these groups is <u>Logical Analysis</u>.

14. The interaction effect between success in problem solving and characteristics of children can also be shown by referring to these two above-mentioned groups. In the former, the students had no correct answers while in the latter the students had two correct solutions.

15. The Profile Analysis was very effective in identifying pupils with similar characteristics which could be used by the classroom teacher for grouping and planning for learning experiences. It could also be used to identify differences within a so-called homogeneous group and utilized as a means for individualizing instruction. Diagnosis, prescription, and sequential learning experiences could all benefit by the use of this approach.

Weaknesses of the Study

The study might have been more comprehensive and conclusive if a more extensive taxonomy of problems had been administered to each child. Selection of strategy and solving the problems accurately would have been more clearly demonstrated. This would, however, have been too great a task for one investigator, especially in regard to the interviews.

A comparison of two or three different grade levels might have revealed variation in findings. However, the investigator was familiar with the fourth grade teachers, procedures and curriculum through the medium of grade-group meetings, membership on mathematics curriculum committees at the fourth grade level, and in exchange of ideas and methods of teaching. This led to good rapport both with teachers and students and a cooperation and willingness on behalf of the faculty to assist the investigator with the administration of the tests and the arranging of the interviews. It is believed that this led to greater validity and reliability of the tests as all teachers tried to adhere implicitly to the instructions.

If the children had been asked to solve the same set of problems, the strategies might have been more clearly demonstrated but there would not have been such a variety of problems.

Children at the fourth-grade level are not very articulate and it was difficult for many to state orally exactly how they solved the problems. Many of them also found it difficult to state in writing exactly how they solved the problems.

Implications

There are many skills which will enable children to be more successful in solving verbal arithmetic problems. Quantitative mathematical vocabulary requires special attention. Even children who are able to read material well, often do not fully understand quantitative vocabulary and symbols.

Children should be taught to restate the problem in their own words, at the same time deciding which figures are necessary to solve it and which figures are redundant. Experts say if children can state the problem in their own words they have more chance of solving it correctly. If the children can write a mathematical sentence, they are almost certain to find the correct answer.

Although many studies have shown that success in reading is not always highly correlated with problem solving, quantitative vocabulary has been demonstrated to be a major factor. Teaching of quantitative vocabulary should be a part of the mathematics instruction so that children become conversant with the terms used in mathematical text books.

Although the relationship between the routine processes of addition, subtraction, multiplication and division has not been shown to have a strong relationship to success in verbal reasoning and problem solving, it is obvious that, unless proficiency in the mechanics is mastered, children will make errors even though they are able to reason through a verbal problem. Many of the children in the study knew how to solve the problems but made many mechanical errors. There was considerable counting on fingers during the interviews, even for simple addition and subtraction facts such as $10 - 3$, $8 + 9$ and others. This is a prop and help and a means to an end, but, by the time children enter the fourth grade, the average child should have mastered the addition and subtraction facts at least up to 10.

As previously mentioned, very few children seemed to estimate the answers before solving the problems. There was also evidence that very little checking of answers was attempted.

It was evident that children need a tremendous amount of practice in solving verbal problems. Some studies have shown that the mere solving of many problems improves children's accuracy and success. It is suggested that the development of divergent thinking would be a great asset in helping children attain this goal. For example, the teacher could write a problem on the board and have a few children read the problem aloud two or three times. Then discuss what figures are necessary for its solution and ask the children to write down as many different ways they can find to solve the problem. After a suitable time has elapsed, have the children relate the different ways the problem could be solved at the same time recording them on the blackboard. Leave plenty of time for discussion and questions. (In the oral interviews, when the children were asked if there was more than one way to solve the problem, most of them replied in the negative. Only those whose method of solution was classified as <u>Creative</u>

showed more than one method or suggested there might be other ways of solving the problem.)

It is suggested that more practice and instruction be given in the solving of one-step problems with familiar settings before proceeding to one-step problems with unfamiliar settings. Most of the errors were made in the latter type. There are many situations involving real-life problems which could be presented to children in order that they gain facility in solving word problems. Text books of this kind would be of great value in improving problem-solving skills.

The teaching of two-step problems both with familiar and unfamiliar settings requires special attention. Many of the two-step problems were not solved accurately and blind guessing ensued as a result. The oral interviews emphasized the lack of direction displayed by many students in attempting to tackle such problems.

More practice in non-paper and pencil problem solving could be of great value to fourth graders. Many of the problems in the test where the figures were relatively small, as in problem 8, could be done mentally. Although in most cases the writing of an algorithm improves the chance of success, there are children who apparently do not require this step and who know the answer without resorting to paper and pencil. This observation was deduced from the interviews.

It was also noticeable during the interviews that, when children were asked to explain how they solved the problem without the use of paper and pencil, their thinking processes seemed to be stimulated and more children volunteered suggestions and correct responses then during the written test. This factor has also been observed by other investigators in the area of problem solving.

Recommendations

Research is a never-ending process and the completion of one study serves only to reveal new areas where answers might be forthcoming. More research is required in this same area to elicit more information in regard to how children think. Further information is needed as to whether we can really teach children to solve verbal problems more effectively and, if so, what approaches we should assume.

This research was limited to fourth graders in a suburban school system, but might have shown very different results if the tests had been administered to children in the fifth and sixth grades. Also a comparison might have been made between children in those grades but in different type communities. For example, children from an inner city school could have been compared with those in a middle class suburban area or even with a more affluent area. It would be interesting to know if there were differences in the thought processes of children from different communities and also if the same relationship would be evidenced between selected variables, such as, Intelligence Quotient and Socio-economic status.

A design for future research might be to have two equated groups of children, the Experimental Group and the Control Group work a set of problems. The Control Group could be asked to solve by <u>Blind Guessing</u> and <u>Trial and Error</u>. The Experimental Group could be asked to solve by other strategies. The difference between the performance would give an indication of the inferiority of <u>Blind Guessing</u> and <u>Trial and Error</u>.

Although most areas now have adopted the so-called new mathematics program, it would be of value to discover if the different texts and curricula had any bearing on children's success in problem solving. For example, some texts are based on the discovery method, whereby children are encouraged to find their own methods of solving problems rather than being given a prescription. One finding which did emerge from the study was that children need to be encouraged to find many solutions to the same problem, and this suggests again, the importance of divergent thinking.

A similar study could be undertaken, employing different variables such as attitudes towards arithmetic, both of children and teachers, study habits, parent interest in the subject, and teacher preparation.

APPENDIX A

DESCRIPTION OF INSTRUMENTS USED

1. Coding Format for Background Data
2. Test Problems
3. Typical Interviews

CODING FORMAT FOR BACKGROUND DATA

Parent Education	Numeric Value
Less than High School	10
High School Graduate	20
College Graduate	30
Socio-economic Status	
Home	
Under $20,000	10
$20,000 - $30,000	20
Above $30,000	30
Employment	
Unskilled worker	10
Skilled worker	20
Professional and Owners of Businesses	30
Estimated Salary	
Under $10,000	10
$10,000 - $20,000	20
Above $20,000	30
Interest in Solving Puzzles	
Yes	20
No	10
Intelligence Quotient	
Low (Stanines 1, 2, and 3)	10
Average (Stanines 4, 5, and 6)	20
High (Stanines 7, 8, and 9)	30
Arithmetic Achievement	
Low (Stanines 1, 2, and 3)	10
Average (Stanines 4, 5, and 6)	20
High (Stanines 7, 8, 9)	30

Number of Problems Correct	Numeric Value
None	10
One	20
Two	30
Strategy Employed to Solve Problems	
Creative or Divergent Thinking	30
Logical Analysis	25
Intuitive	20
Insight	15
Trial and Error	10
Blind Guessing	5
Hobbies	
Academic Subjects	30
Art and Crafts	20
Sports	10

Test Problems

1. Aunt Mary had 160 pretty buttons. She divided them equally among her four daughters. How many buttons did each girl get?

2. John's father bought a new car. He drove it 500 miles the first week and 267 miles the second week. How many more miles did he drive the first week than the second week?

3. Jerry has read 90 pages in a book. He has 34 more pages to read in order to finish the book. How many pages are in the book?

4. Ralph and his father have a garden, which is a rectangle, 30 feet long and 25 feet wide. How many feet of fence do they need to build a fence around the garden?

5. Helen had 54 United States stamps and 39 foreign stamps in her book. She also had 114 loose stamps. She gave Sally 25 stamps. How many stamps in all did she have left?

6. Mary wants to buy two books for $1.25 and $1.65. She has only $2.38. How much more does she need?

7. At a school luncheon each student was served half a grapefruit. How much did the grapefruit cost if 16 were bought and each cost 10 cents?

8. John needs airplane glue. It costs 8 cents a tube. How many tubes of glue can he get for 32 cents?

9. John bought some thin wire so that he could hang his airplanes from the ceiling in his room. He first cut pieces 45 inches long, but he found that pieces 19 inches long would be better. Can he cut two pieces from each of the longer pieces (Yes or No)? How much wire will be left over from each 45 inch piece?

10. Ann saw three of her friends at the flower show, 15 other friends at the 4H Building, and 18 other friends at the Livestock Building. How many of her friends did she see in all?

11. Betty bought two big pieces of cardboard and pasted leaves on them. There were 26 leaves on one board and 45 leaves on the other. How many leaves were in her collection?

12. During one week Fred sold 68 papers at 5 cents each. How much did he get for all the papers?

13. Bob has a collection of 24 model cars. If he arranges them on some shelves with 4 cars on each shelf, how many shelves will he need?

14. Eight frankfurters (hot dogs) cost 48 cents. How much does one frankfurter cost?

15. Brand A frankfurters are sold at 8 for 48 cents. Brand B are sold at 6 for 42 cents. Which would you buy to save money - 3 packages of Brand A or 4 packages of Brand B?

16. Objects weigh 6 times as much on earth as they do on the moon. If Glen's earth weight is 56 lb. how much will his moon weight be?

17. Ted had 45 cents. What is the greatest amount of candy bars at 5 cents each could he buy for his money?

18. Tom had 8 bags with 6 marbles in each bag. He had 29 extra marbles on the ground. How many marbles did he have in all?

19. Ted had 35 rocks. Jim had 63 rocks. Each boy decided to store his rocks, 7 rocks in one can. Who used more cans, Jim or Ted?

20. Jack's breakfast consists of juice 85 calories, cereal 95 calories, strawberries 43 calories, ½ cup of milk 85 calories, and sugar 15 calories. How many calories does Jack eat at breakfast?

21. Carol got $5 for her birthday. She decided to buy some candy. She bought 6 candy bars at 10 cents each. How much did she have left?

22. A sleeping bag cost $18.98 and a tent cost $98.50. How much more does the tent cost than the sleeping bag?

23. Helen gave the clerk a $10 bill to pay for $4.73 worth of groceries. Part of her change was a $5 bill, a $1 bill, and a dime. Explain how you know the clerk made a mistake. What should the correct change be?

24. If one phonograph record cost 89 cents, how much would six records cost?

25. Jeff helps his father at the grocery store after school. He put 32 cans of peaches on the shelf in 8 rows of equal size. How many cans did he put on each row?

Typical Interviews

I **Questioner**: Tell me in your own words exactly how you solved the problem. (Problem referred to is Problem #5.)

Student: Well . . . I read the problem and it said Helen had 54 stamps and 39 stamps and also 114 stamps, so I added the three numbers together. Helen gave Sally 25 stamps and the question said how many stamps did she have left so I subtracted 25 and that is how I found the answer.

II **Questioner**: Tell me in your own words exactly how you solved the problem. Problem referred to is Problem #8.)

Student: One tube of glue was 8¢ and it asked how many tubes John could get for 32¢. I counted 8, 16, 24, 32 and got 4 so I think the answer is 4 tubes.

III **Questioner**: Tell me in your own words exactly how you solved the problem. (Problem referred to is Problem #24.)

Student: I multiplied because there were six records and one cost 89¢ so I said 6 x 89¢. I said 6 x 9 is 54 so I put down 4 and carried 5. Then I multiplied 6 x 8 = 48 and added 5 making 53. So my answer is $5.34.

IV **Questioner**: Tell me in your own words exactly how you solved the problem. (Problem referred to is Problem #2.)

Student: I don't know. It said how many more so I thought it meant take away so I wrote down

$$\begin{array}{r} 500 \\ \underline{-287} \end{array}$$

I was not sure so I said 7 take away 0 is 7. Eight take away 0 is 8 and 5 take away 2 is 3. The answer is 387.

V <u>Questioner</u>: Tell me in your own words exactly how you solved the problem. (Problem referred to is Problem #16.)

<u>Student</u>: The problem says objects weigh 6 times as much on earth as they do on the moon, so I knew it was multiplication so I multiplied 56 x 6. I said 6 times 6 is 36 and I put down 6 and carried 3. Then I said 6 times 5 is 30 and added 3 and got 336.

VI <u>Questioner</u>: Tell me in your own words exactly how you solved the problem. (Problem referred to is Problem #18.)

<u>Student</u>: I added 8 plus 8 plus 29 like this:

$$\begin{array}{r} 8 \\ 8 \\ \underline{29} \\ 45 \end{array}$$

VII <u>Questioner</u>: Tell me in your own words exactly how you solved the problem. (Problem referred to is Problem #19.)

<u>Student</u>: I divided 35 by 7 and got 5, so Ted used 5 cans. Then I divided 63 by 7 and said 7 times what makes 63 and got 9. So Jim used 9 cans. So Jim used more. I wanted to show all the work because it said so on the test but I knew Jim was the right answer because Jim had more rocks to begin with.

VIII <u>Questioner</u>: Tell me in your own words exactly how you solved the problem. (Problem referred to is Problem #23.)

<u>Student</u>: Helen gave $10 bill to the clerk to pay for the groceries. The groceries cost

$4.73 so I knew I should take away $10 from $4.73 to find out what change she should get. I wrote down $10 - $4.73. I knew I could not take three from zero so I regrouped my $10 like this -

$$\begin{array}{r} 9\ 9 \\ \$1\cancel{0}.\cancel{0}0 \\ \underline{-4.73} \\ \$5.27 \end{array}$$

Then I looked at my answer and saw it was $5.27 but the problem said she got change of a $5 bill, a $1 bill and a dime (part of the change). I knew this was too much. She should not have got the $1 bill.

IX Questioner: Tell me in your own words exactly how you solved the problem. (Problem referred to is Problem #4.)

Student: To find the perimeter I said the length is 30 feet and the width is 25 feet, so I multiplied the two numbers and got 750.

X Questioner: Tell me in your own words exactly how you solved the problem. (Problem referred to is Problem #17.)

Student: The question said what is the greatest amount of candy bars that Ted could buy at 5 cents if he had 45 cents. I made a number line and I marked off jumps of 5 starting at 0. Then I drew arrows showing jumps of 5 until I reached 45. Then I counted how many jumps. There were nine so I knew he could get 9 candy bars.

XI Questioner: Tell me in your own words exactly how you solved the problem. (Problem referred to is Problem #20.)

Student: I read the problem and knew I had to add all the calories up to find out how many Jack ate for breakfast. I added 85, 95, 43, ½, 85 and 15. I did not know how to add ½ so I think I added wrong.

APPENDIX B

PROFILE DIMENSIONS AND CLASSIFICATION OF PROFILES FOR ALL DATA

TABLE 10

PROFILE DIMENSIONS AND CLASSIFICATION
OF PROFILES FOR ALL DATA

Profile	Mean	Standard Deviation	Skewness	Kurtosis	Type
1	22.50000	4.62910	5.04163	3.64993	VIII
2	26.25000	5.17549	4.97071	3.65115	XIII
3	21.25000	3.53553	4.92815	3.70660	None
4	16.87500	7.98995	4.82476	4.02820	XXXV
5	25.00000	5.34522	4.86918	3.71060	XIV
6	25.00000	5.34522	5.00001	3.57807	XIII
7	19.37500	6.78101	4.92934	3.98793	None
8	23.75000	5.17549	4.96749	3.73166	XIV
9	21.25000	6.40870	4.99869	3.56693	XXVII
10	14.37500	6.23212	4.84653	3.72605	XXV
11	16.87500	5.93867	5.26204	3.71554	None
12	19.37500	6.78101	5.16134	3.89452	XXXII
13	19.37500	8.63444	4.96235	3.76901	LXII
14	20.00000	7.55929	4.97089	3.75328	LII
15	19.37500	8.63444	4.77448	3.96961	LXVII
16	21.25000	6.40870	4.99162	3.75707	XXVII
17	23.75000	5.17549	4.80406	3.79993	XVI
18	15.62500	6.23212	4.83533	3.87703	XXIII
19	15.62500	8.21040	4.93077	3.92700	XXXIV
20	23.75000	5.17549	4.80406	3.79993	XVI
21	21.25000	6.40870	4.99162	3.75707	XXVII
22	20.62500	7.76324	5.32610	3.72886	LXIII
23	21.25000	6.40870	4.99162	3.75707	XXVII
24	25.00000	5.34522	4.91228	3.70306	XIV
25	16.87500	7.98995	5.01543	4.03183	XXXVII
26	21.25000	6.40870	4.99162	3.75707	XXVII
27	19.37500	8.63444	4.81555	4.19539	LXXI
28	23.75000	5.17549	4.80406	3.79993	XVI
29	21.25000	6.40870	4.85463	3.73390	XXVII
30	22.50000	4.62910	4.85425	3.82366	None
31	25.00000	5.34522	5.06893	3.69506	XI
32	23.75000	5.44024	5.03092	3.50630	XLV
33	22.50000	7.07107	4.95717	3.53495	LXIX
34	22.50000	4.62910	4.90499	3.80635	III
35	25.00000	5.34522	4.91228	3.70306	XIV
36	26.25000	5.17549	4.97071	3.65115	XIII
37	25.00000	5.34522	5.06893	3.69506	XI

TABLE 10--Continued

Profile	Mean	Standard Deviation	Skewness	Kurtosis	Type
38	21.25000	3.53553	4.96104	3.86374	None
39	19.37500	8.63444	4.81555	4.19539	LXXI
40	25.00000	5.34522	5.06893	3.69506	XI
41	22.50000	4.62910	4.90499	3.80635	III
42	20.62500	5.62996	5.05141	3.89682	XIX
43	22.50000	7.07107	4.95920	3.72800	LXVIII
44	20.00000	7.55929	4.78093	3.54912	None
45	15.62500	6.23212	4.93909	3.81401	XXIII
46	20.00000	5.34522	4.95264	3.60755	XXII
47	18.12500	7.52970	5.05392	3.83770	None
48	18.75000	8.34523	4.66326	4.03128	XXXVI
49	22.50000	7.07107	4.97286	3.65448	LXVIII
50	21.25000	8.34523	4.75779	3.86049	LXI
51	19.37500	8.63444	5.24500	3.63086	LXIII
52	25.00000	5.34522	5.09976	3.76193	XI
53	19.37500	4.17261	5.09225	3.79783	X
54	18.12500	6.51235	4.96869	4.09265	None
55	16.87500	5.93867	5.12522	3.74523	XXI
56	23.75000	5.17549	4.96749	3.73166	XIV
57	18.12500	5.30330	4.96302	3.86386	II
58	17.50000	4.62910	4.90276	3.73415	None
59	22.50000	4.62910	5.00002	3.64691	VII
60	20.00000	7.55929	4.62151	3.81353	None
61	21.25000	6.40870	4.86519	3.66798	XXVII
62	23.12500	7.03943	5.04914	3.93997	None
63	23.75000	7.44024	4.96910	3.50628	XLIV
64	23.75000	5.17549	5.01284	3.73414	XI
65	16.87500	7.98995	5.00860	3.71988	None
66	23.75000	5.17549	4.96749	3.73166	XIV
67	26.25000	5.17549	4.97071	3.65115	V
68	27.50000	4.62910	4.89674	3.68571	V
69	14.37500	4.95515	5.09275	3.60386	None
70	22.50000	7.07107	4.95717	3.53495	LXIX
71	23.75000	7.44024	4.83086	3.64573	XLIX
72	22.50000	7.07107	4.75006	3.74747	LXX
73	15.62500	6.23212	5.01201	3.60999	XXIV
74	20.00000	7.55929	4.75689	4.05044	None
75	21.25000	6.40870	4.78438	3.93437	XXX
76	22.50000	7.07107	4.59402	3.85386	None
77	25.00000	5.34522	4.83572	3.77508	XIV

TABLE 10--Continued

Profile	Mean	Standard Deviation	Skewness	Kurtosis	Type
78	23.75000	5.17549	5.01284	3.73414	XI
79	22.50000	7.07107	4.77325	3.80757	LXX
80	25.00000	7.55929	5.12001	3.65754	XLVIII
81	22.50000	4.62910	4.90499	3.80635	VII
82	25.00000	5.34522	5.00001	3.57807	XIII
83	16.87500	7.03943	5.10196	4.10232	LVII
84	23.75000	5.17549	5.01284	3.73414	XI
85	15.62500	8.21040	4.93077	3.92700	XXXIV
86	21.25000	8.34523	4.61467	3.92438	LXI
87	22.50000	4.62910	4.84456	3.76750	VII
88	25.00000	7.55929	4.95760	3.63814	None
89	23.75000	7.44024	5.04186	3.61307	XLVIII
90	17.50000	8.86405	4.49193	4.24825	XXXIII
91	16.25000	7.44024	4.90344	3.49932	None
92	23.75000	7.44024	5.03092	3.50630	XLV
93	23.75000	7.44024	4.83086	3.64573	XLIX
94	15.62500	6.23212	5.01201	3.60999	XXIV
95	26.25000	5.17549	5.14386	3.74069	IV
96	28.75000	3.53553	4.97254	3.69593	V
97	26.25000	5.17549	5.05195	3.66357	IV
98	21.25000	6.40870	5.07222	3.56693	None
99	23.75000	7.44024	5.27650	3.70283	L
100	21.87500	8.42509	5.32862	3.76693	None
101	27.50000	4.62910	5.00001	3.62777	V
102	18.75000	8.34523	4.62844	3.86558	XXXIX
103	26.25000	7.44024	5.03582	3.66519	None
104	25.00000	7.55929	5.10963	3.54677	None
105	18.87500	7.98995	4.73885	3.92522	XXXIX
106	23.75000	5.17549	5.01284	3.73414	XI
107	20.62500	9.42546	4.89421	3.93764	LXIV
108	27.50000	4.62910	5.04646	3.67635	IV
109	16.87500	7.98955	4.73448	3.99786	XXXIX
110	26.25000	5.17549	4.90501	3.67561	V
111	25.00000	5.34522	5.06893	3.69506	XI
112	21.25000	3.53553	4.88349	3.72247	VII
113	19.37500	6.78101	5.16134	3.89452	XXXII
114	18.75000	6.40870	4.75313	3.92688	XVIII
115	27.50000	4.62910	5.00001	3.62777	V
116	22.50000	4.62910	4.90499	3.80635	III
117	16.87500	7.98995	5.00359	3.91522	XXXIV

TABLE 10--Continued

Profile	Mean	Standard Deviation	Skewness	Kurtosis	Type
118	16.87500	7.98995	4.82476	4.02820	XXXV
119	25.62500	7.28869	5.18566	3.86537	None
120	21.25000	6.40870	4.99162	3.75707	XXVII
121	18.75000	8.34523	4.60512	4.21966	XXXIII
122	13.12500	5.93867	4.98013	3.52038	XXVI
123	15.62500	4.95515	5.18256	3.86224	None
124	24.37500	7.28869	5.09245	3.88012	None
125	21.25000	6.40870	4.78438	3.93437	XXX
126	22.50000	7.07107	4.94460	3.59879	LXIX
127	23.75000	5.17549	4.80406	3.79993	XVI
128	22.50000	4.62910	4.90499	3.80635	III
129	25.00000	5.34522	4.91228	3.70306	XIV
130	22.50000	4.62910	4.85425	3.82366	None
131	23.75000	5.17549	4.77025	3.88536	XV
132	18.75000	6.40870	4.81681	4.70300	XXV
133	23.75000	5.17549	4.83572	3.77508	XVII
134	25.00000	5.34522	4.83572	3.77508	XIII
135	21.25000	6.40870	4.85463	3.73390	XXVII
136	22.00000	4.62910	4.90499	3.80635	III
137	20.00000	7.55929	4.75689	4.05044	LXVI
138	20.62500	5.62996	5.05141	3.89682	XIX
139	22.50000	4.62910	4.90499	3.80635	III
140	26.25000	5.17549	4.97071	3.65115	V
141	20.00000	7.55929	4.75689	4.05044	LXVI
142	23.75000	5.17549	4.82496	3.86735	XVII
143	20.00000	5.34522	4.95264	3.60755	XXII
144	21.25000	8.34523	5.08897	3.70520	None
145	18.12500	7.52970	4.91070	4.10503	LIV
146	20.00000	7.55929	4.75689	4.05044	LXVI
147	18.12500	7.52970	4.91070	4.10503	LIV
148	25.00000	5.34522	5.06893	3.69506	XI
149	21.25000	3.53553	4.92815	3.70660	XII
150	26.25000	7.44024	4.86446	3.62401	XLIII
151	23.75000	5.17549	4.77025	3.88536	XV
152	20.00000	7.55929	4.75689	4.05044	LXVI
153	22.50000	4.62910	4.90499	3.80635	III
154	19.37500	6.78101	5.16134	3.89452	XXXII
155	22.50000	7.07107	5.05600	3.70522	LXVIII
156	22.50000	7.07107	5.10998	3.55270	None
157	23.75000	5.17549	4.80406	3.79993	XVI

TABLE 10--Continued

Profile	Mean	Standard Deviation	Skewness	Kurtosis	Type
158	16.87500	7.98995	4.82476	4.02820	XXXV
159	23.75000	5.17549	4.80406	3.79993	XVI
160	23.75000	5.17549	4.80406	3.79993	XVI
161	22.50000	7.07107	4.76014	3.68318	LXX
162	22.50000	4.62910	4.90499	3.80635	III
163	20.62500	6.78101	5.15746	3.83867	LI
164	19.37500	6.78101	5.01029	3.79079	None
165	19.35500	7.76324	4.85059	4.12055	LXXI
166	27.50000	4.62910	5.06138	3.74106	IV
167	23.12500	4.58063	4.84819	3.88457	None
168	18.12500	7.52970	4.96069	3.86130	LV
169	25.00000	7.55929	4.93984	3.58183	LVXII
170	22.50000	4.62910	4.90499	3.80635	III
171	17.50000	8.86405	4.49991	4.12767	XXXVIII
172	23.75000	5.17549	4.80406	3.79993	XVI
173	22.50000	7.07107	4.75006	3.74747	LXX
174	18.12500	7.52970	4.91070	4.10503	LIV
175	21.25000	6.40870	4.85463	3.73390	XXVII
176	23.75000	5.17549	4.80406	3.79993	XVI
177	20.00000	5.34522	4.95264	3.60755	XXII
178	23.75000	7.44024	5.07393	3.62395	XLVIIIX
179	19.37500	6.78101	5.25737	3.71100	None
180	27.50000	4.62910	5.04636	3.67635	IV
181	18.12500	5.30330	5.13618	3.70177	I
182	19.37500	5.62996	4.94878	3.63384	None
183	22.50000	4.62910	4.90499	3.80635	III
184	18.12500	5.30330	5.13618	3.70177	I
185	20.62500	5.62996	5.05141	3.89682	XIX
186	21.25000	6.40870	4.85463	3.73390	XXVII
187	20.62500	7.76324	4.90860	3.83167	LXIV
188	15.62500	6.23212	4.93909	3.81401	XXIII
189	21.87500	6.41235	4.95207	3.92290	XXVIII
190	21.25000	6.40870	4.76515	3.64818	None
191	26.25000	5.17549	4.97071	3.65115	V
192	15.62500	6.23212	4.93909	3.81401	XXIII
193	20.62500	5.62996	5.05141	3.89682	XIX
194	18.12500	5.30330	5.13618	3.70177	I
195	22.50000	7.07107	4.94460	3.59879	LXIX
196	21.25000	8.34523	5.06014	3.50379	LIX
197	20.00000	7.55929	4.70083	3.73913	XLII

TABLE 10--Continued

Profile	Mean	Standard Deviation	Skewness	Kurtosis	Type
198	16.87500	7.98995	4.52388	4.24397	XXXVIII
199	16.87500	7.98995	4.91165	3.92954	None
200	17.50000	7.07107	4.74901	3.63924	LVI
201	19.37500	5.62996	5.23109	3.87937	None
202	25.00000	5.34522	5.00001	3.57807	XIII
203	22.50000	7.07107	4.95717	3.53495	LXIX
204	22.50000	7.07107	4.95717	3.53495	LXIX
205	25.00000	5.34522	4.91228	3.70306	XIII
206	23.75000	7.44024	4.81648	3.71773	XLIX
207	15.62500	6.23212	4.93909	3.81401	XXIII
208	18.12500	5.30330	4.96302	3.86386	II
209	16.87500	5.93867	5.26204	3.71554	XXI
210	20.00000	7.55929	4.65709	4.01361	LXVI
211	23.75000	7.44024	5.22185	3.62158	L
212	26.25000	7.44024	5.02885	3.56081	XLIII
213	26.25000	7.44024	5.02885	3.56081	XLIII
214	20.00000	5.34522	4.95264	3.60755	XXII
215	21.25000	8.34523	5.45223	3.93352	None
216	12.50000	4.62910	5.24881	4.06426	None
217	21.25000	6.40870	5.01692	3.84761	XXIX
218	15.62500	6.23212	4.93909	3.81401	XXIII
219	25.00000	5.34522	5.00001	3.57807	XIII
220	23.75000	5.17549	5.10537	3.62521	XI
221	26.25000	5.17549	4.92280	3.61076	V
222	25.00000	5.34522	5.01873	3.63860	XII
223	18.12500	5.30330	4.96302	3.86386	II
224	23.75000	5.17549	4.92239	3.68615	XIV
225	14.37500	6.23212	5.06900	3.57429	XX
226	22.50000	7.07107	4.75006	3.74747	LXX
227	20.00000	7.55929	4.78093	3.54912	XLI
228	20.00000	7.55929	4.70083	3.73913	XLII
229	15.62500	7.28869	4.79416	3.96158	None
230	21.25000	8.34523	4.48505	4.08900	None
231	21.25000	8.34523	4.77046	3.76374	None
232	18.12500	5.30330	5.20958	3.80256	I
233	20.00000	7.55929	4.95472	3.80247	LXII
234	23.75000	7.44024	4.67743	3.83374	XLVI
235	20.00000	7.55929	4.79875	3.82000	LXVII
236	20.00000	7.55929	4.94281	3.67129	LXII
237	25.00000	5.34522	5.09976	3.76193	XI

TABLE 10--Continued

Profile	Mean	Standard Deviation	Skewness	Kurtosis	Type
238	21.25000	6.40870	4.83741	3.60692	None
239	22.50000	4.62910	4.84456	3.76750	III
240	21.25000	6.40870	4.76515	3.64818	XXXI
241	27.50000	4.62910	4.88081	3.75459	V
242	22.50000	4.62910	4.82033	3.70338	III
243	16.87500	5.93867	5.17165	3.67571	XXI
244	18.12500	7.52970	4.87469	3.88822	LV
245	20.00000	7.55929	4.62151	3.81353	LXVII
246	14.37500	7.28869	4.76687	3.74150	LVI
247	20.00000	7.55929	4.78093	3.54912	XLI
248	22.50000	7.07107	4.75006	3.74747	LXX
249	19.37500	4.17261	5.09225	3.79783	X
250	15.62500	4.95515	5.02247	3.71264	None
251	15.62500	6.23212	4.97381	3.53508	XXVI
252	20.00000	9.25820	4.49659	4.29818	None
253	22.50000	4.62910	4.90499	3.80635	VII
254	20.62500	6.78101	4.82946	3.75424	LII
255	25.00000	5.34522	5.01873	3.63860	XII
256	25.00000	5.34522	5.00001	3.57807	XIII
257	21.25000	8.34523	5.10390	3.56131	LIX
258	19.37500	7.76324	5.19502	3.85768	XL
259	19.37500	4.17261	5.09225	3.79783	X
260	25.00000	5.34522	5.00001	3.57807	XIII
261	20.62500	7.76324	5.12990	3.75254	None
262	19.37500	7.76324	5.19502	3.85768	XL
263	18.12500	5.30330	5.13618	3.70177	I
264	25.00000	7.55929	4.93984	3.58183	LVIII
265	17.50000	7.07107	5.08553	3.80686	None
266	16.87500	5.93867	5.09517	3.58570	XX
267	25.00000	5.34522	5.00001	3.57807	XIII
268	21.87500	7.52970	5.07605	3.55058	None
269	23.75000	7.44024	4.77376	3.87370	XLVI
270	23.75000	5.17549	4.82495	3.86735	XVII
271	25.00000	5.34522	4.91228	3.70306	XIII
272	25.00000	5.34522	4.91228	3.70306	XIII
273	20.00000	5.34522	4.80018	3.81866	None
274	25.00000	5.34522	5.01873	3.63860	XII
275	23.75000	7.44024	4.87834	3.58501	XLIII
276	25.00000	7.55929	4.95016	3.53664	LVIII
277	16.87500	5.93867	4.99915	3.74136	XXV

TABLE 10--Continued

Profile	Mean	Standard Deviation	Skewness	Kurtosis	Type
278	20.00000	7.55929	4.77041	3.67267	XLII
279	21.25000	6.40870	4.80014	3.84537	XXX
280	22.50000	4.62910	5.04163	3.64993	VIII
281	21.25000	6.40870	4.90025	3.63651	XXVII
282	18.75000	8.34523	4.80645	3.77881	None
283	19.37500	6.78101	4.92934	3.98793	LIII
284	22.50000	7.07107	5.20008	3.62913	None
285	18.75000	6.40870	4.75313	3.92688	XVIII
286	18.12500	6.51235	4.96422	3.96664	None
287	18.12500	5.30330	4.96302	3.86386	II
288	23.75000	5.17549	4.80406	3.79993	XVI
289	25.00000	5.34522	5.00001	3.57807	XIII
290	22.50000	7.07107	4.68271	3.95628	None
291	25.00000	5.34522	4.91229	3.70306	XIII
292	23.75000	5.17549	4.82496	3.86735	XVII
293	25.00000	5.34522	4.91228	3.70306	XIII
294	23.75000	5.17549	4.80406	3.79993	XVI
295	20.00000	9.25820	4.49659	4.29818	None
296	20.00000	9.25820	4.79084	4.15076	LXVI
297	21.25000	8.34523	4.68282	4.16538	LX
298	15.62500	4.95515	4.89937	3.93738	None
299	25.00000	5.34522	4.86918	3.71060	XIII
300	25.00000	5.34522	5.02438	3.68770	None
301	26.25000	5.17549	4.97071	3.65115	V
302	18.75000	6.40870	4.73429	3.75819	None
303	22.50000	4.62910	4.90499	3.80635	III
304	26.25000	5.17549	4.95398	3.71607	V
305	23.75000	7.44024	4.61244	4.05897	XLVII
306	22.50000	4.62910	4.90499	3.80635	IIII
307	25.00000	5.34522	5.06893	3.69506	XI
308	16.87500	7.98995	5.01543	4.03183	XXXVII
309	23.12500	8.83883	5.39902	3.09176	None
310	22.50000	7.07107	5.22402	3.69121	L
311	14,37500	7.28869	5.01645	4.07423	LVII
312	20.62500	8.63444	5.18996	3.85199	LXV
313	22.50000	4.62910	5.02924	3.71806	IX
314	19.37500	5.62996	5.09741	4.00753	XIX
315	20.0000	7.55929	5.11629	3.82438	LXV
316	22.50000	7.07107	4.76611	3.87834	XLVI
317	26.25000	5.17549	5.14386	3.74069	IV

TABLE 10--Continued

Profile	Mean	Standard Deviation	Skewness	Kurtosis	Type
318	20.62500	7.76324	4.70423	4.18904	LXVI
319	22.50000	4.62910	4.90499	3.80635	III
320	18.12500	5.30330	5.13618	3.70177	I
321	26.25000	7.44024	5.02885	3.56081	XLIII
322	20.00000	7.55929	4.65709	4.01361	LXVI
323	25.00000	7.55929	4.78295	3.77803	None
324	27.50000	4.62910	5.04646	3.67635	IV
325	26.25000	5.17549	4.93919	3.70425	V
326	16.87500	7.98995	4.82476	4.02820	XXXV
327	21.25000	6.40870	4.78438	3.93437	XXX
328	23.75000	5.17549	4.96749	3.73166	XIV
329	27.50000	4.62910	5.00001	3.62777	V
330	20.62500	9.42546	4.94599	4.11399	LXI
331	25.00000	5.34522	5.06893	3.69506	XI
332	19.37500	1.76777	5.02817	3.79366	None
333	20.62500	6.78101	5.21206	3.97980	None
334	19.37500	6.78101	4.92934	3.98793	LIII
335	18.75000	8.34523	4.79161	4.01832	XXXVI
336	25.00000	5.34522	5.00001	3.57807	XIII
337	18.12500	7.52970	4.91070	4.10503	LIV
338	27.50000	4.62910	4.89674	3.68571	V
339	20.00000	5.34522	5.01535	3.85743	None
340	20.62500	5.62996	5.05141	3.89682	XIX
341	25.00000	5.34522	5.00001	3.57807	XIII
342	17.50000	8.86405	4.49193	4.24825	XXXIII
343	26.25000	5.17549	4.83141	3.82940	III
344	21.25000	8.34523	4.75779	3.86049	LXI
345	20.00000	7.55929	4.89685	3.94469	LXIV
346	26.25000	5.17549	4.83960	3.75409	V
347	21.87500	6.51235	5.14868	3.94137	XXIX
348	21.25000	6.40870	4.99162	3.75707	XXVII
349	26.25000	5.17549	4.97071	3.65115	V
350	26.25000	5.17549	4.92280	3.61076	V
351	26.25000	5.17549	4.93919	3.70425	V
352	26.25000	5.17549	4.97071	3.65115	V
353	25.00000	5.34522	5.00001	3.57807	XIII
354	21.87500	6.51235	4.95207	3.92990	XXVIII
355	25.00000	5.34522	4.73250	3.87442	VI
356	25.00000	5.34522	5.00001	3.57807	XIII
357	23.75000	7.44024	5.08344	3.55735	None

TABLE 10--Continued

Profile	Mean	Standard Deviation	Skewness	Kurtosis	Type
358	25.00000	5.34522	4.73250	3.87442	VI
359	18.75000	6.40870	4.74523	3.83173	XVIII
360	22.50000	7.07107	4.88078	3.73342	LXVIII
361	16.87500	7.98995	5.00359	3.91522	XXXIV
362	21.25000	6.40870	4.87141	3.87334	XXVIII
363	19.37500	6.78101	5.02548	3.95996	LI
364	25.00000	5.34522	4.88589	3.77168	XIII
365	20.00000	5.34522	4.95264	3.60755	XXII
366	21.87500	6.51235	5.11048	3.83523	XXIX
367	18.75000	8.34523	4.60512	4.21966	XXXIII
368	19.37500	6.78101	5.02548	3.95996	LI
369	21.25000	8.34523	4.68282	4.16538	LX
370	14.37500	6.23212	4.76892	3.88623	XVIII
371	18.75000	8.34523	4.47656	4.18725	XXXVIII
372	26.25000	5.17549	4.92280	3.61076	V
373	26.25000	5.17549	4.92280	3.61076	V
374	23.75000	5.17549	4.80406	3.79993	XVI
375	17.50000	8.86405	4.68304	4.07283	XXXVIII
376	22.50000	4.62910	4.82033	3.70338	III
377	18.12500	5.30330	4.98205	3.75861	I
378	17.50000	7.07107	4.59186	4.08996	None
379	22.50000	4.62910	4.90499	3.80635	VII
380	16.87500	5.93867	5.07985	3.72019	XXI
381	22.50000	7.07107	4.92146	3.54059	None
382	22.50000	4.62910	4.90499	3.80635	III
383	20.00000	7.55929	4.75689	4.05044	LXVI
384	20.62500	5.62996	5.05141	3.89682	XIX
385	20.00000	5.34522	4.95264	3.60755	XXII
386	21.25000	6.40870	4.86519	3.66798	XXVII
387	22.50000	4.62910	4.79547	3.78341	III
388	21.87500	6.51235	4.95207	3.92290	XXVIII
389	22.50000	4.62910	4.90499	3.80635	III
390	21.25000	6.40870	4.87141	3.67334	XXVIII
391	19.37500	6.78101	5.02548	3.95996	LI
392	18.75000	6.40870	5.04026	3.56116	XXIV
393	26.25000	5.17549	4.97071	3.65115	V
394	14.37500	6.23212	4.98677	3.65104	XXV
395	19.37500	6.78101	4.92934	3.98793	LIII
396	22.50000	4.62910	4.90499	3.80635	III
397	19.37500	6.78101	5.02548	3.95996	LI

TABLE 10--Continued

Profile	Mean	Standard Deviation	Skewness	Kurtosis	Type
398	25.00000	5.34522	5.00001	3.57807	XIII
399	22.50000	7.07107	4.70577	4.02597	XLVII
400	23.75000	5.17549	5.01284	3.73414	XI
401	19.37500	6.78101	4.92934	3.98793	LIII
402	19.37500	8.63444	4.77448	3.96961	LXVII
403	23.75000	7.44024	4.61244	4.05897	XLVII
404	21.25000	6.40870	5.01692	3.84761	XXIX
405	27.50000	4.62910	4.89674	3.66571	V
406	22.50000	4.62910	4.90499	3.80635	III
407	16.87500	7.03943	4.95241	4.25075	None
408	26.25000	5.17549	4.97071	3.65115	V
409	23.75000	5.17549	5.07762	3.68616	XII
410	21.25000	8.34523	4.65000	4.08498	LX
411	18.75000	6.40870	4.90049	3.52800	XXVI
412	22.50000	7.07107	4.75006	3.74747	None
413	23.75000	7.44024	4.83086	3.64573	XLIX
414	26.25000	7.44024	4.86446	3.62401	XLIII
415	25.00000	5.34522	4.91228	3.70306	XIII
416	22.50000	4.62910	4.90499	3.80635	III
417	23.75000	5.17549	4.96749	3.73166	XIV
418	23.75000	5.17549	4.80406	3.79993	XVI
419	25.00000	5.34522	5.06893	3.69506	XI
420	22.50000	7.07107	4.76611	3.87834	XLVI
421	26.25000	7.44024	4.86446	3.62401	XLIII
422	21.25000	6.40870	4.85463	3.73390	XXVII
423	25.00000	5.34522	4.91228	3.70306	XIII
424	20.62500	5.62996	5.05141	3.89682	XIX
425	21.25000	6.40870	4.87141	3.87334	XXVIII
426	20.62500	5.62996	5.05141	3.89682	XIX
427	20.00000	7.55929	4.98864	3.90422	LXIV
428	22.50000	4.62910	4.90499	3.80635	III
429	26.25000	5.17549	4.97071	3.65115	V
430	26.25000	5.17549	4.92280	3.61076	V
431	18.75000	6.40870	4.81681	3.70300	XXV
432	23.75000	5.17549	4.80406	3.79993	XVI
433	20.00000	9.25820	4.55295	4.09954	LXVI
434	20.00000	7.55929	4.73801	3.76879	XLII
435	23.12500	4.58063	5.04464	3.76677	IX
436	17.50000	8.86405	5.09264	3.52841	None
437	20.00000	7.55929	5.00000	3.47026	None

TABLE 10--Continued

Profile	Mean	Standard Deviation	Skewness	Kurtosis	Type
438	21.25000	8.34523	5.11764	3.47026	LIX
439	25.00000	5.34522	5.00001	3.57807	XIII
440	25.00000	7.55929	4.95016	3.53664	None
441	26.25000	5.17549	4.97071	3.65115	V
442	23.75000	7.44024	4.67743	3.83374	XLVI
443	23.75000	7.44024	5.03092	3.50630	XLV
444	26.25000	5.17549	4.92280	3.61076	V
445	26.25000	5.17549	4.97071	3.65115	V
446	26.25000	5.17549	4.97071	3.65115	V
447	25.00000	5.34522	5.00001	3.57807	XIII
448	21.87500	6.51235	5.16077	3.85905	XXIX
449	23.75000	7.44024	5.03092	3.50630	XLV
450	27.50000	4.62910	4.89674	3.68571	V
451	20.62500	7.76324	4.92774	4.00876	LXIV
452	21.25000	3.53553	5.07188	3.70664	VIII
453	25.00000	5.34522	5.00001	3.57807	XIII
454	26.25000	5.17549	4.97071	3.65115	V
455	23.75000	7.44024	5.03092	3.50630	XLV

APPENDIX C

DESCRIPTION OF PROFILES

Type I:

1. All children have average Intelligence Quotients.

2. All children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are arts and crafts.

6. The parents are high school graduates.

7. The subjects received a zero score on their two problem test.

8. The strategy used to solve the problems was Blind Guessing.

Type II:

1. All children have average Intelligence Quotients.

2. All children have low arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The subjects received a zero score on their two problem test.

8. The strategy used to solve the problems was Blind Guessing.

Type III:

1. All children have average Intelligence Quotients.

2. All children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had one correct problem on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type IV:

1. All the children have high Intelligence Quotients.

2. All children have high arithmetic achievement.

3. The socio-economic status of the parents is high.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had two correct problems on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type V:

1. All the children have high Intelligence Quotients.

2. All the children have high arithmetic achievement.

3. The socio-economic status of the parents is high.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are college graduates.

7. The children had two correct problems on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type VI:

1. All the children have average Intelligence Quotients.

2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are college graduates.

7. The children had two correct problems on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type VII:

1. All the children have average Intelligence Quotients.

2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The children had one correct problem on their two-problem test.

7. The parents are high school graduates.

8. The strategy used to solve the problems was Logical Analysis.

Type VIII:

1. All the children have average Intelligence Quotients.

2. All the children have high arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The children's hobbies are academic subjects.

6. The parents are high school graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problem was Logical Analysis.

Type IX:

1. All the children have average Intelligence Quotients.

2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is high.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are academic subjects.

6. The parents are high school graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

<u>Type X</u>:

1. All the children have average Intelligence Quotients.

2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children received a zero score on their two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

<u>Type XI</u>:

1. All the children have high Intelligence Quotients.

2. All the children have high arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XII:

1. All the children have high Intelligence Quotients.

2. All the children have high arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are academic subjects.

6. The parents are college graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XIII:

1. All the children have high Intelligence Quotients.

2. All the children have high arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are academic subjects.

6. The parents are high school graduates.

7. The children had two correct answers on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XIV:

1. All the children have high Intelligence Quotients.
2. All the children have average arithmetic achievement.
3. The socio-economic status of the parents is average.
4. The children are interested in solving all kinds of puzzles.
5. The hobbies of the children are games and sports.
6. The parents are high school graduates.
7. The children had one correct answer on their two-problem test.
8. The strategy used to solve the problems was Logical Analysis.

Type XV:

1. All the children have average Intelligence Quotients.
2. All the children have average arithmetic achievement.
3. The socio-economic status of the parents is average.
4. The children are interested in solving all kinds of puzzles.
5. The hobbies of the children are games and sports.
6. The parents are college graduates.
7. The children had two correct answers on their two-problem test.
8. The strategy used to solve the problems was Creative or Divergent Thinking.

Type XVI:

1. All the children have average Intelligence Quotients.

2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had two correct answers on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XVII:

1. All the children have average Intelligence Quotients.

2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are college graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XVIII:

1. All the children have low Intelligence Quotients.

2. All the children have low arithmetic achievement.

3. The socio-economic status of the parents is low.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are arts and crafts.

6. The parents are high school graduates.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

Type XIX:

1. All the children have average Intelligence Quotients.

2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

Type XX:

1. All the children have average Intelligence Quotients.

2. All the children have low arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are not interested in solving puzzles.

5. The hobbies of the children are art and crafts.

6. The parents have less than a full high school education.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

<u>Type XXI</u>:

1. All the children have average Intelligence Quotients.

2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are art and crafts.

6. The parents are high school graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Insightful.

<u>Type XXII</u>:

1. All the children have average Intelligence Quotients.

2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are art and crafts.

6. The parents are high school graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XXIII:

1. All the children have low Intelligence Quotients.
2. All the children have low arithmetic achievement.
3. The socio-economic status of the parents is average.
4. The children are interested in solving all kinds of puzzles.
5. The hobbies of the children are art and crafts.
6. The parents are high school graduates.
7. The children had a zero score on their two-problem test.
8. The strategy used to solve the problems was Blind Guessing.

Type XXIV:

1. All the children have low Intelligence Quotients.
2. All the children have average arithmetic achievement.
3. The socio-economic status of the parents is average.
4. The children are not interested in solving puzzles.
5. The hobbies of the children are art and crafts.
6. The parents are high school graduates.
7. The children had a zero score on their two-problem test.
8. The strategy used to solve the problems was Blind Guessing.

Type XXV:

1. All the children have average Intelligence Quotients.
2. All the children have low arithmetic achievement.
3. The socio-economic status of the parents is average.
4. The children are interested in solving all kinds of puzzles.
5. The hobbies of the children are art and crafts.
6. The parents are high school graduates.
7. The children had one correct answer on their two-problem test.
8. The strategy used to solve the problems was Logical Analysis.

Type XXVI:

1. All the children have low Intelligence Quotients.
2. All the children have low arithmetic achievement.
3. The socio-economic status of the parents is low.
4. The children are not interested in solving puzzles.
5. The hobbies of the children are art and crafts.
6. The parents have less than a high school education.
7. The children had a zero score on their two-problem test.
8. The strategy used to solve the problems was Blind Guessing.

Type XXVII:

1. All the children have average Intelligence Quotients.
2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

<u>Type XXVIII</u>:

1. All the children have average Intelligence Quotients.

2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

<u>Type XXIX</u>:

1. All the children have average Intelligence Quotients.

2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving puzzles.

5. The hobbies of the children are sports.

6. The parents are college graduates.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XXX:

1. All the children have average Intelligence Quotients.

2. All the children have low arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XXXI:

1. All the children have average Intelligence Quotients.

2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are not interested in solving puzzles.

5. The hobbies of the children are academic subjects.

6. The parents are high school graduates.

7. The children had two correct answers on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XXXII:

1. All the children have average Intelligence Quotients.

2. All the children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents have less than a high school education.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

Type XXXIII:

1. All the children have low Intelligence Quotients.

2. All the children have low arithmetic achievement.

3. The socio-economic status of the parents is low.

4. The children are interested in solving all kinds of puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XXXIV:

1. All the children have low Intelligence Quotients.
2. All the children have low arithmetic achievement.
3. The socio-economic status of the parents is low.
4. The children are not interested in solving puzzles.
5. The hobbies of the children are games and sports.
6. The parents have less than a high school education.
7. The children had a zero score on their two-problem test.
8. The strategy used to solve the problems was Blind Guessing.

Type XXXV:

1. All the children have low Intelligence Quotients.
2. All the children have low arithmetic achievement.
3. The socio-economic status of the parents is average.
4. The children are not interested in solving puzzles.
5. The hobbies of the children are games and sports.
6. The parents are high school graduates.
7. The children had zero score on their two-problem test.
8. The strategy used to solve the problems was Blind Guessing.

Type XXXVI:

1. All the children have low Intelligence Quotients.
2. All the children have low arithmetic achievement.
3. The socio-economic status of the parents is average.

4. The children are interested in solving puzzles.

5. The hobbies of the children are arithmetic achievement.

6. The parents are college graduates.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XXXVII:

1. All the children have low Intelligence Quotients.

2. All the children have low arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving puzzles.

5. The children's hobbies are games and sports.

6. The parents have less than a high school education.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

Type XXXVIII:

1. All the children have low Intelligence Quotients.

2. All the children have low arithmetic achievement.

3. The socio-economic status of the parents is low.

4. The children are interested in solving puzzles.

5. The hobbies of the children are art and crafts.

6. The parents of the children are high school graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XXXIX:

1. All the children have average Intelligence Quotients.

2. All the children have low arithmetic achievement.

3. The socio-economic status of the parents is low.

4. The children are not interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

Type XL:

1. All the children have average Intelligence Quotients.

2. All the children have high arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are not interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Trial and Error.

Type XLI:

1. All the children have average Intelligence Quotients.
2. All the children have average arithmetic achievement.
3. The socio-economic status of the parents is average.
4. The children are not interested in solving puzzles.
5. The hobbies of the children are art and crafts.
6. The parents are high school graduates.
7. The children had two correct answers on their two-problem test.
8. The strategy used to solve the problems was Logical Analysis.

Type XLII:

1. All the children have average Intelligence Quotients.
2. All the children have low arithmetic achievements.
3. The socio-economic status of the parents is average.
4. The children are interested in solving puzzles.
5. The hobbies of the children are art and crafts.
6. The parents are high school graduates.
7. The children had two correct answers on their two-problem test.
8. The strategy used to solve the problems was Logical Analysis.

Type XLIII:

1. All the children have high Intelligence Quotients.
2. All the children have high arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are not interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had two correct answers on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

<u>Type XLIV</u>:

1. All the children have high Intelligence Quotients.

2. The children have high arithmetic achievement.

3. The socio-economic status of the parents is high.

4. The children are interested in solving puzzles.

5. The hobbies of the children are art and crafts.

6. The parents are college graduates.

7. The children had two correct answers on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

<u>Type XLV</u>:

1. All the children have high Intelligence Quotients.

2. The children have high arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving puzzles.

5. The hobbies of the children are art and crafts.

6. The parents are high school graduates.

7. The children had two correct answers on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

<u>Type XLVI</u>:

1. The children have low Intelligence Quotients.

2. The children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The children had two correct answers on their two-problem test.

7. The parents are high school graduates.

8. The strategy used to solve the problems was Logical Analysis.

<u>Type XLVII</u>:

1. All the children have average Intelligence Quotients.

2. The children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are not interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had two correct answers on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XLVIII:

1. All the children have high Intelligence Quotients.

2. The children have high arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are not interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type XLIX:

1. All the children have high Intelligence Quotients.

2. The children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are not interested in puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had two correct answers on their two problem test.

8. The strategy used to solve the problems was Logical Analysis.

<u>Type L</u>:

1. All the children have high Intelligence Quotients.

2. The children have high arithmetic achievement.

3. The socio-economic status of the parents is high.

4. The children like to solve puzzles.

5. The hobbies of the children are academic subjects.

6. The parents have less than a high school education.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

<u>Type LI</u>:

1. All the children have low Intelligence Quotients.

2. The children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

<u>Type LII</u>:

1. All the children have average Intelligence Quotients.

2. The children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are not interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had two correct answers on their two-problem test.

8. The strategy used to solve the problems was Trial and Error.

Type LIII:

1. All the children have average Intelligence Quotients.

2. The children have low arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

Type LIV:

1. All the children have low Intelligence Quotients.

2. The children have low arithmetic achievement.

3. The socio-economic status of the children is average.

4. The children are interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents have a high school education.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

Type LV:

1. All the children have low Intelligence Quotients.

2. The children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are not interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

Type LVI:

1. All the children have average Intelligence Quotients.

2. The children have low arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are not interested in solving puzzles.

5. The hobbies of the children are art and crafts.

6. The parents are high school graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type LVII:

1. All the children have average Intelligence Quotients.

2. The children have low arithmetic achievement.

3. The socio-economic status of the children is average.

4. The children are interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents have less than a high school education.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Trial and Error.

Type LVIII:

1. All the children have high Intelligence Quotients.

2. The children have high arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are not interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had two correct answers on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

<u>Type LIX</u>:

1. All the children have high Intelligence Quotients.

2. The arithmetic achievement of the children is high.

3. The socio-economic status of the parents is low.

4. The children are interested in solving puzzles.

5. The hobbies of the children are art and crafts.

6. The parents are high school graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

<u>Type LX</u>:

1. All the children have low Intelligence Quotients.

2. The arithmetic achievement of the children is low.

3. The socio-economic status of the parents is average.

4. The children are interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are college graduates.

7. The children had one correct answer in their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

<u>Type LXI</u>:

1. All the children have average Intelligence Quotients.

2. The children have low arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents have less than a high school education.

7. The children had two correct answers on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type LXII:

1. All the children have low Intelligence Quotients.

2. The children have average arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents have less than a high school education.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type LXIII:

1. All the children have high Intelligence Quotients.

2. The arithmetic achievement of the children is high.

3. The socio-economic status of the parents is average.

4. The children are interested in solving puzzles.

5. The hobbies of the children are art and crafts.

6. The parents are high school graduates.

7. The children had a zero score on their basic two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

Type LXIV:

1. All the children have low Intelligence Quotients.

2. The children have low arithmetic achievement.

3. The socio-economic status of the parents is low.

4. The children are interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had two correct answers on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type LXV:

1. All the children have high Intelligence Quotients.

2. The children have high arithmetic achievement.

3. The socio-economic status of the parents is low.

4. The children are interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Trial and Error.

Type LXVI:

1. All the children have average Intelligence Quotients.

2. The children have low arithmetic achievement.

3. The socio-economic status of the parents is low.

4. The children are interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had one correct answer on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

Type LXVII:

1. All the children have average Intelligence Quotients.

2. The children have low arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are not interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are college graduates.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

Type LXVIII:

1. All the children have average Intelligence Quotients.

2. The children have average arithmetic achievement.
3. The socio-economic status of the parents is average.
4. The children are interested in solving puzzles.
5. The hobbies of the children are games and sports.
6. The parents have less than a high school education.
7. The children had two correct answers on their two-problem test.
8. The strategy used to solve the problems was Logical Analysis.

Type LXIX:

1. All the children have average Intelligence Quotients.
2. The children have high arithmetic achievement.
3. The socio-economic status of the parents is average.
4. The children are interested in solving puzzles.
5. The hobbies of the children are art and crafts.
6. The parents are high school graduates.
7. The children had two correct answers on their two-problem test.
8. The strategy used to solve the problems was Logical Analysis.

Type LXX:

1. All the children have average Intelligence Quotients.
2. The children have average arithmetic achievement.
3. The socio-economic status of the parents is average.
4. The children are not interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are high school graduates.

7. The children had two correct answers on their two-problem test.

8. The strategy used to solve the problems was Logical Analysis.

<u>Type LXXI</u>:

1. All the children have low Intelligence Quotients.

2. The children have low arithmetic achievement.

3. The socio-economic status of the parents is average.

4. The children are interested in solving puzzles.

5. The hobbies of the children are games and sports.

6. The parents are college graduates.

7. The children had a zero score on their two-problem test.

8. The strategy used to solve the problems was Blind Guessing.

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Education:	Attended Kirkcaldy North Public School and Kirkcaldy High School. Graduated from Warton College England, 1969. Attended Wayne State University and received Master's Degree in 1960.
Positions Held:	Taught French and Mathematics in High School in Sale, Cheshire for 2½ years prior to coming to the United States. Worked in law and certified public accountant's office in Detroit before beginning to teach again. Changed from High School to Elementary and have taught at that level since 1955.
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