ROBUSTNESS AND COMPARATIVE STATISTICAL POWER OF THE REPEATED MEASURES ANOVA AND FRIEDMAN TEST WITH REAL DATA

By

OPEOLUWA BOLU FADEYI

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DEDICATION

To my husband, children, and parents.

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CHAPTER ONE

INTRODUCTION

OVERVIEW OF THE PARAMETRIC TESTS

Parametric tests are those which based the necessary assumptions on the parameters of the underlying population distribution from which the samples are drawn. It is generally believed that parametric tests are robust to the violation of some of the assumptions, this means that the tests have the power to control the probability of rejecting the false null hypothesis. For example, ANOVA can be used to analyze ordinal scale data such as Likert scales, even without any consequences (Leys & Schumann, 2010; Nanna & Sawilowsky, 1998; Zimmerman & Zumbo, 1993). Another peculiar characteristic of a parametric test is that it is uniformly most powerful unbiased (UMPU). "This means that when all underlying assumptions are met based on the inference from the samples, no other test has greater ability to detect a true difference for a given sample" (Bridge & Sawilowsky, 1999, p. 229). For example, the t-test is uniformly most powerful unbiased when the assumptions of independence, homoscedasticity, and normality are met (Bradley, 1968b; Kelley & Sawilowsky, 1997). However, a "light shift" in the shapes of the distribution of the variables when the number of samples in each treatment group gets close to 30 or more still generates robust results (Glass, Peckham, & Sanders, 1972; Leys & Schumann, 2010; Lix, Keselman, & Keselman, 1996; Lumley, Diehr, Emerson, & Chen, 2002). Studies have been carried out to examine the violation of the assumption of homogeneity of variances which may have a severe impact on the type I error rate of F-tests, and It has been established that F-test will yield statistically significant results when the group sample sizes are equal and size of the groups greater than seven (Box, 1954; David & Johnson, 1951; Horsnell, 1953; Hsu, 1938; Linquist, 1953; Norton, 1952; Scheffé, 1959). Another procedure that can be performed when there is the heterogeneity of variance is to transform or change the form of the data involved. Examples of this procedure are Log transformation, square root transformation, or inverse transformation (Blanca, Alarcón , Arnau, Bono, & Bendayan, 2017; Keppel, 1991; Leys & Schumann, 2010; Lix, Keselman, & Keselman, 1996; Saste, Sananse, & Sonar, 2016). This procedure works well in stabilizing the variances and improve the normality of the dataset. Parametric tests are used to analyze interval and ratio scale data (Bridge & Sawilowsky, 1999; Shah & Madden, 2004). Other examples of parametric tests are the t-test, the Chi-squared test, test-of-goodness of fit, analysis of variance or F-test, analysis of covariance, multiple linear regression, and discriminant function analysis (Weber & Sawilowsky, 2009).

The robustness property in the normal distribution test signifies the ability of a test to retain its Type I error rate close to its nominal alpha, as well as its Type II errors for data sampled from non-normal distributions at a similar rate as those datasets sampled from a normal distribution (Bridge & Sawilowsky, 1999; Hunter & May, 1993). However, parametric tests are not always tolerant to extreme violations of their underlying assumptions. Outliers are the major causes of shifts in the shapes of the distribution. Outliers can render the results of the parametric tests inaccurate and misleading by inflating or deflating the error rates. This problem of error inflation is made worse by how frequent outliers are present in a group of scores (Geary, 1947; Hunter & May, 1993; Micceri, 1989; Nunnally, 1978; Pearson, 1895; Pearson & Please, 1975; Sawilowsky & Blair, 1992; Tan, 1982). "When the assumption of normality is not met, ANOVA loses its distinct ability of being uniformly most powerful unbiased (UMPU) test, as does the t-test" (Sawilowsky, 1990, p. 100). This emphasizes the importance of rank-based nonparametric alternative approaches, specifically concerning the treatment models of shift in location parameter. The alternative solutions to the problem of severe violation of underlying assumptions in parametric tests are nonparametric tests, robust procedures, data transformation, resampling, simulations and bootstrapping, etc (Feys, 2016).

Origin of Nonparametric Tests

Nonparametric tests are distribution-free tests that do not base their requirements on fulfilling the assumptions of their parent distributions such as F-test or Chi-square distribution (Kruskal & Wallis, 1952). Such assumptions include normality and independence of observation. Meanwhile, there are other assumptions of the nonparametric tests that are generally considered weak because they are not connected to the validity of the nonparametric tests' results. The assumption could be ignored since they do not interfere with the functionality of the tests. Such assumptions relating to the population distributions from which they are drawn are generally weak. Those assumptions are not restrictive for the results to be valid (Gibbons, 2003). There are three main types of nonparametric tests, namely categorical, sign, and rank-based tests (Gleason, 2013; Sawilowsky, 1990). Nonparametric tests are usually robust to nonnull distribution and are good alternatives to handling the occurrence of outliers in statistical analysis. Many studies have been carried out on comparing the robustness and the comparative power advantages of the parametric tests with their nonparametric counterparts. In the two-group layout, it is assumed that the data are independently and identically distributed (I.I.D). Sign test, Wilcoxon-Sign Rank test (WSR), and Manny-

Whitney tests are some of the examples in this group. These tests are competitors with the student t-test, paired sample t-test, and the independent t-test. However, when the number of groups is increased to 3 or more $(\mathrm{i.e.k} \geq 3)$, the Kruskal-Wallis test competes well with the regular one-way ANOVA while Friedman's test can be applied as an alternative to the one-way repeated measures ANOVA (Friedman, 1937). One of the assumptions of the Friedman test is that "samples are dependent under all levels" (Ingram & Monks, 1992, p. 827)

Historically, nonparametric tests were viewed as being useful only when the assumptions of the parametric tests were not met (Lehmann, 1975; Marascuilo & McSweeney, 1977). Subsequently, it was proved that when testing for the differences in location parameters, if the distribution shapes are not normal or are heavy-tailed, the nonparametric tests are robust and present considerable power advantages over their parametric counterparts (Blair & Higgins, 1985; Sawilowsky, 1990).

Nonparametric statistics were popular in the 1950s but began to wane for three reasons in the 1970s. Those three reasons were summarized by (Sawilowsky, 1990, p. 92). (Boneau, 1960; Box, 1954; Glass, Peckham, & Sanders, 1972; Linquist, 1953)

> First, it is usually asserted that parametric statistics are extremely robust with respect to the assumption of population normality (Boneau, 1960; *Box, 1954; Glass, Peckham, & Sanders, 1972; Linquist, 1953)*, precluding the need to consider alternative tests. Second, it is assumed that nonparametric tests are less powerful than their parametric counterparts *(Kerlinger, 1964, 1973; Nunnally, 1975)*, apparently regardless of the shape of the population from which the data were sampled. Third, there has been a paucity of nonparametric tests for the more complicated research designs *(Bradley, 1968)*.

One of the goals of performing a statistical test is to investigate some claims using samples and make inferences about the general populations from which the samples are drawn. Therefore, researchers need to understand the criteria for making the right choice of tests that will yield accurate and clear results for decision-making purposes. The statistical power of a test will determine if such a test carries the ability to detect a significant statistical effect when such an effect is present. The significant level at which a test will commit a false rejection is called Type I error, denoted by the Greek small letter Alpha (α). A default value of 0.05 is commonly used in research.

Statistical power

Statistical power efficiency refers to the minimum size of the samples required to determine whether there is an effect due to an intervention. This is the ability to reliably differentiate between the null and the alternative hypothesis of interest. To measure the statistical power of a test effectively, Relative Efficiency (RE) and the Asymptotic Relative Efficiency (ARE) will be considered. The relative efficiency of a statistical test is the index that measures the power of a test, by comparing the sample size required of one parametric test to the sample size of its nonparametric counterpart. To achieve an unbiased estimate, the two tests must be subjected to equal conditions, that is, the significant level and the hypothesis under which they are both compared must be equal (Sawilowsky, 1990).

Asymptotic Relative Efficiency (ARE) of a statistical test for both parametric and nonparametric tests, is the ratio of two tests as compared to 1 when the sample sizes are large and the treatment effect is very small. Thus, if the ARE of a parametric test over the nonparametric alternative is greater than 1, the parametric test has a power advantage over its nonparametric counterpart (Pitman, 1948; Sawilowsky,1990). The ARE is also called the Pitman efficiency test.

The parametric test that employs the analysis of a complete block design when comparing only two group means or treatments is the paired t-test. The two nonparametric alternatives in the same category are the Wilcoxon signed ranks (WSR) test and the sign test. The sign test uses the information based on the within-block rankings to assign ranks to the absolute values of observations when the number of the groups is 2, $(\mathrm{k} = 2)$. Friedman's test design has extended the procedure of the sign test to a randomized block design involving more than two comparisons, $(\mathrm{k} \geq 3)$. Therefore, the Friedman test is considered an extension or generalization of the sign test (Hodges & Lehmann, 1960; Iman, Hora, & Conover, 1984; Zimmerman & Zumbo, 1993).

Observations generated by subjecting the same set of participants to three or more different conditions are termed repeated measures or the within-subjects data. The parametric statistical design that is used to analyze this type of observation is the usual F-test for block data or the One-Way Repeated Measures ANOVA. "The ARE of the Friedman test as compared to the F test is $(3/\pi)\mathrm{k}/(\mathrm{k}+1)$ for normal distributions, and [≥.864k/(k+1)] for other distributions" (Hager, 2007; Iman, Hora, & Conover, 1984; Potvin & Roff, 1993; Sen, 1967, 1968; Zimmerman & Zumbo, 1993)**.**

"The ARE of a test is related to large sample sizes and very insignificant treatment effects, this is highly impractical in the real-world experiment. However, Monte Carlo simulations have been confirmed to play very significant role in calculating the ARE and RE for small sample sizes" (Sawilowsky, 1990, p. 93; see also Potvin & Roff, 1993; Zimmerman & Zumbo, 1993).

Problem of the Study

Several Monte Carlo studies were conducted on the comparative power of the univariate repeated measures ANOVA and the Friedman test (Hager, 2007; Hodges & Lehmann, 1960; Iman, Hora, & Conover, 1984; Mack & Skillings, 1980; Potvin & Roff, 1993; Zimmerman & Zumbo, 1993). However, conclusions based on simulated data were limited to data sampled from specific distributions. This is a disadvantage in the ability to generalize the results to the population from which samples were drawn. Real-life data have been found to deviate from the normality assumptions more drastically than those patterns found in the mathematical distributions (Blanca, Arnau, López-Montiel, Bono, & Bendayan, 2013; Harvey & Siddique, 2000; Kobayashi, 2005; Micceri, 1989; Ruscio & Roche , 2012; Van Der Linder, 2006). As a case in point, most of what is known regarding the comparative statistical power of the one-way repeated measures ANOVA and the Friedman tests were tied to specific mathematical distributions, and it is not well known how the two tests compare with common, real-world data.

Purpose of this study

The results from previous research have shown that the parametric statistics have a little power advantage over their nonparametric alternatives when the assumption of normality holds. However, under varying non-symmetric distributions, the nonparametric tests yielded comparable power advantages over the parameter-based tests. It is therefore the goal of this study to examine the robustness and comparative statistical power properties of the one-way repeated measure ANOVA to its nonparametric counterpart, Friedman's test, to the violations of normality using the real-world data, which has not been extensively studied.

Research questions

The research questions addressed in this study are as follows:

- Will the results of previous simulation studies about the power advantage of parametric over nonparametric be generalizable to real-world situations?
- Which of these tests will yield a comparative power advantage under varying distribution conditions?

Relevance to Education and Psychology

Research helps to make inferences about the general population through the samples drawn from them. The tool for reaching this goal is statistical analysis. To generate accurate conclusions and avoid misleading decisions, necessity is laid on the researchers to choose the statistical tools that have appropriate Type I error properties and comparative statistical power in real-life situations. Studies have shown that the nonparametric statistics have greater power advantages both in the normal distribution models and the skewed and kurtosis characterized distributions.

Limitations of the study

The study is limited to the one-way repeated measures layouts and did not consider the higher-order procedures that include interactions. The treatment alternatives were restricted to shift in location for various sample sizes and measure combinations. This research work uses the real-life data, (mortality count from the COVID-19 data), and it is analyzed using the SPSS 26.01 and G*Power for the calculation of the power analysis as a function of the shift in the location parameter. Therefore, it is assumed that the results are replicable under these situations.

Definitions of Terms

Robustness

Hunter and May (1993) defined the robustness of a test as "the extent to which violation of its assumptions does not significantly affect or change the probability of its Type 1 error" (p. 386). Sawilowsky (1990) stated, "the robustness issue is related not only to Type 1 error but also to Type II error, the compliment of the power of a statistical test" (p. 98).

Power

Bradley (1968) wrote, "the power of a test is the probability of it's rejecting a specified false null hypothesis" (p. 56). Power is calculated as 1-β, where β signifies the Type II error (Cohen, 1988). As β increases, the power of a test decreases.

Power Efficiency

Power efficiency is defined as the least sample size needed to notice a true treatment difference or to identify the false null hypothesis (Sawilowsky, 1990)

Interaction

Interaction is present when the pattern of differences associated with either one of the independent variables changes as a function of the levels of the other independent variable (Kelley, 1994).

Asymptotic Relative Efficiency (ARE)

The Asymptotic Relative Efficiency (also known as Pitman Efficiency) compares the relative efficiency of two statistical tests with large samples and small treatment effects (Sawilowsky, 1990). Blair and Higgins (1985) defined ARE as the "limiting value of b/a as "a" is allowed to vary in such a way as to give test A the same power as test B while “b” approaches infinity and the treatment effect approaches zero” (p. 120). This means that the efficiency of the competing nonparametric statistic is divided by that of the parametric statistic. If the ratio is found to be less than one, the nonparametric test is predicted to be less powerful than the parametric counterpart (Kelley, 1994).

Type I Error

This is when the result of a statistical test shows that there is an effect in the treatment when there is none, the decision to reject the null hypothesis is made. It is denoted by the Greek small letter alpha (α)

Type II Error

The decision of a test to fail to reject a null hypothesis (there is no treatment effect) when it is false is known as the Type II error. It is called beta (β).

CHAPTER TWO

THEORETICAL FOUNDATIONS AND LITERATURE REVIEW

Introduction

Researchers and organizations are often faced with the decision of choosing an intervention that yields a better result from between two conditions or treatments. The T-test is the statistical tool that has been very effective in solving this problem. However, this tool is not relevant in situations of choosing the most effective intervention among groups that are more than two. In that case, the perfect substitute to the t-test is the Analysis of Variance (ANOVA). “Analysis of variance may be defined as a technique whereby the total variation present in a set of data is partitioned into two or more components. Associated with each of these components is a specific source of variations so that in the analysis it is possible to ascertain the magnitude of the contributions of each of these sources to the total variation” (Daniel, 2009, p. 306). ANOVA model is an extension of the t-test therefore, it can fit into many different statistical designs based on the numbers of factors and levels. Factors are independent variables that can affect some outcomes of interest. Levels are those specific values attached to factors. ANOVA models test the hypotheses about population means and population variances. Invariably, it analyzes variances to make conclusions about the population means (Methods, 2020; Lane, 2019).

ANOVA is divided into different groups based on the different types of experimental designs, for example, one-way designs, mixed factor or mixed-method designs, repeated measures ANOVA and two-way ANOVA, etc. This research work focused on comparing the robustness and power of Repeated Measures ANOVA with its nonparametric counterpart- the Friedman test, and how each test behaves with the real-world dataset. Higher-order designs that involve interactions are not covered in this research study.

ANOVA was developed by Sir Ronald Fisher in 1918 (Stevens, 1999). It is an analytical tool used in statistics, that splits the total variance in a dataset into two parts: 1. Systematic factors or errors, and 2. Random factors or errors. Error is not a mistake but a part of the measuring process. It is called observational or experimental error.

Random errors are statistical alterations (in either direction) in the measured data due to the characteristics of different measurements. These errors are due to the peculiar attributes of different participants in the experiment. Random error in a statistical sense is defined in terms of mean error, the correlation between the error and true scores, where the correlation between errors is assumed to be zero. The direction of these types of errors is not predictable in an experiment and its distribution usually follows a normal distribution. Random errors do not have a statistical impact on the dataset, only the last significant digit of a measurement is altered. Random errors can be eliminated by increasing the number of samples taken and taking the average value of the sample sizes.

Systematic errors follow a single direction multiple times due to factors that interfere with the instrument used in generating data. Systematic errors have a statistical impact on the results of the given experiment. For example, if an experimenter wants to know the effects of two teaching methods on the results of students in different classes, one class was well lit and the other poorly lit. The means (averages) of these two classes will be statistically different because the two studies are not conducted under the same environmental conditions. Therefore, the system is biased. Systematic errors can occur due to faulty human interpretations, change in the environment during the experiments

(Khillar, 2020). Researchers can control for this type of error by randomization or blocking technique, by using proper techniques, calibrating equipment, and employing standards, etc. Unlike the random errors, systematic errors cannot be analyzed by generating a mean value for the samples because these types of errors are reproduced each time a similar study is conducted. Invariably, this type of error can be more dangerous, and the results generated from this type of observation will lead to inaccurate decisions.

ANOVA is used to determine the effects of the independent variables on the dependent variables in an experiment. Some assumptions need to be verified before ANOVA can be an appropriate tool for analysis:

- Homogeneity of the variance of each group of the dataset
- The observations /data groups are independent of each other.
- The data set is normally distributed on the dependent variable.

The F-test is conceptualized as a ratio of systematic error to random error: I.e., Variance Ratio is another name for F-test.

$$F = \frac{MST}{MSE} \approx \frac{systematic\ error}{random\ error} \tag{1}$$

where MST is Mean Square Total, and MSE is Mean Square Error. F is equal to the mean square total divided by the mean square error which is equivalent to the systematic error divided by the random error. F-values range from 0 to positive infinity (0 to +∞) and it depends on a pair of degrees of freedom (df), i.e., df for the numerator and df for the denominator. The ANOVA F-test allows the comparison of 3 or more groups of observations to determine the between sample errors and within samples errors.

This was not possible with the two-sample group t-test. In ANOVA, there are two types of hypotheses in the Neyman-Pearson frequentist approach to experiments, which includes the null and alternative hypotheses. The null hypothesis, denoted by H_o indicates that there is no statistically significant difference in the group means, while the alternative hypothesis (H_a) is the exact opposite of the claim stated in the null hypothesis.

The hypothesis tested in one-way ANOVA is $\mathrm{Ho}\text{: } \mu_1 = \mu2 = \ldots \mu_n$, which seeks to determine if there are differences among at least one of the sample means, as opposed to whether such differences are due to sampling error (Chan & Walmsley, 1997). The ANOVA is relatively robust to departures from population normality when testing for a shift in location (Hecke, 2010). However, in situations where the normality assumption is violated, the nonparametric alternatives, which are completely robust, offer additional power in detecting a false null hypothesis. Rank-based nonparametric alternatives employ a ranking technique to convert the original data into ranks.

There are divergent views concerning information integrity when data are ranked. Some researchers opined data converted into ranks results in the loss of information and less powerful test (Adams & Anthony, 1996; Borg, 1987; Chase, 1976; Garrett, 1966; Gravetter & Wallanu, 1985; Kerlinger, 1964). Wolfowitz (1949) asserted “the only kind of information a nonparametric procedure is likely to waste is information that is unavailable anyway” (p. 175). Others affirmed that the ranking of scores removes noise and increases the statistical power of a test (Blair, Higgins, & Smitley, 1980; Blair & Higgins, 1985; Langhehn, Berger, Higgins, Blair, & Mallows, 2000; Sawilowsky, 1993). "Transformation techniques are typically performed in order to stabilize error variance, improve normality of the datasets, and simplify the model" (Saste, Sananse, & Sonar, 2016, p. 654).

Solomon & Sawilowsky (2009) also note “rank-based transformations not only attempt to equate the means and homogenize the variance of test-score distributions, they also aim to create conformity in the third and fourth moments, skewness and kurtosis.” (p. 449).

Repeated Measures ANOVA

Repeated measures ANOVA is a technique of analyzing the mean differences that exist among groups of observations when the number of subjects is few, changes in participants’ behavior (variable) need to be measured over long periods. This model subjects the same group of participants multiple times to different conditions or interventions, to see how they perform at different times and also if there are noticeable improvements beyond those due to chance. Improvements or changes in the performance of the subjects can either be in the positive or negative direction. For example, when a group of obese women is randomly assigned into 3 different diet plans, to monitor the effect on their body weight for 4-weeks, the improvement is expected to be in the negative direction, (to lose some body fat). However, when a group of cancer patients is given three different brands of medication, the expected change will be in the positive direction, (good health). When the same group of individuals is repeatedly assessed over a specific period, it is called the within-subject or (RM) Repeated Measures ANOVA (Stevens, 1999). Repeated measures ANOVA is termed the within-subject measures because researchers compare the means of the varying observations from the same subject, each subject representing a block, and provides control values against which to compare. The repeated observations which are taken from the same subject, tend to be dependent among each other. Since repeated scores are highly correlated among groups, it takes very little variation in the treatment means to detect any effect that is present. This makes the within-subject design possess more power advantage over the between-subjects designs. However, when different participants are exposed to the same level of treatments then the situation is the between-subject design, also variabilities among participants are present. The within-subject ANOVA has a greater power advantage over the between-subject design because the random error is reduced drastically. The within-subject ANOVA identifies two types of variations present in the analysis: (a) variation that is due to the subjects and (b) variation that is due to the treatments. RM removes the variation that is due to the subjects from the *MS* error, which brings *MS* error to a smaller value and makes the F ratio to be large. The large F-value will result in rejecting the false null hypothesis.

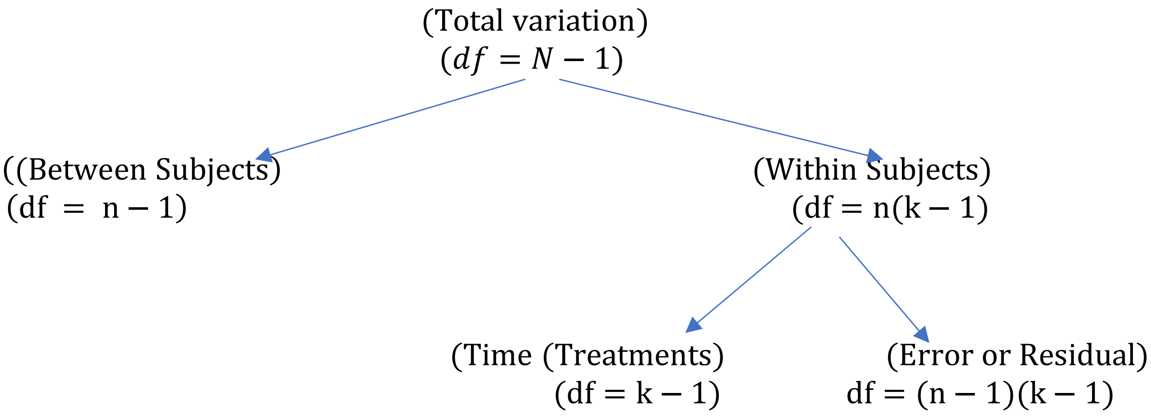

(Total variation)
$(df = N - 1)$
((Between Subjects)
$(\mathrm{df} = \mathrm{n} - 1)$
(Within Subjects)
$(\mathrm{df} = \mathrm{n}(\mathrm{k} - 1)$
(Time (Treatments)
$(\mathrm{df} = \mathrm{k} - 1)$
(Error or Residual)
$\mathrm{df} = (\mathrm{n} - 1)(\mathrm{k} - 1)$

Figure 1. Partition of Errors for One-factor Repeated Measures ANOVA.

N: the number of subjects
K: the number of repeated assessments (e. g. , time points)
$\mathrm{k} * \mathrm{n}$: total number of measurement

Test statistic F=Variation Over Time or Between Treatments/Error or Residual Variation
Note: Adapted from Sullivan (Sullivan L. M., 2008, p. 1239).

Randomized Block Design

In a randomized block design, each subject serves as a block and their responses serve as different conditions. This design eliminates the equivalency problem before the interventions and removes participant variability from the error term. By that, fewer participants can be tested at all levels of the experiment making each subject serve as its own control against which to compare the other variables. This technique is best appreciated in the medical world where large numbers of participants are not accessible. Repeated measures design could also have some shortcomings. These may include:

- **the carryover effect**: when the residue of the first treatment affects the experimental outcomes.
- **the latency effect** is the effect that is present but did not manifest until the subsequent treatments are administered, and
- **fatigue** is because of the stress participants experienced by involving in series of experiments which can affect the result of subsequent interventions (Girden, 1992; Stevens, 1999).

When a researcher faces a situation of exposing the same subjects to several treatments at a time, caution needs to be taken in the order of administering the treatments. The counterbalancing procedure of administering the treatments was proposed by Girden (1992), to alleviate the problem of treatment ordering effect. For example, "Carry-over effect can be minimized by lengthening the time between treatments; latency, however, is harder to control" (p.3). Also, holding the extraneous variable constant can help reduce some of the latency effects; administering short and interesting (activities) conditions can eliminate fatigue in the participants during the experimental process. However, when any of the effects due to the patterns of treatments influence the outcomes of the experiment, there are threats to the internal validity of the test. Some factors that pose threats to the internal validity of RM are listed below:

"***Regression threat,*** (when subjects are tested several times, their scores tend to regress towards the means), ***a maturation threat*****,** (subjects may change during the course of the experiment), and **a *history threat,*** (events outside the experiment that may change the response of subjects between the repeated measures)" (Lumen Boundless, 2020).

Statistical analyses always have some assumptions to be met before their applications can be valid. Of no exception is the repeated measures ANOVA.

The univariate assumptions of the repeated measures ANOVA are listed below:

I. The dependent variables of each level of the factor must follow a multivariate normal distribution pattern.

II. the variances of the difference scores between each level of factor must be equal across levels.

III. correlations between any pair of the levels must be the same across levels, e.g., $\rho(\mathrm{L1}, \mathrm{L2}) = (\mathrm{L2}, \mathrm{L3}) = (\mathrm{L1}, \mathrm{L3})$, (II & III constitute circularity or sphericity assumption).

IV. subject scores should be independent of each other.

V. Participants must be randomly sampled.

Parametric and Nonparametric Tests

The term parameter is generally used to categorize unknown features of the population. "A parameter is often an unspecified constant appearing in a family of probability distributions, but the word can also be interpreted in a broader sense to include almost all descriptions of populations characteristics within a family" (Gibbons, 2003, p. 1). In a distribution-free inference, either hypothesis testing or estimation, the methods of testing are based on sampled data whose underlying distributions are completely different from distributions of the population from which the samples were drawn. Therefore, the assumptions about the parent distribution are not needed (Gibbons, 2003). Nonparametric test connotes the claim of the hypothesis test which has nothing to do with parameter values. “Nonparametric statistic is defined as the treatment of either nonparametric types of inferences or analogies to standard statistical problems when specific distribution assumptions are replaced by very general assumptions and the analysis is based on some function of the sample observations whose sampling distribution can be determined without knowledge of the specific distribution function of the underlying population. Perhaps the chief advantage of nonparametric tests lies in their very generality, and an assessment of their performance under conditions unrestricted by, and different from, the intrinsic postulates in classical tests seems more expedient" (Gibbons, 1993, p. 4; Gibbons, 2003, p. 6-7).

Corder & Foreman, (2009) state “specifically, parametric assumptions include samples that:

- are randomly drawn from a normally distributed population,

- consist of independent observations, except for paired values,
- have respective populations of approximately equal variances,
- consist of values on an interval or ratio measurement scale,
- are adequately large and approximately resemble a normal distribution." (p. 1-2).

However, different researchers have defined the minimum sample size for using a parametric statistical test differently, e.g., Pett (1997) and Salkind (2004) suggest $\mathrm{n} > 30$ as common in research while, Warner (2008) consider a sample of greater than twenty $(\mathrm{n} > 20)$ as a minimum and a sample of more than ten $(\mathrm{n} > 10)$ per group as an absolute minimum.

When a dataset does not satisfy any of the above-listed assumptions, then violation occurs. In the situation of assumption violations, few corrections may be considered before parametric statistics can be used for such analysis. First, with detailed explanations, extreme values or occurrences that may shift the distribution shapes can be eliminated or dropped. Second, the application of rank transformation techniques can be used to change the observations from interval or ratio scale to (ranks) ordinal scales (see Conover & Iman, 1981 for details). Although, this method has been seriously criticized and termed "controversial method" (Thompson, 1991, p. 410; see also Akritas, 1991; Blair & Higgins, 1985; Sawilowsky, Blair, & Higgins, 1989). All the alterations or modifications must be displayed in the discussion section of the analysis. Fortunately, another body of statistical tests has emerged that does not require the form of the dataset to be changed before analysis. These are the Nonparametric Tests (Corder & Foreman, 2009).

Jacob Wolfowitz first coined the term nonparametric by saying "we shall refer to this situation (where a distribution is completely determined by the knowledge of its finite parameter set) as the parametric case, and denote the opposite case, where the functional forms of a distribution are unknown as the nonparametric case" (Wolfowitz, 1942, p. 264). Hollander & Wolfe (1999) stated explicitly “in the 60+ years since the origin of nonparametric statistical methods in the mid-1930s, these methods have flourished and have emerged as the preferred methodology for statisticians and other scientists doing data analysis” (p. xiii).

The drastic success of nonparametric statistics over the era of six years can be credited to the following merits:

- Nonparametric methods require less and unrestrictive assumptions about the underlying distributions of the parent populations from which the data are sampled.
- “Nonparametric procedures enable the users to obtain exact statistical properties. E.g., exact *P*-values for tests, exact coverage probabilities for confidence intervals, exact experimental-wise error rates for multiple comparison procedures, and exact coverage probability for confidence bands even in the face of nonnormality” (Siegel, 1956, p. 32).
- Nonparametric techniques are somewhat easy to understand and easier to apply.
- Outliers, which distort the distribution shapes cannot influence the nonparametric techniques since score ranks are only needed.
- “Nonparametric tests are applicable in many statistical designs where normal theory models cannot be utilized.” (Hollander & Wolfe 1999, p. 1).

How Rank Transform Techniques Work

“A problem that applied statisticians have been confronted with virtually since the inception of parametric statistics is that of fitting real-world problems into the framework of normal statistical theory when many of the data, they deal with are clearly non-normal. From such problems have emerged two distinct approaches or schools of thought: (a) transform the data to a form more closely resembling a normal distribution framework or (b) use a distribution-free procedure.” (Conover and Iman,1981, p. 124). The application of rank transform techniques to change the form of data from interval or ratio to ordinal scales before applying the parametric model for analysis is what Conover (1980) proposed as the rank transformation (RT) approach. He termed this approach as a bridge between the parametric and nonparametric tests by simply replacing the data with their ranks, then apply the usual parametric tests to the ranks.

Research showed that rank-based tests yield a comparable power advantage over the classical counterparts (Hodges & Lehmann, 1960; Iman, Hora, and Conover, 1984; Sawilowsky, 1990). Hajek & Sidak (1967) stated rank tests are derived from the family of permutation tests and were developed “to provide exact tests for wide (nonparametric) hypothesis, similar to those developed for parametric models in the small sample theory.” (p. 11). Rank tests “maintain the properties of the parent permutation test in being nonparametric exact tests, and yet these procedures are often easy to compute” (Sawilowsky, 1990, p. 94).

The ranking of observations carries some merits:

- The methods of calculation are very simple,

- Only very general assumptions are made about the kind of distributions from which the observations arise,
- Rank tests have the chance of detecting the kinds of differences of real interest.
- "If there are multiple samples, the mean ranks for any of them are jointly distributed approximately according to a multivariate normal distribution, provided that the sample sizes are not too small" (Chan & Walmsley, 1997, p. 1757).
- "Rank transformation techniques results in a class of nonparametric methods that includes the Wilcoxon-Mann-Whitney test, Kruskal-Wallis test, the Wilcoxon signed ranks test, the Friedman test, Spearman's rho, and others. It also furnishes useful methods in multiple regression, discriminant analysis, cluster analysis, analysis of experimental designs, and multiple comparisons." (Conover & Iman 1981, p. 124).
- "Variance estimates based on ranks are less sensitive to the values of outliers than are those based on the original data.
- The use of RT methods protects the practitioner against making the false decisions than can result from a distorted significance level due to nonnormality." (Potvin & Roff, 1993, p. 1621).

Methods of Ranking

Four ways of ranking data were suggested by Conover and Iman.

- "Rank Transform (RT)1 is when the entire observation is ranked together from smallest to the largest, with the smallest observation having rank 1, second smallest having rank 2, and so on. Average ranks are assigned in case of ties.

- In RT 2- the observations are partitioned into subsets and each subset is ranked within itself independently of the other subsets. This is the case of the Friedman test.
- RT 3 – this rank transformation is RT-1 applied after some appropriate re-expression of the data.
- RT 4- the RT-2 type is applied to some appropriate re-expression of the data” (p. 124).

Friedman: A Nonparametric Alternative to the Repeated Measures ANOVA

Friedman’s ANOVA is a nonparametric test that examines whether more than two dependent groups mean ranks differ. It is the nonparametric version of *one-way repeated-measures ANOVA*. The Friedman test is perhaps the most popular among the rank tests for analyzing *k*-related samples. The method of ranking random block data was discussed in detail by Friedman (1937).

The test statistic for the Friedman test involves grouping observations together based on their similar characteristics, which forms the blocks of data. The summary of the test procedure is as follows:

I. Arrange the scores in a table that have *K* columns (conditions or treatments) and *N* rows (subjects or groups).

II. Rank the variables across the levels of the factor (row) that is, from 1 to *K.*

III. Determine the sum of the ranks for each level of the factors and divide the value by the number of the subjects, $(\frac{\boldsymbol{Rj.}}{\boldsymbol{n}})$. This is termed $\bar{R}_{j.}$

IV. Determine the sum of the variables across the levels of the factor (row) that is, from 1 to *K,* multiply this value by half, $\frac{1}{2}(K+1)$*,* that is the grand mean. Label this value $\bar{R}$.

V. "The test statistics is a function of the sum of squares of the deviations between the treatment rank sums $\bar{R}_{j.}$, and the grand mean $\bar{R}$." (Gibbons, 1993, p. 55).

The formula is written as follows:

$$S = \sum_{j=1}^{k} (\bar{R}_j \; - \bar{\bar{R}})^2 \equiv \; S = \sum_{j=1}^{k} (\frac{\boldsymbol{Rj.}}{\boldsymbol{n}}) - (\frac{\mathrm{k}+1}{2})^2 \qquad (2)$$

$$M = \frac{12n}{k(k+1)} S \qquad (3)$$

Where *n* is the number of rows or subjects, *k* is the number of columns, and *S* is a function of the sum of squares of the deviations between the treatment rank sums $\bar{R}_{j.}$, and the grand mean $\bar{R}$. Or "the sum of the squares of the deviations of the mean of the ranks of the columns from the overall mean rank."

An alternate formula that does not use *S* was the test statistic as proposed by Friedman and it is as follows:

$$M = \left[\frac{12}{nk(k+1)} \; \sum_{j=1}^{k} \; R_j{}^2\right] - 3n(k+1) \qquad (4)$$

Where *n* is the number of rows, *k* is the number of columns and $\bar{R}_{j.}$ Is the rank sum for the *J^th^* column. J = 1,2, 3, ... *k*" (Fahoom & Sawilowsky, 2000, p. 26, See also, Pereira, Afonso, & Medeiros, 2015; Siegel & Castellan Jr, 1988). Note: All these statistics will arrive at the same result. "When the number of treatments and blocks is large, it is generally assumed that *S*, with the degree of freedom *k-1*, tends to be asymptotically distributed according to the Chi-squared (x^2) approximation" (Siegel,1956, p. 168)

The model for this test statistic was developed by Friedman (1937). The design assumed that the additive model holds as follows:

$$X_{ij} = \mu + \beta_i + \tau_j + E_{ij} \tag{5}$$

where X_{ij} is the value of each treatment (j^{th}) in the (i^{th}) block, μ is the grand mean, $\tau_j$is the (j^{th}) treatment effect, β_i is the (i^{th}) block effect. The errors E_{ij} are assumed to be independent and identically distributed, (i.i.d) with continuous distribution function *F*(x) (Skillings & Mack, 1981, p. 171). Friedman's test is an analog to the one-way repeated measures ANOVA where the same participants are subjected to different treatments or conditions.

Hypothesis Testing and Errors in Statistical Analysis

Statistical inference is in two major forms: estimation and hypothesis testing. "The purpose of hypothesis testing is to aid the clinician, researcher, or administrator in reaching a conclusion concerning a population by examining a sample from that population" (Daniel, 2009, p. 216). Hypothesis testing and power go hand in hand. In statistical analysis, two hypotheses are highlighted; the null hypothesis or the statistical hypothesis which is the hypothesis of no effect of treatment or intervention or zero difference among the sample means. It contains a statement of equality and its "claim may be evaluated by the appropriate statistical technique" (Daniel, 2009, p. 217). Then, the alternative hypothesis, counters whatever is stated in the null hypothesis, it is the claim that is believed to be true if the statistical results reject the null hypothesis.

Friedman's test examines the null hypothesis of whether the total value for each treatment group is equal across treatments. Hollander & Wolfe (1999) state it as follows: "that no differences among the additive treatments effect $\tau_1 \dots \dots \dots . \tau_k$, namely,"

$$H_o: [\tau_1 = \cdots = \tau_k] \qquad (6)$$

versus the general alternative hypothesis

$$H_a: [\tau_1 \dots \dots ., \tau_k] \qquad (7)$$

The significance level (α) is set at 0.05, "the H_0 is rejected if $S \geq \chi^2_{k-1;\alpha}$, otherwise do not reject, where $\chi^2_{k-1;\alpha}$ is the upper alpha percentile point of a chi-square distribution with $\mathrm{k}-1$*, df"* (p.272-273). Iman & Davenport (1980) noted that the Chi-square approximation quickly falls off as *k* increases with fixed *b.* Therefore, he proposed F approximation, which improves as *k* increases and the error rate is liberal, but still dominates the Chi-square approximation based on +/- 10%. He then advised the researchers "to choose F approximation over the Chi-square approximation for small samples" (p. 584). F-approximation is distributed with $(\mathrm{K}-1)$ and $(\mathrm{K}-1)(\mathrm{n}-1)$ degrees of freedom (Pereira et al., 2015, p. 2639). Because Friedman's test is an omnibus test, it can only indicate that significance exists between the groups but does not specify the exact pair or groups. Therefore, it is necessary to perform post hoc tests, such as the Wilcoxon-sign test, to determine where the significance lies.

Type I and Type II Errors

In hypothesis testing, an alpha level of 0.05 signifies there is a 5% chance that the test result will yield a false alarm or that the test will display an effect that is not present. This can lead the researcher to making an erroneous decision of rejecting the null hypothesis. Studies show that given reasonably large sample size (> 30), the results of a test will always yield a significant effect, even if the effect is due to sampling errors (Akbaryan, 2013; Johnson, 1995; Kim, 2015; Steidl, Hayes, & Schauber, 1997; Thomas & Juanes, 1996). This is the first type of error (Type I error) in hypothesis testing. The second type of error is the Type II error, denoted by β. This error is committed when the result of a test fails to reject the false null hypothesis. Then, "the power analysis (retrospective or *posteriori* power analysis)" of such test needs to be performed in order to provide explanation and confirmation to the validity of the test results" (Steidl, Hayes, & Schauber, 1997, p. 271). To reduce the rate of error, alpha can be set at a very small value (stringent alpha). Beta (β) is directly related to the power of a test. Statistical power is the probability that the result will find a true effect that is present in the analysis, and then, reject the false null hypothesis of no difference (Bridge & Sawilowsky, 1999; Cohen, 1962, 1969; Faul, Erdfelder, & Buchner, 2007; Kim, 2015; Kupzyk, 2011; Park & Schutz, 1999; Potvin, 1996; Steidl et al., 1997; Thomas & Juanes, 1996).

Table 1. Hypothesis Table.

		Condition of Ho (Reality)	
		Ho is True	Ho is False
Decision & result	Fail to reject Ho	Correct (1-α)	Type II error (β)
	Reject Ho	Type I error (α)	Correct (1-β)

Adapted from (David, 2009; Steidl et al., 1997, 271)

Important Assumptions of the Univariate One-Way Repeated Measures ANOVA are elaborated in detail below:

Sphericity Assumption

Before the univariate method of analyzing block-designs can be the appropriate choice of the test statistic for any observation, the degree of variability (variances) within each level of intervention must be equal. Generally, there is always some level of interrelationships among observations, scores are dependent on each other. Therefore, it is assumed that the variances of the differences (covariances) between each pair of the variables of within-factor level must be equal across treatments. These two patterns of variabilities are called compound symmetry (Box, 1954) and are later termed sphericity or circularity assumption (Huynh & Feldt, 1970). Sphericity is equivalent to the homogeneity of variance assumption in the between factor or independent measures ANOVA. For the two-sample t-test, the assumption of homogeneity of variances is always a work-over since there is only one covariance present. Invariably, covariance is the deviations from the mean of each of two measures for each person, this connotes that the means and the variances of the differences can be obtained by subtracting the first observation from the second observation, and the result must be the same for the difference between the first observation and third observation. Simply put “sphericity requires that variances of differences for all treatment combinations be homogeneous i.e., $\sigma_{y_1-y_2}^2 = \sigma_{y2}^2 - y_3, etc$” (Girden, 1992, p.16; Lamb, 2003, p. 14). Therefore, in situations where these values are not similar across levels, the assumption of sphericity has been violated.

There are many other viable options to solve this dilemma, some of which are insensitive to the assumption of variance equality. Multivariate analysis of variance (MANOVA, e.g., Hotelling’s T^2) can be used to analyze the repeated observations with violated sphericity. This design requires either first to transform the original scores into a new form of J-1 differences and the analysis is performed. Or second, by creating the matrix of orthonormal coefficients, then, use the coefficients to perform the analysis. The assumption of sphericity does not affect this test. These two methods of correction will generate the same result (Girden, 1992; see also Stevens, 1999 for details). However, MANOVA design is beyond the scope of this study.

There are many methods of estimating the homogeneity of variances assumption in two or more group samples data: Levene's test, Bartlett's test, Brown-Forsythe test, Flinger-Killeen test (a nonparametric test) Cochran's Q test (for dichotomous data of more than 2 dependent groups), Hartley test (compares variance ratios to the F-critical value), O'Brien test (tests homogeneity for several samples at once), Mauchly's W (tests the sphericity assumption in a repeated measures or matched group samples design).

For independent group ANOVA, there is an assumption of independence of observation. While, for the repeated measures ANOVA, there are interrelations among the response variables, hence the test for sphericity needs to be carried out. This is to determine the extent to which the sphericity has shifted. Epsilon (ε) is the parameter used for correcting the sphericity violation. Epsilon is always set at 1, which indicates perfect sphericity. The farther away from 1 epsilon is the more the violation (Box, 1954; Bryan, 2009; Girden, 1992; Greenhouse & Geisser, 1959; Lamb, 2003). Assumption of sphericity is hardly met or often violated in the real-life data. When the dataset violates this assumption, it implies that the test is liberal, (i.e., Type I error rate is increased or inflated) (Vasey & Thayer, 1987). To avoid a test that lacks power, the degree of violation of sphericity (ε) is estimated. Mauchly (1940) proposed a test that displays the results of homogeneity alongside the significance level (i.e., P-value). When Mauchly's W gives a significant result (P-value < α), then the hypothesis which states that the variances of the differences between the levels of the responses are equal will be rejected (Bryan, 2009). Three values of (ε) are generated by Mauchly's test: the first is the (G-G) Greenhouse & Geisser (1959), the second is for (H-F) Huynh & Feldt (1976) and the last value is for Lower bound. The first two results are always referenced in research.

The significant F-value indicates large values for the two degrees of freedom (*df*), and the post hoc test procedure is the adjustment of the two degrees of freedom by the value of (ε) generated. Therefore, the correction is to reduce the numerator and denominator *df* by multiplying both by the (ε) value (Bryan, 2009; Girden, 1992; Lamb, 2003; Stevens, 1996).

The (ε) is calculated by two formulae: epsilon hart ($\hat{\varepsilon}$) and epsilon tilde ($\tilde{\varepsilon}$)

$$\hat{\varepsilon} = \frac{\mathrm{J}^2\overline{(\mathrm{D}}-\overline{\mathrm{Cov}_\mathrm{T}})^2}{(\mathrm{J}-1)(\sum \mathrm{Cov}_{\mathrm{ij}}{}^2-2\mathrm{J}\sum \overline{\mathrm{Cov}_{\mathrm{i.}}{}^2}+\mathrm{J}^2\overline{\mathrm{Cov}_\mathrm{T}}^2)} \qquad (8)$$

Where, $\overline{D}$: mean of variances along the diagonal,

$\overline{Cov_T}$: mean of all entries in the matrix,

$Cov_{ij}{}^2$: a squared entry in the matrix, and

$\overline{Cov_{i.}}$: mean of the entries of a row in the matrix.

This $\hat{\varepsilon}$ adjustment is known as the G-G's correction parameter and it ranges from 1/J-1, indicating the worse spherical shift to 1 a perfect spherical pattern (Box, 1954; Bryan, 2009; Greenhouse & Geisser, 1959; Lamb, 2003). This $\hat{\varepsilon}$ adjustment is accurate when it is kept below 0.75.

However, studies have shown that If the value of $\hat{\varepsilon}$ is greater than 0.75, ($\hat{\varepsilon} > 0.75$) then the adjustment will be conservative and tends to underestimate epsilon, meaning that many nonnull will falsely be retained. Therefore, to further correct for this conservativeness, Huynh and Feldt (1976), introduced a less conservative epsilon parameter called epsilon tilde ($\tilde{\varepsilon}$), and it is calculated by this formula:

$$\tilde{\varepsilon} = \frac{[N(J-1)\hat{\varepsilon}]-2}{(J-1)[N-k-(J-1)\hat{\varepsilon}]} \qquad (9)$$

k: number of groups, or 1 for a single − factor study,

N: the total number of subjects,

J: the number of treatment conditions

(Bryan, 2009; Girden, 1992; Keselman, Algina, & Kowalchuk, 2001; Lamb, 2003).

This alternative works great in correcting the degrees of freedom (df) when it is greater than 0.75, otherwise, it tends to overestimate epsilon and produces a liberal adjustment (Bryan, 2009; Lamb, 2003; Maxwell & Delaney, 1990).

Since these two estimates are heading in the opposite directions, Huynh & Feldt (1976) suggested “the difference between $\hat{\varepsilon}$ and $\tilde{\varepsilon}$ tends to decrease as the number of sample size N is increasing” (p. 75). To get a near unbiased figure of epsilon, it was recommended that the mean of the two figures be taken (Bryan, 2009; Girden, 1992; Lamb, 2003; Stevens, 1992, 1996).

Girden (1992, p. 21) summarized the whole process as follows:

1. If epsilon is greater than 0.75, adjust df by less conservative epsilon tilde.
2. If epsilon is less than 0.75, adjust df by the more conservative epsilon hart.
3. If nothing is known about epsilon, adjust df by the conservative epsilon.

Robustness

From the previous studies, it has been confirmed that normality is a very rare, almost unattainable, and difficult assumption in the real-world dataset. Micceri (1989) analyzed 440 distributions from ability and Psychometric measures and discovered that most of those distributions have extreme shifts from the normal distribution shape, including different tail weight and different classes of asymmetry. (Blanca, Arnau, López-Montiel, Bono, & Bendayan (2013) analyzed “693 distributions derived from natural groups formed in institutions and corresponding to 130 different populations, with sample sizes ranging from 10 to 30. 39.9% of distributions were slightly non-normal, 34.5% were moderately non-normal, and 2.6% distributions showed high contamination. The displayed skewness and kurtosis values were ranging between 0.26 and 1.75. He, therefore, assert “these results indicate that normality is not the rule with small samples” (p. 5,10). Other studies such as the works of (Harvey & Siddique, 2000; Kobayashi, 2005; Van Der Linder, 2006), have also established this fact. Therefore, researchers are faced with the task of deciding whether the F-test is the best fit to analyze the real-world data. Robustness is the insensitivity of test statistics to the violation of the underlying assumptions. I.e., robustness is when a statistical test still retains its properties of rejecting a false null hypothesis, and also the beta properties in the situation of assumption violation. However, there should be a degree or an extent of violation of assumptions a test statistic can reach before its type I error rate is inflated.

Over the years, there have been several ambiguous and very broad interpretations given to the term “robustness” of a test statistic, which made it difficult for researchers to determine the extent to which the F-test can be used when the distributions are non-normal. For example, phrases like slight/moderate shift from normal distribution cannot influence the results of the fixed-effects ANOVA (Montgomery, 1991). Keppel, (1982) puts the same phrase as the violations of normality should not be a thing of worry, unless the violations are really to the extreme or F test is robust to moderate shift in location, provided the sample sizes are fairly large and equal across the treatment groups (Winer, Brown, & Michels, 1991). Some opined that F-test is insensitive to a little shift in the location of distribution shape (Berenson & Levine, 1992; Bridge & Sawilowsky, 1999; Harwell, 1998; Kelley, 1994; Sawilowsky & Blair, 1992). All the interpretations given to the term robustness were relative to the basis of the research study. This ambiguity problem also made the study comparisons across different fields to be impossible (Blanca, Alarcón, Arnau, Bono, & Bendayan, 2017). Bradley (1978) summed the situation up in this statement, “Not only is there no generally accepted, and therefore standard, quantitative definition of what constitutes robustness but, worse claims of robustness are rarely accompanied by any quantitative indication of what the claimer means by the term. In order to provide a quantitative definition of robustness (of significance level), you would have to state for a given alpha value the range of *p*-values for which the test would be regarded as robust” (p. 145-146).

Therefore, Bradley (1978) proposed a criterion that remedied the problem and defined robustness as follows: "a test is robust if the type I error rate is .025 and .075 for a nominal alpha level of 0.05" (Blanca, Alarcón , Arnau, Bono, & Bendayan, 2017, p. 533).

Bradley finally proposed liberal and stringent meanings of robustness. The liberal criterion which he defined as 0.5 alpha ≤ π ≤ 1.5 alpha, alpha being the nominal significance level, π being the actual type I error rate. Therefore, a nominal alpha level of

.05 would generate a *p*-value ranging from 0.025 to 0.075, and for the nominal alpha of 0.01, there would be a p-value range from 0.005 to 0.015. The stringent definition of robustness is as follows; “0.9 alpha ≤ π ≤ 1.1 alpha, thus a nominal alpha level of 0.05 would yield a p-value ranging from 0.045 to 0.055” (Bridge, 1996; Kelly, 1994).

Power Analysis

It is important to carry out *a priori* statistical power analysis for the repeated measures design. However, “complicated procedures, lack of methods for estimating power for designs with two or more RM factors, and lack of accessibility to computer power programs are among some of the problems which have discouraged researchers from performing power analysis on these designs” (Potvin, 1996, p. ii). Statistical power is defined as the probability of finding a significant effect, or a magnitude of any size of differences when there exists a true effect among the population means (Park & Schutz, 1999).

Power analysis performed at the outset of an experimental study carries with it the following benefits:

I. Power analysis helps researchers to determine the necessary number of subjects needed to detect an effect of a given size. Stevens (1999) noted “the poor power may result from small sample size (e.g., <20 samples per group) and/or from small effect size.” (p. 126)

II. Power analysis is performed before an experiment to determine the magnitude of power a study carries given the effect size and the number of samples (Kupzyk, 2011; Potvin, 1996; Steidl, Hayes, & Schauber, 1997).

III. It helps the researcher to answer such a question as: does the study worth the money, time, and the risk involved, given the number of participants needed, and the effect sizes assumed (Potvin, 1996)**.**

IV. Low power studies may “cut off further research in areas where effects do exist, but perhaps are more subtle e.g., social or clinical psychology” (Stevens, 1999, p. 126).

V. “It also helps researchers to be familiar with every aspect of the study” (UCLA, 2020).

The concept of power had existed for about four decades, (Halow, 1997) before Cohen brought the concept to the limelight through his publications (Cohen, 1962; 1969)**.** The power of a statistical test was not thought of as a concept that can bridge the gap between statistical significance and physical significance of a test (Thomas & Juanes, 1996). As soon as it is well known the significant contribution of power analysis to the research process, efforts have been made towards making its calculations very easy and accessible. Also, practical methods for calculating statistical power, and all its components have been generated. For some simple statistical designs, several computer software programs and power calculation tables, have been made available to the researchers (Borenstein & Cohen, 1988; Bradley, 1978, 1988; Cohen, 1988; Elashoff, 1999; Erdfelder, Faul, & Buchner, 1996, 2007; Goldstein, 1989). However, for complex designs, analytical methods of estimating power are not easy to come by because more factors result in higher interactions among the factors. The methods of analyzing power for the repeated measures ANOVA incorporates all factors that constitute the power concept, such as the correlations among the samples, sample size, the number of treatment levels, the population mean differences, error variances, the significance (α) level, and the effect sizes (Bradley, 1978; Cohen, 1988; Lipsey, 1990; Potvin & Schutz, 2000; Winer, Brown, & Michels, 1991). Hence, "this method of estimating power function is mathematically very complex." (Park & Schutz, 1999, p. 250). In RM ANOVA, the response variables are interdependent of each other, the higher the correlations among the variables, the higher the power (Bryan, 2009; Girden, 1992; Keselman, Algina, & Kowalckuk, 2001; Lamb, 2003). "The outcome of the effect of all the factors that correlate and affect power function, in ANOVA designs can be described by what is called *the non-centrality parameter (NCP).* The non-centrality parameter (NCP), is the magnitude of the size of the differences between population means that represents the degree of inequality between an *F*-distribution and the central (null hypothesis) *F*-distribution when the observed differences in population means are not due to chance or sampling bias (Winer et al., 1991). There are quite a few methods of calculating a non-centrality parameter (e.g., f, δ^2, Φ, λ) but all are closely related to each other, and they all signify standardized effect sizes. This makes generalizability possible and comparable across studies, (meta-analysis) (Cohen, 1988; Kirk, 1995; Park & Schutz, 1999; Barcikowski & Robey, 1984; Tang, 1938; Winer, Brown, & Michels, 1991). The non-centrality parameter λ, for the one-way **RG** ANOVA, can be represented as:

$$\lambda = \frac{n \sum(\mu_i - \mu)2}{\sigma^2} \qquad (10)$$

Where *n* is the sample size per group, μ_i represents the marginal (group) means, μ is the grand mean, and σ^2 is the error variance (Bradley,1978; Winer, Brown, & Michels, 1991). "The power is a nonlinear function of lambda (λ), the numerator and denominator degrees of freedom of the F-test, and the alpha level. For an RM design, the error variance decreases as the degree of correlations among the levels of the RM factor increases.” This Lambda, the unit of non-centrality for Repeated Measures design can be derived by the following equations. For the one-way RM ANOVA (j= 1, 2,….q),

$$\lambda = \frac{n\sum(\mu j-\ \mu)2}{\sigma 2\ (1-\bar{\rho})} \qquad (11)$$

(Park & Schutz, 1999, p.251)

The non-centrality parameter measures the degree to which a null hypothesis is false (Carlberg, 2014; Kirk, 2012). Invariably, it relates to the statistical power of a test. For instance, if any test statistic has a distribution with a non-centrality parameter that is zero, the test statistic, (T-test, Chi-square, F-test) will all be central (Glen, 2020). NCP is represented by lambda (λ), and all the factors that affect power also affect lambda. When the null hypothesis is not true, the one-way RM ANOVA has shifted from being centrally distributed (Howell, 1992, 1999; Potvin, 1996; Winer, Brown, & Michels, 1991). Therefore, power correlates with lambda in a quadratic manner, that is, nonlinear association.

Path to Effect Sizes

When researchers are thrilled by the curiosity of knowing whether a difference exists among groups because of an intervention or treatment given or not given, they embark on null hypothesis significance testing (NHST). Thompson (2003) puts it this way: “NHST evaluates the probability or likelihood of the sample results, given the sample size, and assuming that the sample came from a population in which the null hypothesis is exactly true” (p. 7) However, studies have shown that this statistical analysis is not an end in itself but a means to an end, (generalization to the population). The sixth edition of the APA, (2010) condemned the sole reliance on NHST by “not only encouraging psychology to shift emphasis away from NHST but also, more fundamentally, to think quantitatively and cumulatively" (Fidler, Thomason, Cumming, Finch, & Leeman, 2004; Fidler, 2010, p. 2). Therefore, "APA stresses that NHST is but a starting point, and that additional reporting elements such as effect sizes, confidence intervals, and extensive description are needed" (APA, 2010a, p. 33).

P-value only gives the probability that an effect exists given that the hypothesis of no effect is true that is, *p* (data **|** hypothesis) (Nakagawa & Cuthill, 2007; Sullivan & Feinn, 2012). Simply put the *p*-value is the probability that any disparity displayed among the groups is only attributable to chance or sampling variations (bias). Statistical significance is the interpretation of a test result given by the *p*-value in comparison to the level of significance (p< alpha), (Kim, 2015)**.**

Statistical significance and *p*-value are a function of both effect size and sample size, therefore, given a large enough number of samples even a very infinitesimal difference can display a misleading result, and lead to waste of resources (Aarts, Akker, & Winkens, 2014; Kim, 2015; Maher, Markey, & Ebert-May, 2013, p. 346; Sullivan and Feinn, 2012), and on the other hand, with fewer sample size, the analysis carries no power to detect significance. Alpha level (level of significance) is the probability of rejecting the null hypothesis when it is true. It is the measure of how compatible the sample data are with the null hypothesis. Also, the results given by the *p*-values make the researchers resolve to a two-way (dichotomous) decision. Either there is an effect, reject the H_o or effect does not exist, fail to reject the null hypothesis. Significant testing alone cannot give information about the size of the difference that exists among groups and also does not give a range of values (precision) around the effect of treatment or intervention, within which the value of the effect should be contained. This is the Confidence Interval. Dependence on statistical significance poses difficulty to the meta-analysis (studies will not be comparable across studies) (Maher, Markey, & Ebert-May, 2013).

All these demerits are found with the use of the NHST, and to overcome these pitfalls, researchers crave a better alternative- *Effect size!*

Meaning and importance of Effect size in Research

The Task Force on Statistical Inference of the American Psychological Association understands the importance of Effect Size (ES) and has suggested that researchers "should always provide some Effect Size estimates when reporting a *p*-value" (WILKINSON & TASKFORCE, 1999, p. 599); it stressed on reporting the effect sizes alongside their interpretation: "Wherever possible, base discussion and interpretation of results on point and interval estimates" (APA, 2010, p. 34); and finally gives detailed standards for reporting meta-analyses: "reporting and interpreting Effect Sizes in the context of previously reported effects is essential to good research" (p.599). Effect size gives information as to whether the observed difference is large enough to make sense in real life or the context of the field of the research (clinical, biological, physical, or educational fields). ES can also signify the direction of the variability between groups or the association between 2 groups of samples. Different fields of knowledge have used the term Effect size to report differences among group means, e.g., education (Baird & Pane, 2019; Kraft, 2018; Lipsey, 2012; Sawilowsky, 2006), medicine, and sciences (Aarts, Akker, & Winkens, 2014; Akbaryan, 2013; Kim, 2015; Maher, Markey, & Ebert-May, 2013; Nakagawa & Cuthill, 2007), psychology (Bakeman, 2005; Durlak, 2009;

Schäfer & Schwarz, 2019). Effect sizes have been defined from various perspectives, but they all boil down to the same meaning: Nakagawa & Cuthill (2007), gave three definitions of ES:

> *“Firstly, the effect size can mean a statistic which estimates the magnitude of an effect (e.g. mean difference, regression coefficient, Cohen’s d, correlation coefficient). It is called ‘effect statistic’ or ‘effect size index’. Secondly, it also means the actual values calculated from certain effect statistics (e.g. mean difference = 30 or r =0.7; in most cases, ES is written as ‘effect size value’). The third meaning is a relevant interpretation of an estimated magnitude of an effect from the effect statistics. This is sometimes referred to as the biological importance of the effect, or the practical and clinical importance in social and medical sciences.” (p. 593).*

Deep insight into the meaning of effect size has provided an answer to the following questions:

- Is there a real effect noticed beyond that which can be attributed to chances?
- If there is truly an effect, what is the size of such treatment effect?
- How physically important is the size of such an effect? (Bakker, et al., 2019; Kirk, 2001)**.**

“Effect size is a way to measure or quantify the effectiveness of an intervention, treatment or program. ES can also be described as the degree of falsity of the null hypothesis” ((Descôteaux, 2007, p. 25). An estimate of ES in conjunction with power analysis is used to determine the sample size needed for the analysis. This must be carried out before the experimental procedures and is called prospective or a priori power analysis.

Reporting the effect sizes for significant *p*-value is believed to be the norm, however, studies have shown that “even the *p*-values that are not significant should have their effect sizes reported” (Thompson, 1996, p. 29)**.**

Methods of calculating Effect Sizes

There are various methods of obtaining or calculating the effect sizes. The first and simple one that comes to mind is *the direct group means comparison*. This is the effect size that is calculated by comparing the raw group means, i.e., $\mu_1 \text{ minus } \mu_2$. However, this estimate is not generalizable to other studies since the variable of each study is scaled according to the intentions of the researcher. Even, studies conducted in the same field of study might not have the same dependent variables, hence the scales are different (Durlak, 2009; Ladesma, Macbeth, & Cortada de Kohan, 2009). Research studies are meant to complement each other; therefore, new knowledge should be developed upon the existing ones. Consequently, researchers clamor for a better effect size estimator – *standardized effect size or standardized group mean difference.*

There are about three possible methods under this group. When comparisons involve only two groups of mean values, Cohen's *d* is the most used effect size estimator. This is a further step from the raw score mean difference estimator by standardizing the difference, through the pooling of the two groups' Standard Deviations (SD). Cohen's *d* is only useful when the groups' SDs are very close, and the distributions of the samples are approximately normal (Cohen, 1988, 1992; Maher, Markey, & Ebert-May, 2013).

$$d = \frac{M_E - M_C}{Sample\ SD\ pooled} X \left(\frac{N-3}{N-2.25}\right) X \sqrt{\frac{N-2}{N}} \qquad (12)$$

And

$$\text{SD pooled } = \frac{\sqrt{(SD_E)^2 + (SD_C)^2}}{2} \qquad (13)$$

However, when the sample sizes significantly vary, Hedge proposed pooling the SDs from data that violate the homogeneity of variance assumption.

$$g = \frac{M_E - M_C}{SD\ pooled} \tag{14}$$

$$S_{pooled} = \sqrt{\frac{((n_E - 1)SD_E^2 + (n_C - 1)SD_C^2)}{(n_E + n_C) - 2}} \tag{15}$$

The third method assumes that the control group SD is closer to the population SD, so it uses the control group SD to standardize the mean difference ((Glass, McGraw, & Smith, 1981). Glass's delta is represented by

$$\Delta = \frac{\mu_1 - \mu_2}{SD_{control}} \tag{16}$$

There are various approaches to converting these metrics among each other e.g., Cohen's d to Hedges' g, point biserial to Cohen's d, etc. (For further readings, see Durlak, 2009; Ladesma, Macbeth, & Cortada de Kohan, 2009; Maher, Markey, & Ebert-May, 2013). Another method that is equally useful in educational research is the correlation coefficient. This measures the relationships between two groups of variables. The magnitude of the association can range from a negative one (-1 indicating perfect inverse proportion) to zero (0 indicating no linear relationship) and a positive one (+1 indicating perfect direct proportion). For this, the Pearson correlation coefficient (r) is used and the formula is $r = \frac{S_{xy}}{SD_x SD_y}$ where r is the Pearson r, S_{xy} is the covariance of the groups, and $SD_x SD_y$ is the product of the group's SDs.

Effect Size for More Than Two Group Means

Cohen’s *f*

When the number of groups has increased to more than two, Cohen (1988), suggested the use of the parameter he proposed to be Cohen’s *f* for estimating the effect size. The flaw in this method is that there is no distinguishing factor among the group means, it is just to reach a dichotomous decision of either the group means are equal or not. The method is to normalize the sum of the deviations of the sample means from the combined sample mean to the combined sample SD. The formula is:

$$f = \frac{\sigma_m}{\sigma} \qquad \sigma_m = \sqrt{\frac{\Sigma(m_i - \bar{m}^2}{k}} \tag{17}$$

k: *the number of sample groups,*

m_i : *mean of group i and*

$\bar{m}$: *mean of k sample means, and*

σ: *pooled Sd of k sample groups*

There are other measures of effect size that are appropriate for ANOVA, ANCOVA, and MANCOVA, which are the Eta-Squared and Partial Eta-Squared. Eta-Squared is the ratio of the between-group sum of squares to the total sum of squares while the partial Eta-squared is the ratio of the between-group sum of squares to the sum of the between-groups sum of squares and the error sum of squares. (Maher, Markey, & Ebert-May, 2013. For further details on these effect sizes and the new methods, see Bakeman, 2005; Cohen, 1973; Kerlinger, 1964).

Eta-Squared is calculated thus:

$$\eta^2 = \frac{SS_{between}}{SS_{total}}, \quad SS: sum\ of\ squares \tag{18}$$

While the partial Eta-Squared is calculated thus:

$$\eta_p^2 \ \frac{SS_{between}}{SS_{between} + SS_{error}} \ , \quad SS: sum\ of\ squares \tag{19}$$

CHAPTER THREE

METHODOLOGY

The goals of this study are: (1) to confirm and establish that the real-world data usually deviate from normality assumptions, no matter the field of study; (2) to compare the Type I error rates and the comparative power, of the statistical methods of comparing the differences in population means, when correlated groups or dependent samples are involved. The two hypothesis tests involved are (1.) The parametric one-way repeated measures, (2.) The nonparametric Friedman’s test.

Background of the Data

Early in 2020, the Centers for Diseases Control and Prevention (CDC) announced the breakout of a virus in the family of severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2). The world health organization (WHO) termed the novel virus the coronavirus disease 2019 or COVID-19 (CDC, 2020). According to CDC, the COVID-19 is highly contagious spreading from human to human. Although research is still ongoing to unravel all the details about the disease, significant information about its nature such as the mode of spread, maturation, and symptoms have been reported. The disease can be contracted through close contact with the droplets (sneezing, coughing) from the infected individuals. The first case of the virus in the United States was confirmed in Washington State in February 2020. Within a month, cases had been reported in about six states. By March 2020, the virus had spread exponentially across all the states in the U.S. Studies showed that the spread of the virus was more rapid in areas with large gatherings of people in close proximities. Examples of such gatherings include religious places, restaurants and cafes, schools, gyms, and other indoor recreation centers.

Consequently, governments imposed some control measures such as stay-at-home orders, lockdowns, and restricted movements to reduce the spread from person to person. Each state implemented the measures differently and at various times. The data about the COVID-19 including daily spread, daily death counts by age, daily death count by gender, and daily death counts by race, hospitalizations, and other underlying conditions relating to COVID-19, are available on the John Hopkins University website and the Centers for Disease and Control prevention website.

The weekly death count data was obtained from the CDC website and were grouped into April and May (9 weeks), June and July (8 weeks), and Aug and Sept (9 weeks). The first nine weeks represent the times of implementation of the lockdown measures by different states, during the eight weeks group, the measures, such as wearing of masks, and social distancing were still in place, but the movement of people had increased, during the last nine weeks group, the people moved freely, restaurants dine-ins were opened for few people to gather, also libraries allow readers to come in and study, people gathered at events, and finally, schools were opened for students, stores were operating normally, etc.

The Method of Sampling

Since the pandemic hit different states at varying degrees, the daily mortality was different among the states. Some states did not experience the breakout at the beginning of the outbreak, some had fluctuations in their case rates, while other states reported increasing cases daily, e.g., New York. Therefore, out of the 50 states, 37 states were first sampled based on the completeness of the weekly death counts data, with very few states having one or two missing counts. These figures were grouped in the following order: April and May total death count data, (9 weeks), June and July total death count data, (8 weeks), Aug and Sept total death counts data, (9 weeks). Afterward, four states out of the 37 previously sampled states, were dropped because of incomplete case records. The total number of samples was 33 states, having complete records of mortality count for 7 months, (April-Oct). A resampling was done after the first sampled data had been plotted in histograms. The shapes of the distributions generated with these grouped data correspond with previous studies on the issue of normality. This confirmed the assumption of normality as a rare concept in real-life data. From these chosen states' data, random sampling of different sizes was performed with replacement (n=5,8,12,18,25,33) using Excel 2016.

Methods of Analysis

The analysis was conducted on differing sizes of samples, randomly selected from the 33 states dataset. The samples are in the following orders, (N=5,8,12,18,25,33). The nominal alpha was set at 0.05, the test statistic used for the repeated measures ANOVA was the F, and the test statistic for Friedman was F_r. There was no treatment administered to any of the groups, however, the data represents a longitudinal observation of the weekly mortality counts, that occurred in each month for different states. The null hypothesis of interest is that of no differences among the group means. The null hypothesis assumes that the distributions of the observations within each block come from the same parent distribution. The alternative hypothesis is that at least one of the group means is different. This design is a randomized complete block with one observation per treatment-block combination.

For the first part of the objective of this study, the selected datasets were grouped in varying orders. For example, six months data out of the seven months were grouped by two months, (April to May total death counts, June to July total death counts, Aug to Sept total death counts). Descriptive analyses were performed on the 37 total samples, to observe the basis of normality assumption and the extent of their violations. Therefore, the distribution results were compared against the normal curve.

Three Major Distributions and their Characteristics.

The normal distribution is represented by a bell shape or curve, with a line dividing the curve into equal halves known as *symmetry*. The first half of the curve mirrors the other half. Gaussian distribution was named after the author, Carl Friedrich Gauss in 1809, who discovered the normal distribution to rationalize the method of least squares (wikipedia.org). This distribution has a mean (μ) of 0 and standard deviation (σ) of 1. This implies that the mean and the median are equal. Skewness and kurtosis are the third and fourth moments of the normal distribution. The skewness for the normal distribution is set at zero (0), and the kurtosis is set at three (3).

The probability density function is:

$$\mathrm{Pu}(\mu) \ = \ (\sqrt{2}\pi)^{-1} \exp(-1/2\mathrm{u}^2) \qquad (20)$$

Skewness and kurtosis are common descriptive properties that quantify violations from normality (Glass et al, 1978).

Skewness signifies how long the tail of the distribution is. It measures how symmetry or not symmetry the shape of the distribution looks. In normal distribution, skewness = 0. Symmetry can be either tilted to the left of the distribution, with a long tail to the right, this is often termed, *the positive skewness*. This happens when the mean of the distribution is greater than the median and the mode, and the bulk of the scores are close to zero. Whereas, when the bulk of the scores are tilting towards the right of the distribution and long tail towards the left, this is called *negative skewness.* The median is greater than the mean in this distribution.

For univariate data, Y_1 , Y_2 , ... Y_N the formula for skewness is:

$$g1 = \frac{\sum_{i=1}^{N}(Y_i - \bar{Y})^3/N}{S^3} \qquad (21)$$

Where $\bar{Y}$ is the mean, *S* is the standard deviation, and *N* is the number of sample size. Note that in computing the skewness, the S is computed with N in the denominator rather than $\mathrm{N} - 1$. “This formula for skewness is referred to as the Fisher- Pearson coefficient of skewness.”

Kurtosis represents the shape of the peak of the distribution i.e. how tall or short the peak looks like. For a normal distribution, kurtosis = 3.00. Any distribution that displays a kurtosis value larger than 3, signifies a very long peak. This is called leptokurtic, and as the kurtosis value falls below 3, the peak becomes smaller/flatter, this is playkurtic.

$$\text{Kurtosis} = \frac{\sum_{i=1}^{N}(Y_i - \overline{aY})^4/N}{S^4} - 3 \qquad (22)$$

Three is subtracted from the value generated by the formula so that the standard normal distribution has a kurtosis of zero.” (Methods, 2020, p. 1.3.5.11).

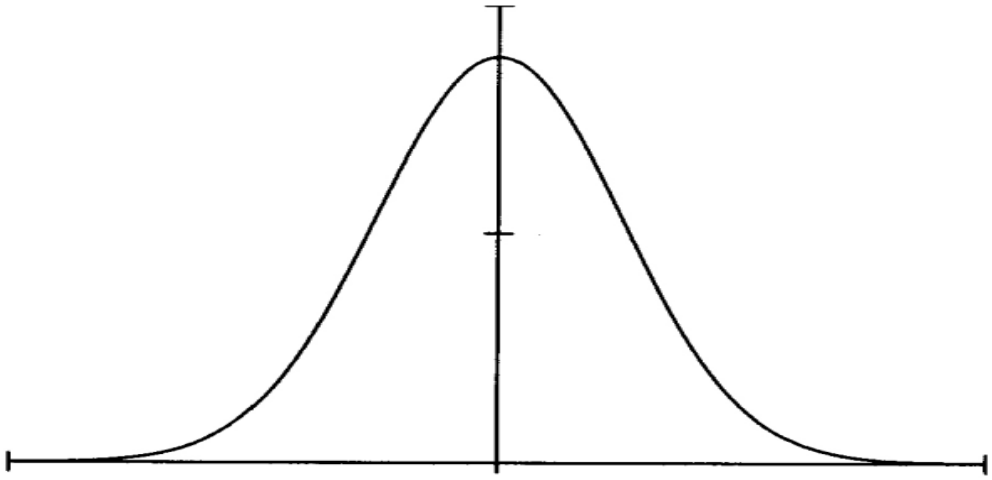


Figure 2.Gaussian (Normal) Distribution.

Chi-Square Distribution: when chi-square carries only two degrees of freedom, it is usually referred to as *exponential.* The chi-square is calculated by subtracting the expected outcome from the observed outcomes. The probability function is:

$$px(X) = \sigma^{-1} \exp\left[-\frac{(x-\theta)}{\sigma}\right] \quad (x > \theta;\ \sigma > 0 \qquad (23)$$

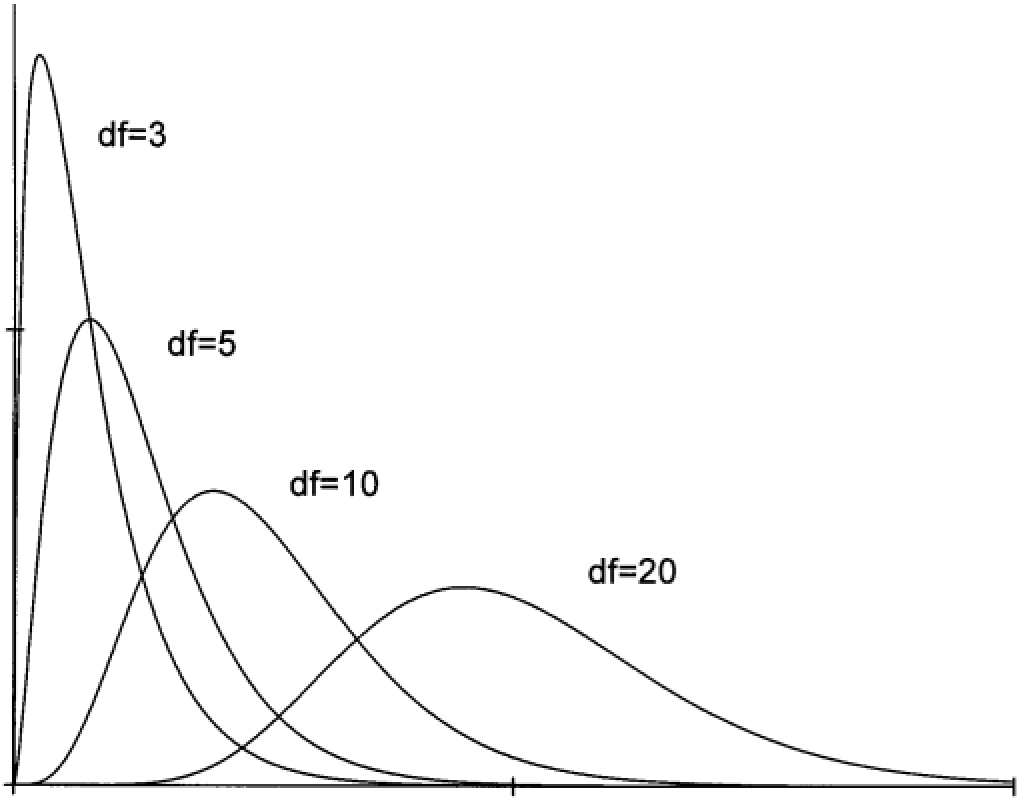


Figure 3.Chi-Square Distribution.

The uniform distribution: The peak of this type of distribution of data, is usually flat at the top. The histogram is rectangular in shape, and all the outputs are equally likely. The probability function of a uniform distribution is as follows:

$$PY(y) = (\beta - \alpha)^{-1} \quad (\alpha \leq y \leq \beta \tag{24}$$

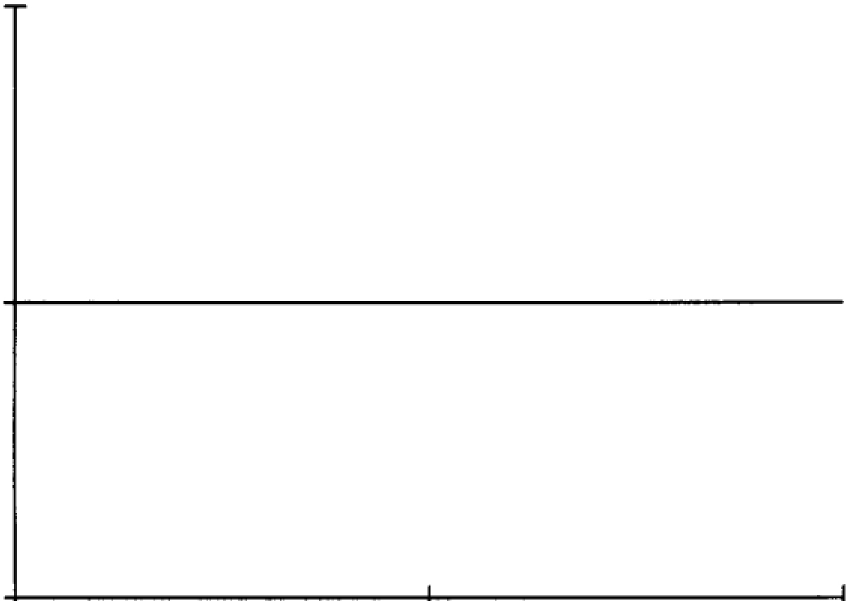


Figure 4.Uniform Distribution.

All three distributions were adapted from Sawilowsky & Fahoome (2003).

The Descriptive Analysis

The April & May dataset has a mean of 2676.19, median = 969.00 standard deviation = 3934.957, IQR = 2210, skewness = 2.654, and kurtosis = 7.799. This dataset has an 11.53% variance (leptokurtic) from normal kurtosis. June & July dataset displays a mean of 1041.03, median of 682.00, standard deviation of 1438.659, IQR = 755, skewness of 2.993, (positively skewed), and kurtosis = 8.841 demonstrating a 194.7%above the normal kurtosis. The August & Sept dataset showed a mean of 1341.38, median of 748, standard deviation of 1966.657, IQR of 1050, positive skewness of 2.834 and kurtosis of 7.445. This is 148.2% more kurtotic than the normal distribution kurtosis. The results of the histograms are displayed in Appendix A.

The outputs of the histograms in figures 2-4, were consistent with the previous findings of Micceri (1989), and Blanca, Arnau, López-Montiel, Bono, & Bendayan (2013) on how real-life datasets have violated the assumption of normality.

Also, the samples were randomly re-selected with replacements, analyzed with the number of repeated measures equals 7 in each number of samples, the results of the analysis showed a little variation from the ones displayed earlier. However, it was also not inconsistent with prior findings. The distributions of output display mostly the Chi-Square distribution. The histograms are displayed in Appendix B.

To check for the multivariate normality assumption, some random numbers were computed with the standardized residuals of the variables, in SPSS 26.01. These values were used to plot the histograms with the normal curves. The results are as follows: Uniform distributions and multimodal distributions seemed to be common in the results. There was a trace of normality as the sample sizes increased from 12 through 33, confirming the central limit theorem. In conclusion, the assumption of normality is hardly met in the real-world distributions.

Charts

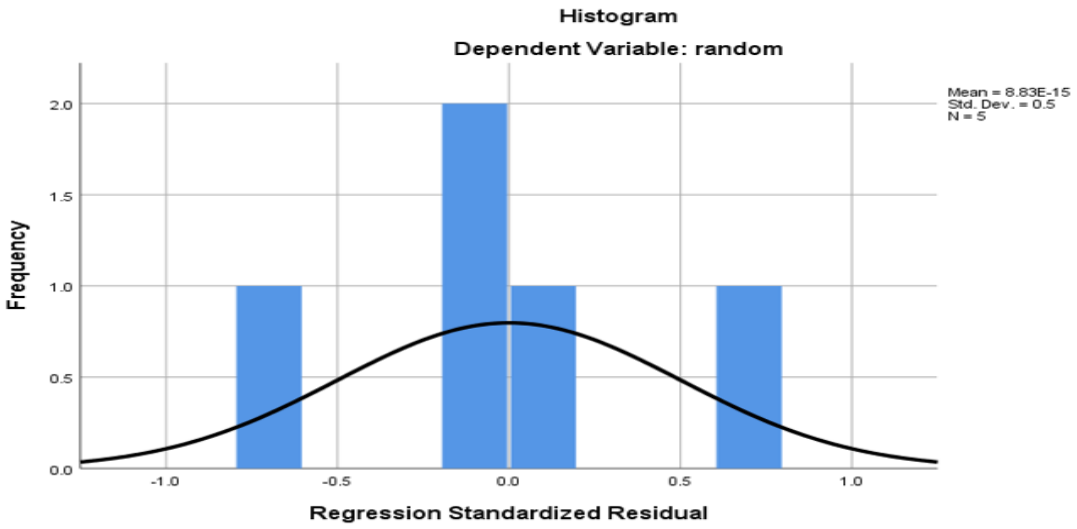


Figure 5. Multivariate Normal Distribution for Sample Size of 5, k=7.

Charts

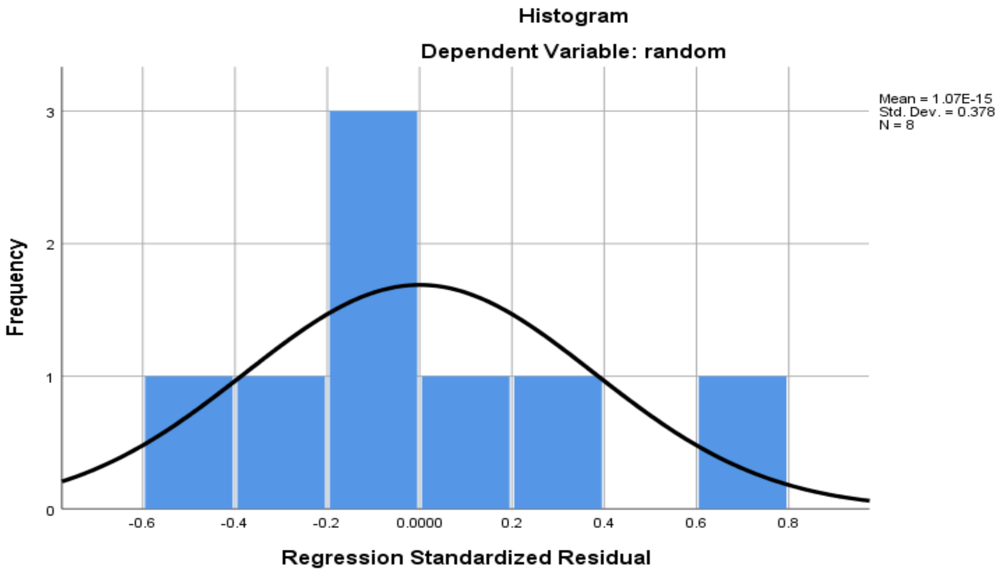


Figure 6. Multivariate Normal Distribution for Sample Size of 8, k=7.

Charts

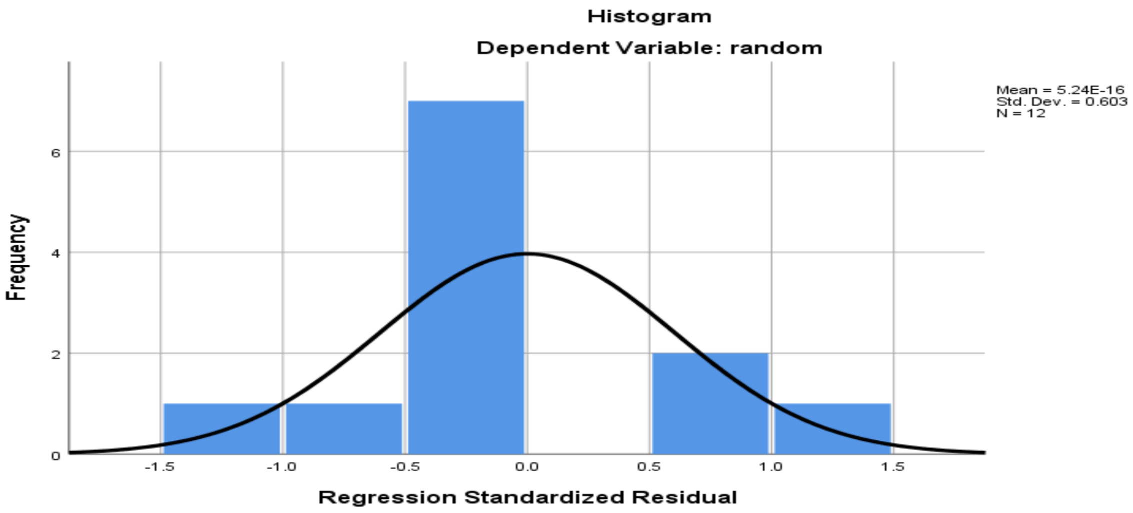


Figure 7. Multivariate Normal Distribution for Sample Size of 12, k=7.

Charts

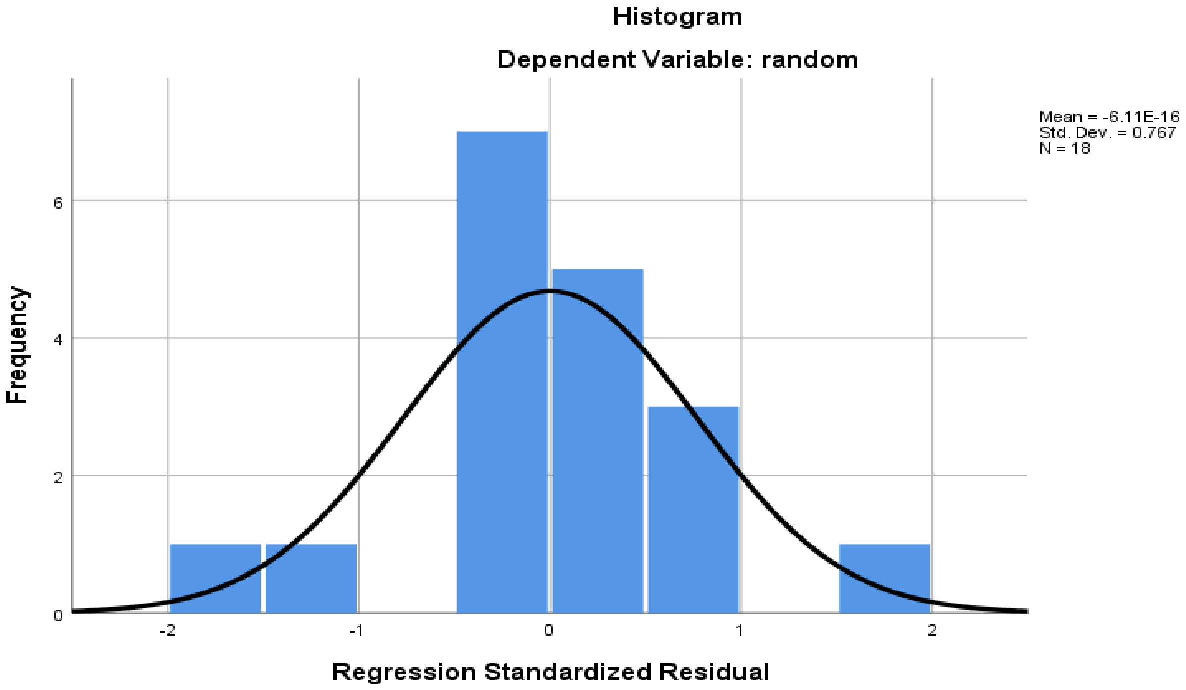


Figure 8. Multivariate Normal Distribution for Sample Size of 18, k=7.

Charts

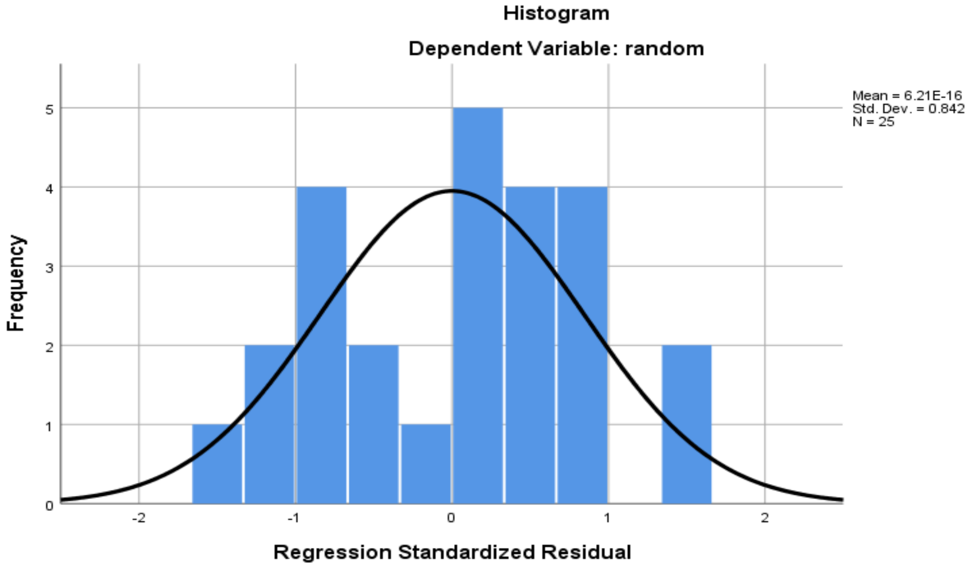


Figure 9. Multivariate Normal Distribution for Sample Size of 25, k=7.

Charts

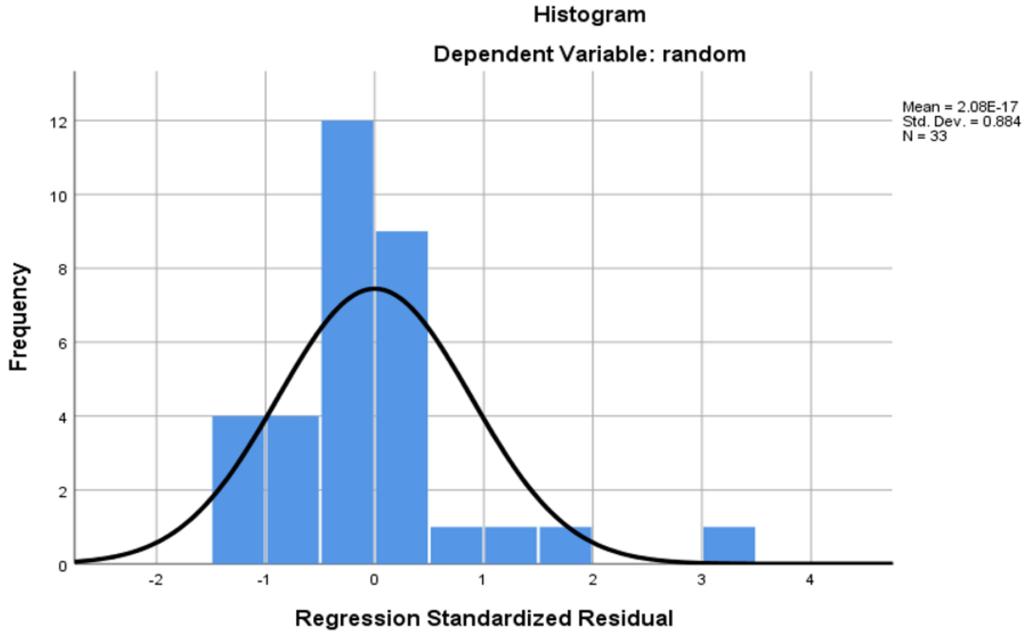


Figure 10. Multivariate Normal Distribution for Sample Size of 33, k=7.

Steps Followed in the Methods of Analysis

Differing combinations of n and k were considered (n=5,8,12,18,25,33, and k= 3,5, &7). To compute F_r statistic, data were ordered from least to greatest within each block independently, r_{ik} is the rank of X_{ik} in the ith block and average rank were assigned via the within-blocks ranking method. The test is significant, if the calculated result is greater than the tabulated values, the decision is to reject the null hypothesis. The current mortality data were analyzed using the Repeated measures ANOVA test according to equation 17:

$$F = \frac{Ms_B}{Ms_E} \tag{25}$$

MS$_B$ is the Mean Square Between or Mean Square for the Treatments or Conditions, and the Mean Square Error is the within mean square that will remain after the MS$_S$, Mean Square Subjects or Participants has been removed. Thereby enhances higher power.

Friedman's test will be analyzed according to either of these two formulae. The two formulae both arrive at the same answer.

$$\mathrm{X}_{\mathrm{r}}^{2} = \frac{12}{\mathrm{bk(k+1)}} \sum \mathbb{R}\mathrm{J} - \frac{\mathrm{b(k+1)}^{2}}{2} \tag{26a}$$

$$X_r^2 = \frac{12}{bk(k+1)} \Sigma R_J^2 - 3b(k+1) \tag{26b}$$

The effect size was calculated using the G*Power 3.1.9.4. "In G*Power, there is an effect size calculator or effect size drawer. "Effect size drawer has the possibility of computing the effect size, *f,* from the variance explained by the tested effect, and error variance" (Faul et al., 2007). Eta squared (η^2) or effect size *f* as defined by Cohen (1988) is used in almost all ANOVA procedures. Cohen stated the values for the effect size *f* as 0.1 for the small effect, 0.25 for the medium effect, and 0.4 for the large effect size. Since this is raw data, tabular visualization of the observation (histogram, normal curve, box plot, etc.) was generated, and the q-q plot was used to determine the normality of the observations. Mauchly's W test was calculated to determine, to what extent the sphericity assumption was met, and if there are cases of sphericity assumption violation, necessary corrections of the degrees of freedom were performed. Shapiro-Wilks test of normality was reported since the Kolmogorov-Smirnov test, is always not strong enough to correctly reject the false null. The non-centrality parameter is the expected joined effect of all the factors on power in ANOVA design. Non-centrality parameter is required to calculate the power for the Friedman’s test in G*Power 3.1.9.4 software.

All computations were carried out using SPSS 26 and G*Power 3.1.9.4. The results of all the analyses are reported in the next chapter.

CHAPTER FOUR RESULTS

RESULTS AND DISCUSSION

The 37 states mortality counts were represented on a histogram to compare the shapes of the outcome variables with the prevailing three distribution shapes: normal bell curve, Uniform, and Chi-Square distributions. Statistical analyses were performed to investigate the Type I error rates, and the comparative power properties of the two tests: the repeated measures and Friedman's test, for differing sample sizes and different condition group combinations. Six sample sizes, $(\mathrm{n} = 5,8,12,18,25,33)$ and number of conditions, $(\mathrm{k} = 3,5,7)$ were used to perform the analysis. SPSS 26.01 was used to compute the effect sizes for the repeated measures in the form of partial eta squared, and the effect size for the Friedman's test, in the form of Kendall's W. The results of all the Type I error rates for the two tests under differing sample sizes and group combinations are presented in the tables below.

Type I Error Rates

Table 2. Type I Error Rates when α=0.05 with G-G correction.

Sample size &number of k	$\mathrm{Rep.M}eas$	$\mathrm{G-G}$ correction	Friedman's Test
$n_k = 5; k = 3$	0.245	Nil	0.091
$n_k = 5; k = 5$	0.547	0.740	0.308
$n_k = 5; k = 7$	0.450	0.557	0.515
$n_k = 8; k = 3$	0.015	Nil	0.008
$n_k = 8; k = 5$	0.236	0.184	0.004
$n_k = 8; k = 7$	0.155	0.044	0.001

$n_k = 12; k = 3$	0.007	Nil	0.028
$n_k = 12; k = 5$	0.183	0.111	0.015
$n_k = 12; k = 7$	0.176	0.072	0.010
$n_k = 18; k = 3$	0.080	0.046	0.000
$n_k = 18; k = 5$	0.061	0.007	0.000
$n_k = 18; k = 7$	0.053	0.001	0.001
$n_k = 25; k = 3$	0.080	0.047	0.000
$n_k = 25; k = 5$	0.126	0.055	0.000
$n_k = 25; k = 7$	0.082	0.008	0.000
$n_k = 33; k = 3$	0.021	0.006	0.000
$n_k = 33; k = 5$	0.065	0.013	0.000
$n_k = 33; k = 7$	0.026	0.000	0.000

The following tables 3-5 below showed the original robustness of the repeated measures with the follow up Greenhouse-Geisser corrections for the significant Mauchley’s W test.

Rates of Errors with the Greenhouse- Geisser Corrections

Table 3. Type I Error Rates for k=3,and G-G corrections.

$Various$ Samples when $\mathrm{k} = 3$	$\mathrm{Mauchly's\ W}$	Error Rates (RM)	Type I Error$(\mathrm{G} - \mathrm{G})$	Error Rates (Fr)
$n_1 = n_2 = n_3 = 5$	$Significant$	0.245	Nill	0.091
$n_1 = n_2 = n_3 = 8$	$Significant$	0.015	0.021	0.008
$n_1 = n_2 = n_3 = 12$	$Significant$	0.007	Nill	0.028
$n_1 = n_2 = n_3 = 18$	$Significant$	0.046	0.080	0.000
$n_1 = n_2 = n_3 = 25$	$Significant$	0.049	0.080	0.000
$n_1 = n_2 = n_3 = 33$	$Significant$	0.006	0.021	0.000

Table 4. Type I Error Rates for k=5,and G-G corrections

$Various$ Samples when $\mathrm{k} = 5$	Mauchley's W significance	Type I Error Rates (RM)	Type I Error Rates (G-G)	Type I Error Rates (*F_r*)
$n_1 = n_2 = n_3 = n_4 = n_5 = 5$	Not Significant	0.740	0.547	0.308
$n_1 = n_2 = n_3 = n_4 = n_5 = 8$	Not Significant	0.184	0.236	0.004
$n_1 = n_2 = n_3 = n_4 = n_5 = 12$	Not Significant	0.111	0.183	0.015
$n_1 = n_2 = n_3 = n_4 = n_5 = 18$	Significant	0.007	0.061	0.000
$n_1 = n_2 = n_3 = n_4 = n_5 = 25$	Significant	0.055	0.126	0.000
$n_1 = n_2 = n_3 = n_4 = n_5 = 33$	Significant	0.013	0.065	0.000

Table 5. Type I Error Rates for k=7,and G-G corrections

$Various$ Samples when $\mathrm{k} = 7$	Mauchley's W significance	Type I Error Rates (RM)	Type I Error Rates (G-G)	Type I Error Rates (F_r)
$n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7 = 5$	Significant	0.557	0.450	0.515
$n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7 = 8$	Significant	0.044	0.155	0.001
$n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7 = 12$	Significant	0.072	0.176	0.010
$n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7 = 18$	Significant	0.001	0.053	0.001
$n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7 = 25$	Significant	0.008	0.082	0.000
$n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7 = 33$	Significant	0.000	0.026	0.000

Effect Sizes: Partial Eta Squared and the Kendall's W

Table 6. Partial Eta squared for RM & Kendall's W for F_r test

Sample size &number of repeated measures (rm)	Partial Eta Squared (η^2)	Kendall's W
$n_1 = n_2 = n_3 = 5, k = 3$	0.296	0.480
$n_k = 5, k = 5$	0.110	0.240
$n_k = 5, k = 7$	0.172	0.174
$n_k = 8, k = 3$	0.451	0.609

$n_k = 8, k = 5$	0.193	0.484
$n_k = 8, k = 7$	0.255	0.478
$n_k = 12, \mathrm{k} = 3$	0.360	0.299
$n_k = 12, \mathrm{k} = 5$	0.154	0.257
$n_k = 12, \mathrm{k} = 7$	0.157	0.233
$n_k = 18, \mathrm{k} = 3$	0.166	0.531
$n_k = 18, \mathrm{k} = 5$	0.185	0.280
$n_k = 18, \mathrm{k} = 7$	0.195	0.207
$n_k = 25, \mathrm{k} = 3$	0.119	0.386
$n_k = 25, \mathrm{k} = 5$	0.091	0.225
$n_k = 25, \mathrm{k} = 7$	0.112	0.188
$n_k = 12, \mathrm{k} = 3$	0.147	0.387
$n_k = 33, k = 5$	0.094	0.190
$n_k = 33, k = 7$	0.124	0.169

The effect sizes generated from the SPSS 26.01, showed that the Kendall's W, which is the effect size for the Friedman's test, displayed higher values than the effect size values of the repeated measures ANOVA. It was only in one situation, ($n1 = n2 = n3 = 5, k = 7$), that the effect size for the two tests nearly overlapped, (η^2 =0.172; Kendall's W = 0.174), but Kendall's W still supersedes. When $n1 = n2 = n3 = 12, k = 3$, RM has slightly higher effect than the Friedman, (η^2 = 0.360, Kendall's W = 0.299). It showed the extent to which Friedman's test has greater power to detect differences among a group of means, even when the parametric assumptions have been violated. the rate at which

Friedman’s test detects differences was as high as 609, whereas, the RM did not pass the level of 451. Table 6 above displays the results.

Table 7. The Sphericity Assumption Results.

Sample size &number of repeated measures (rm)	Mauchly’s W
$n_1 = n_2 = n_3 = 5, k = 3$	0.138
$n_k = 5, k = 5$	0.007
$n_k = 5, k = 7$	0.000
$n_k = 8, k = 3$	0.592
$n_k = 8, k = 5$	0.000
$n_k = 8, k = 7$	0.000
$n_k = 12, k = 3$	0.753
$n_k = 12, k = 5$	0.000
$n_k = 12, k = 7$	0.000
$n_k = 18, k = 3$	0.000
$n_k = 18, k = 5$	0.000
$n_k = 18, k = 7$	0.000
$n_k = 25, k = 3$	0.000
$n_k = 25, k = 5$	0.000
$n_k = 25, k = 7$	0.000
$n_k = 33, k = 3$	0.000
$n_k = 33, k = 5$	0.000
$n_k = 33, k = 7$	0.000

Table 7 displayed the results of the assumption of sphericity, (equal variances and equal standard deviations across groups). It was obvious that this assumption was met in only three sample groups out of the total of eighteen sampled groups. The groups were N= 5, 8, and 12, with the combinations of three groups of repeated measures. This assumption was violated in the subsequent sample groups. This is an evidence that it is only in smaller samples that the assumption of either homogeneity or sphericity is established.

Comparative Power Analysis

The summaries of the power analysis as a function of the three Effect sizes, as stated by Cohen (1988), *f* for small effect is 0.1, medium effect is 0.25, and large effect is 0.4 were given in details below. As previously stated, there were six samples of equal sizes of n= 5,8,12,18,25,33, each of which was combined with different numbers of repeated measures (k= 3,5,7). With each combination, power of the repeated measures ANOVA and the nonparametric alternative, Friedman's tests were computed. The y-axis represents the power (1-Beta) label, and it ranges from 0 to 1.00. When a test displays a power of zero, (0), it signifies that such a test has no power to detect differences among means. Whereas, a power level equivalent to one, (1), means that the test has maximum power to find even the slightest significance among group means. The x-axis displayed the effect sizes labels 0.10sd, 0.25sd, & 0.40sd, the "sd" is the standard deviation of each sample group. The actual effect size is the standard deviation of the groups multiplied by the constants, (effect size benchmarks), before them. The power curve was obtained through the G*Power 3.1.9.4. To compute the power curve for Friedman's test, a non-centrality parameter corresponding to each sample size from the repeated measures ANOVA was used. The values from the two power curves were obtained and plotted on both the line graph and bar graphs for different alpha levels (0.01, 0.05, 0.1). Although the results for the three significant levels were displayed in a table in this study, the power curves for only α= 0.05 were presented in the result, since 0.05, alpha level is prevalent in research studies. The bar graphs will be referenced in Appendices A-B

Differing Sample Sizes and Differing Condition Groups

Sample $n_1 = n_2 = n_3 = 5$

The first group of number sampled was n_1=n_2=n_3=5, with the number of treatments to be three, (n=5, k=3). The alpha level is set at 0.05. This sample group yielded powers of 0.1 and .099 with shift of 0.1σ for the Repeated Measures ANOVA (RMA) and the Friedman’s test. At shift 0.25σ, the powers were .211 for RMA, and .219 for Friedman’s test, and at the 0.4σ, there was .384 power for RMA, and .396 for Friedman’s test. Except for the .1σ, that gave about the same power, the Friedman’s test gave more power than the RMA.

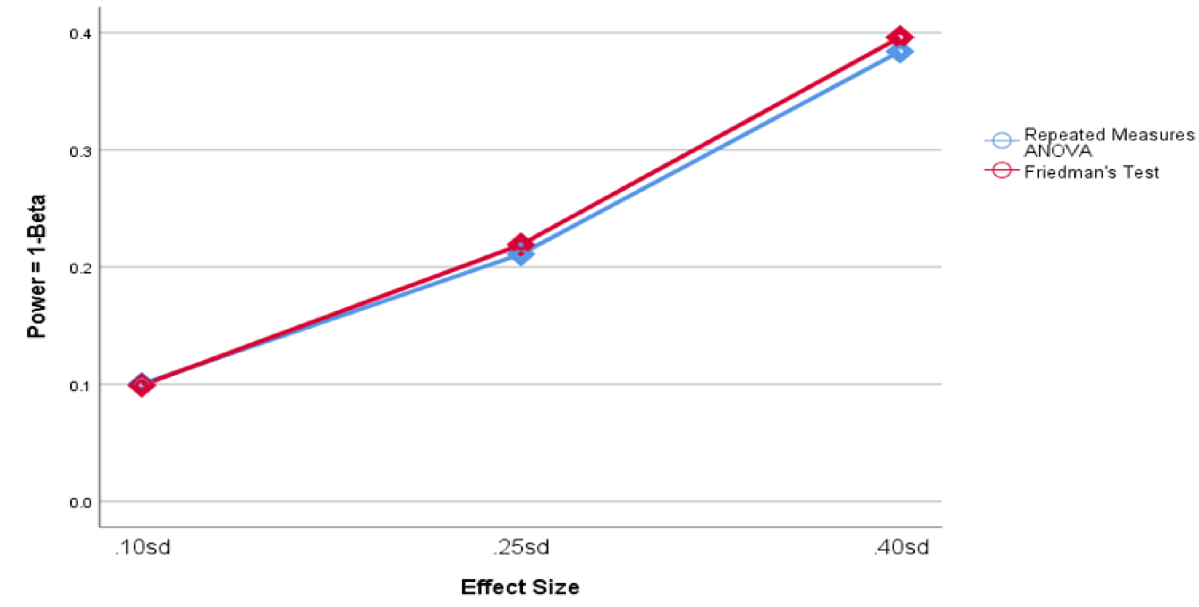


Figure 11. Comparative Power rate for the RM & F_r for n=5,k=3.

Sample $n_1 = n_2 = n_3 = n_4 = n_5 = 5$

The next sample group was kept unchanged, and the treatment groups increased to 5. At .1σ, the power for RMA was .128, whereas, it was .156 for the Freidman's test. For effect size of .25σ, the powers were .322 for RMA and .421 for Friedman's test, and for 0.4σ, the power was .605 for RMA, and .747 for Friedman's test. The Friedman's test demonstrated power advantage all through over the RMA.

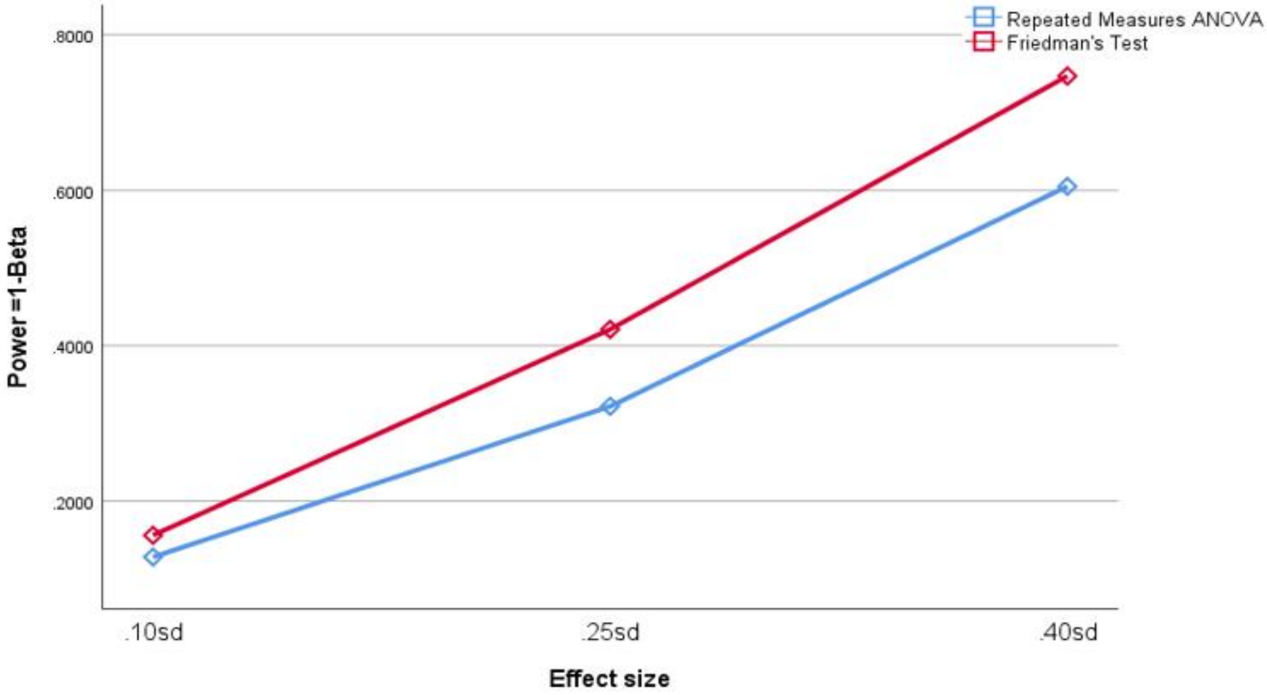


Figure 12. Comparative Power rate for the RM & F_r for n=5,k=5.

Sample n_1=n_2=n_3=n4=n_5=n_6=n_7=5

The next group that was explored was the same number of groups, but the repeated measures were increased again to 7. At .1σ, the power for RMA was .152, whereas, it was .219 for the Freidman's test. For effect size of .25σ, the powers were .420 for RMA and .605 for Friedman's test, and for 0.4σ, the power was .756 for RMA, and

.912 for Friedman’s test. Again, the Friedman’s test demonstrated higher power rates over the RMA for all the effect sizes.

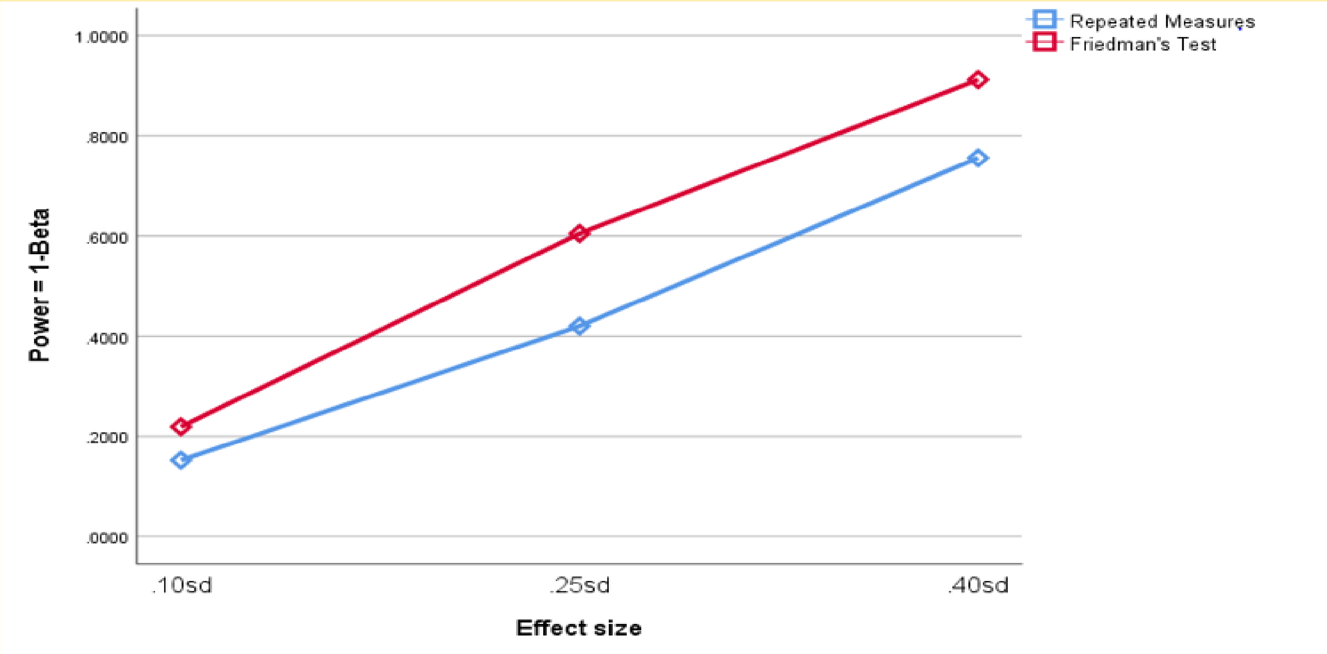


Figure 13. Comparative Power rate for the RM & F_r for n=5,k=7.

Sample $n1 = n2 = n3 = 8$

Another group of random samples that was explored was sample of eight, and treatments of three (Sample n_1=n_2=n_3=8). In this group, the effect size of .1σ gave the power of .156 for RMA, and .115 for the Freidman’s test. For effect size of .25σ, the powers were .394 for RMA and .294 for Friedman’s test, and for 0.4σ, the power was .687 for RMA, and .578 for Friedman’s test. Conversely, the RMA demonstrated higher power rates over the Friedman’s test across all the effect sizes.

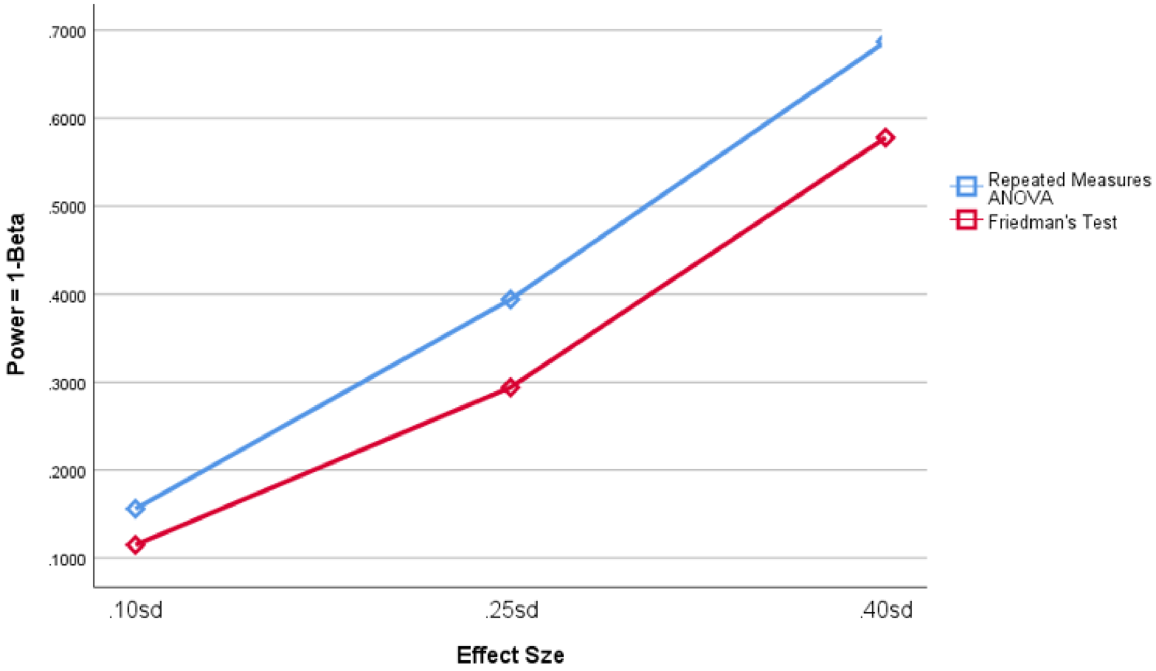


Figure 14. Comparative Power rate for the RM & F_r for n=8,k=3.

Sample $n_1 = n_2 = n_3 = n_4 = n_5 = 8$

The next shift was calculated for sample groups of eight and treatment groups of five. For .1σ, the power for RMA yielded .218, whereas, it was .199 for the Freidman's test. And for effect size of .25σ, the powers were .599 for RMA and .578 for Friedman's test, then, for 0.4σ, the power was .907 for both RMA and Friedman's test. Except for the 0.4σ, where both tests tallied, for the remaining two shifts, the RMA was a little bit higher.

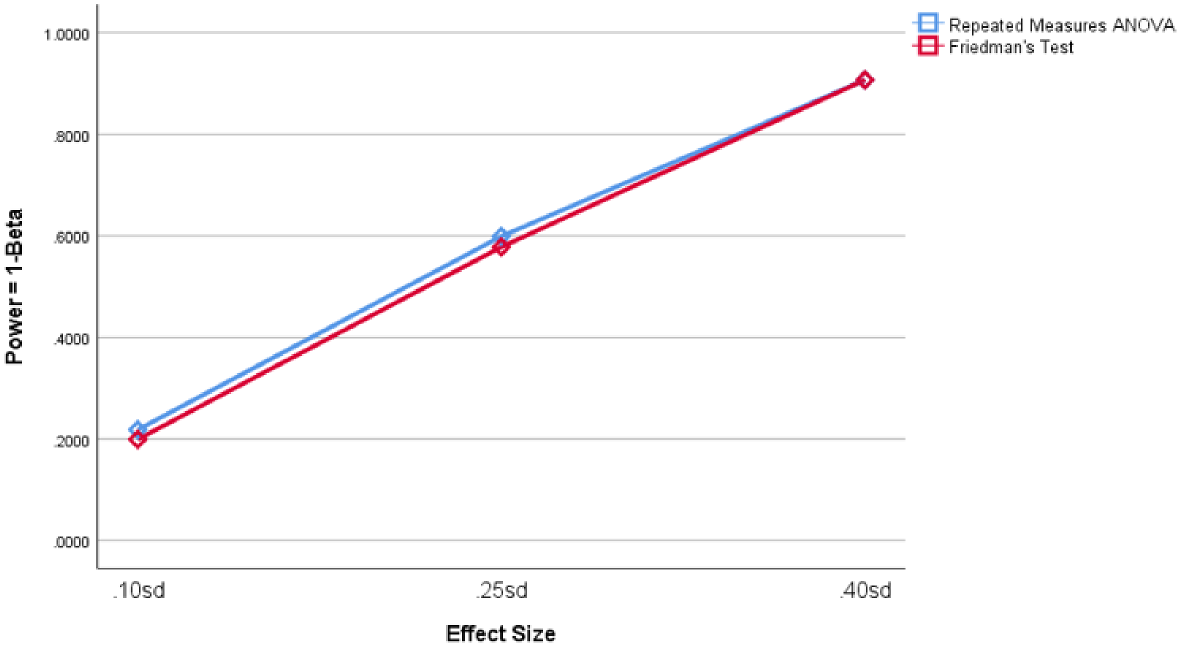


Figure 15. Comparative Power rate for the RM & F_r for n=8,k=5.

Sample $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7 = 8$

The next group was the same number of groups, but the repeated measures were increased to 7. At .1σ, the power for RMA yielded .273, whereas, it was .294 for the Freidman's test. For effect size of .25σ, the powers were .742 for RMA and .788 for Friedman's test, and for 0.4σ, the power displayed .975 for RMA, and .987 for Friedman's test. Again, the Friedman's test gave higher power rates over the RMA for all the effect sizes.

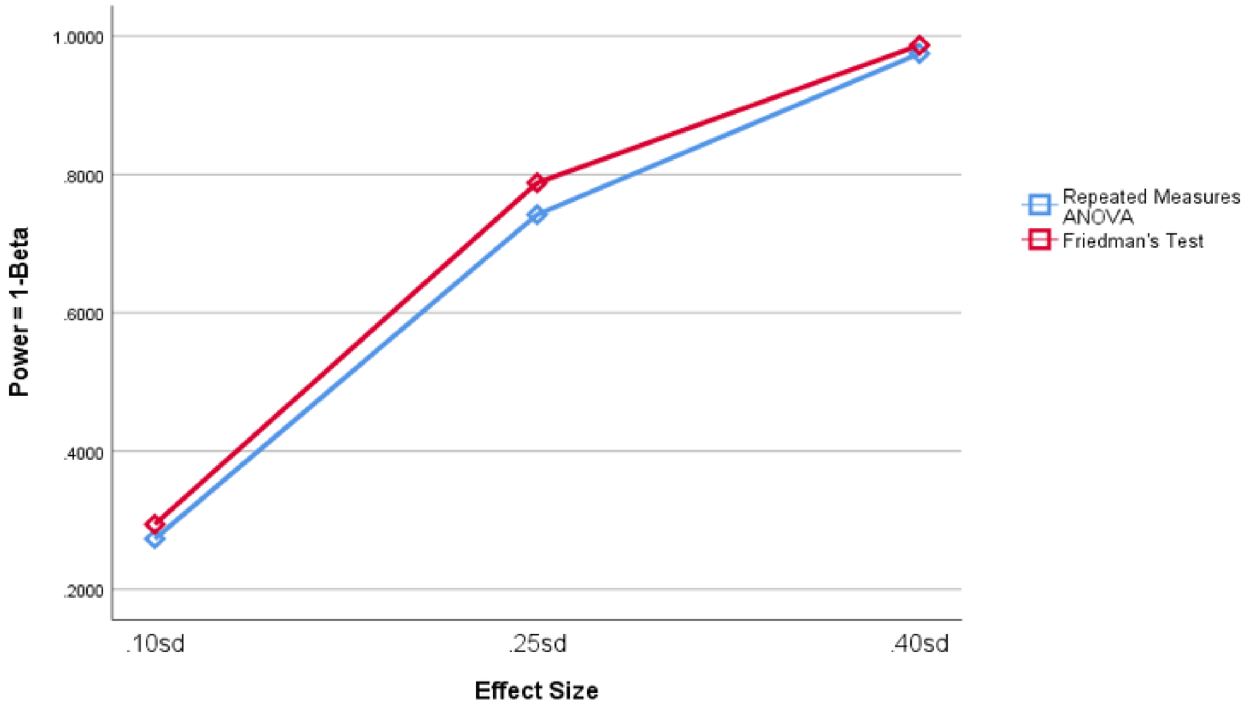


Figure 16. Comparative Power rate for the RM & F_r for n=8,k=7.

Sample $n_1 = n_2 = n_3 = 12$

For this group of samples, at the .1σ shift, the power for RMA was .238, whereas, it was .135 for the Freidman's test. For effect size of .25σ, the powers were .613 for RMA and .381 for Friedman's test, and for 0.4σ, the power was .902 for RMA, and .730 for Friedman's test. Again, the Friedman's test performed lower in terms of power than the RMA for all the effect sizes, with the differences ranging from 0.103 to 0.232.

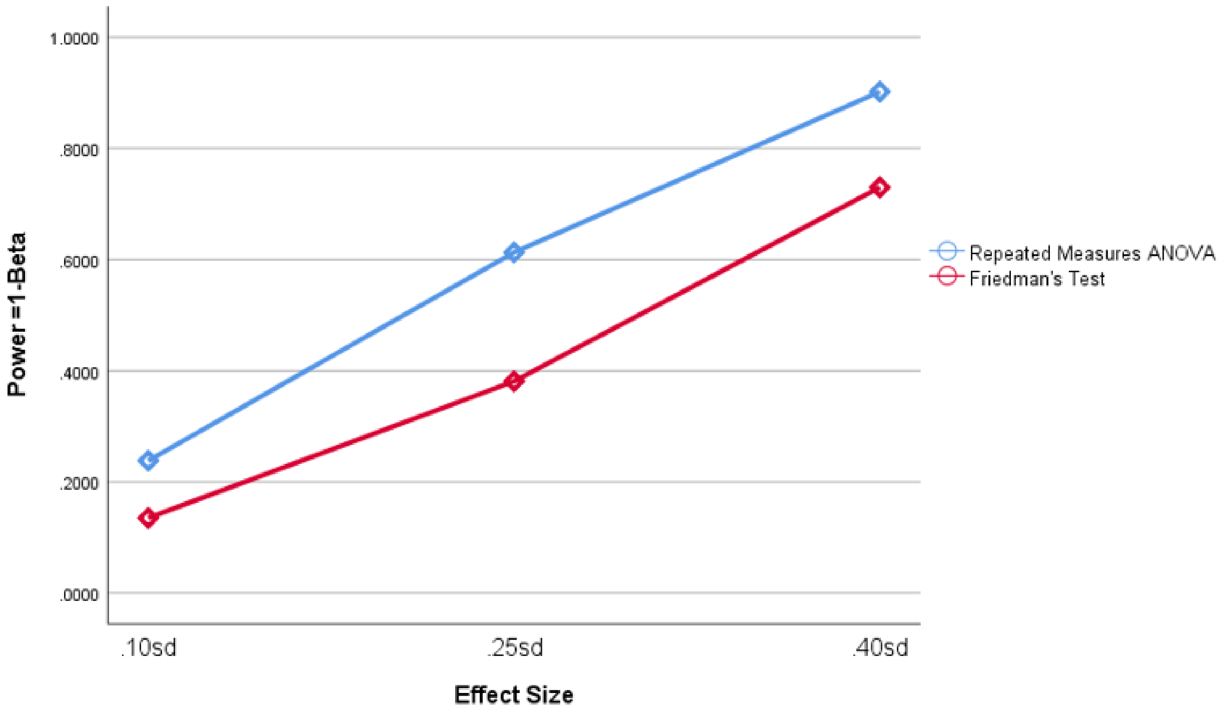


Figure 17. Comparative Power rate for the RM & F_r for n=12,k=3.

Sample $n_1 = n_2 = n_3 = n_4 = n_5 = 12$

For this group of samples, at the .1σ shift, the power for RMA was .351, whereas, it was .250 for the Freidman's test. For effect size of .25σ, the powers were .842 for RMA and .730 for Friedman's test, and for 0.4σ, the power was .992 for RMA, and .978 for Friedman's test. Again, the Friedman's test performed lower in terms of power than the RMA for all the effect sizes.

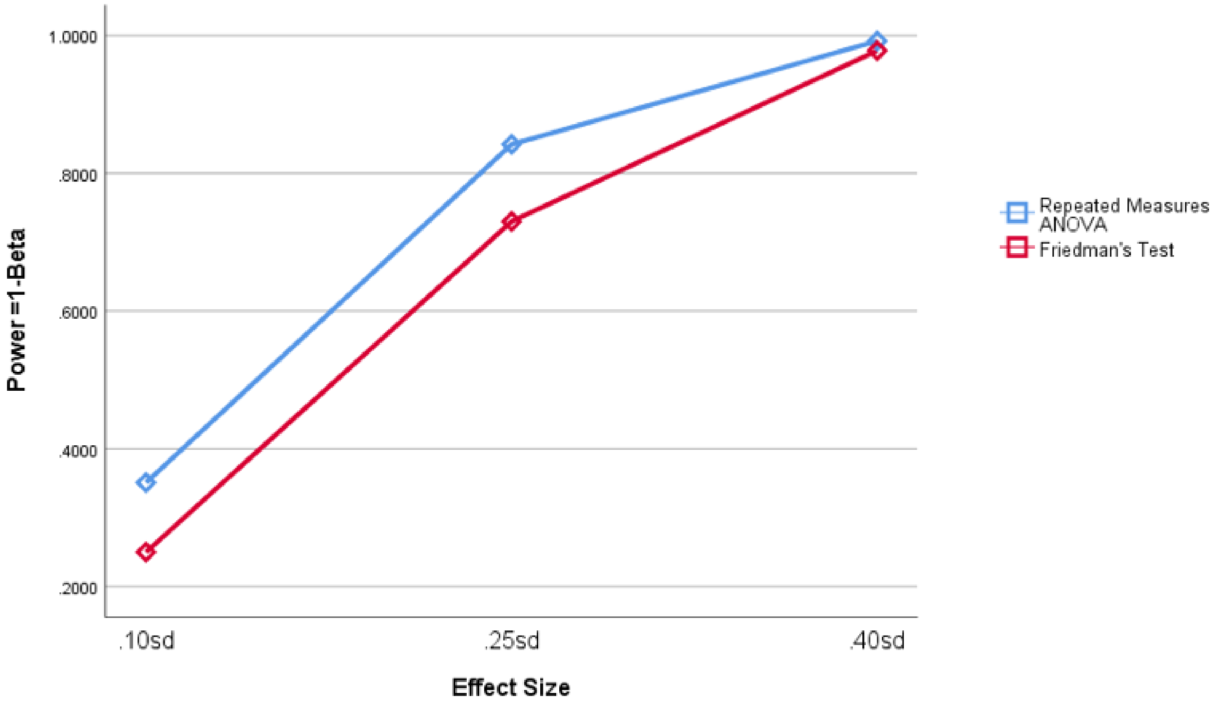


Figure 18. Comparative Power rate for the RM & F_r for n=12,k=5.

Sample $\boldsymbol{n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7 = 12}$

This sample group has the same number as the last two groups, but the number of repeated measures was increased to 7. At .1σ, the power for RMA yielded .448, and, .381 for the Freidman’s test. For effect size of .25σ, the powers were .996 for RMA and .979 for Friedman’s test, and for 0.4σ, the power displayed 1.000 for RMA, and .999 for Friedman’s test. Here, the RMA gave higher power rates over the Friedman’s test for all the effect sizes.

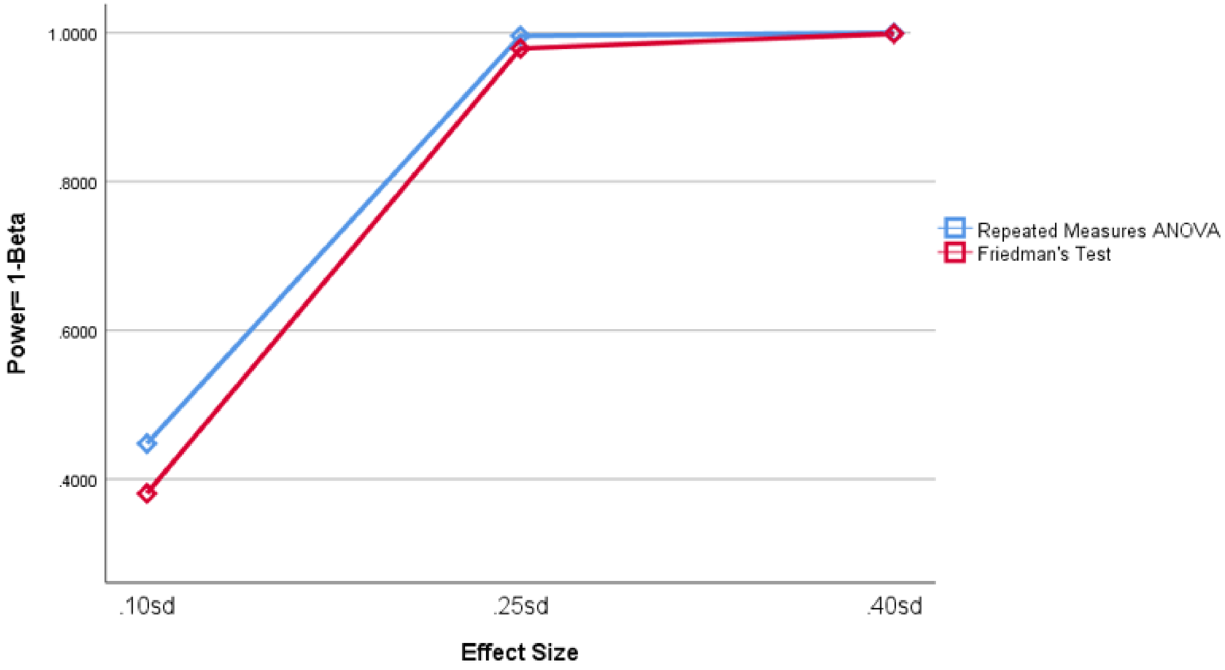


Figure 19. Comparative Power rate for the RM & F_r for n=12,k=7.

Sample $\boldsymbol{n_1 = n_2 = n_3 = 18}$

This group comprises of eighteen equal samples in three different groups, (**n_1=n_2=n_3=18).** This sample group yielded powers of .365, .161 with shift of 0.1σ for the Repeated Measures ANOVA (RMA) and the Friedman's test. At shift 0.25σ, the powers were .830 for RMA, and .495 for Friedman's test, and at the 0.4σ, there was .988 power for RMA, and .866 for Friedman's test. At 0.4σ level of shift, the power discrepancy between the RMA and Friedman's test was 0.122. but over the RMA gave higher power across all shifts.

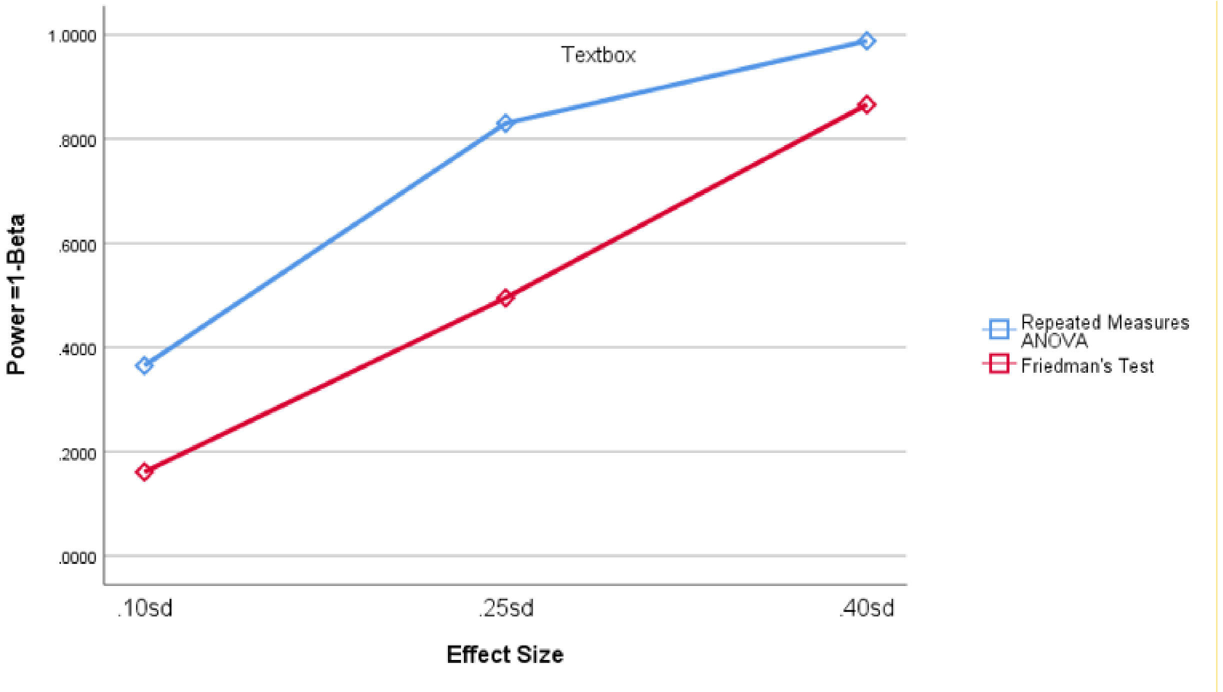


Figure 20. Comparative Power rate for the RM & F$_r$ for n=18,k=3.

Sample $n_1 = n_2 = n_3 = n_4 = n_5 = 18$

This group of samples demonstrated the power of .543 at the .1σ shift for RMA, and .321 for the Freidman's test. For effect size of .25σ, the powers were .972 for RMA and .866 for Friedman's test, and for 0.4σ, the power was 1.000 for RMA, and .998 for Friedman's test. The power difference was large at .1σ shift and decreased to about half the difference .25σ shift. The Friedman's test rose sharply to 0.998 at the 0.4σ shift, trailing the RMA of 1.000.

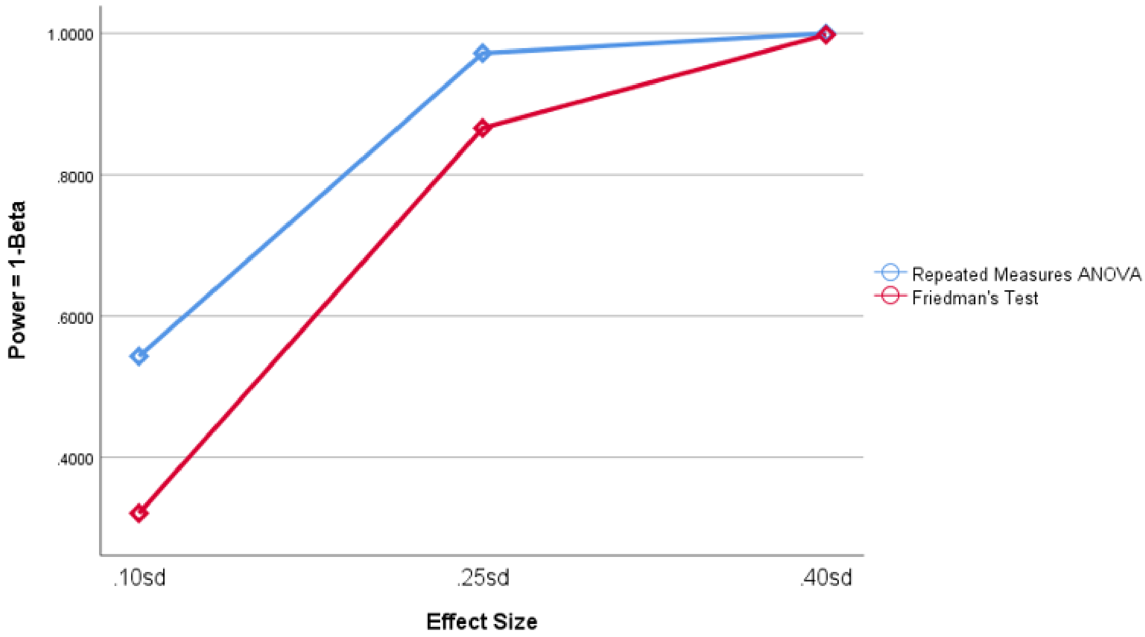


Figure 21. Comparative Power rate for the RM & F_r for n=18,k=5.

Sample $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7 = 18$

This is eighteen equal samples with seven number of measures. It gave the power of .677 at the .1σ shift for RMA, and .495 for the Freidman's test. For effect size of .25σ, the powers were almost the same for the two tests, .996 for RMA and .979 for Friedman's test. And at 0.4σ, the power was 1.000 for both RMA, and Friedman's test. The power discrepancy very was large at .1σ shift and decreased to almost negligible figure at .25σ shift. The RMA and Friedman's test rose to 1.000 at the 0.4σ shift.

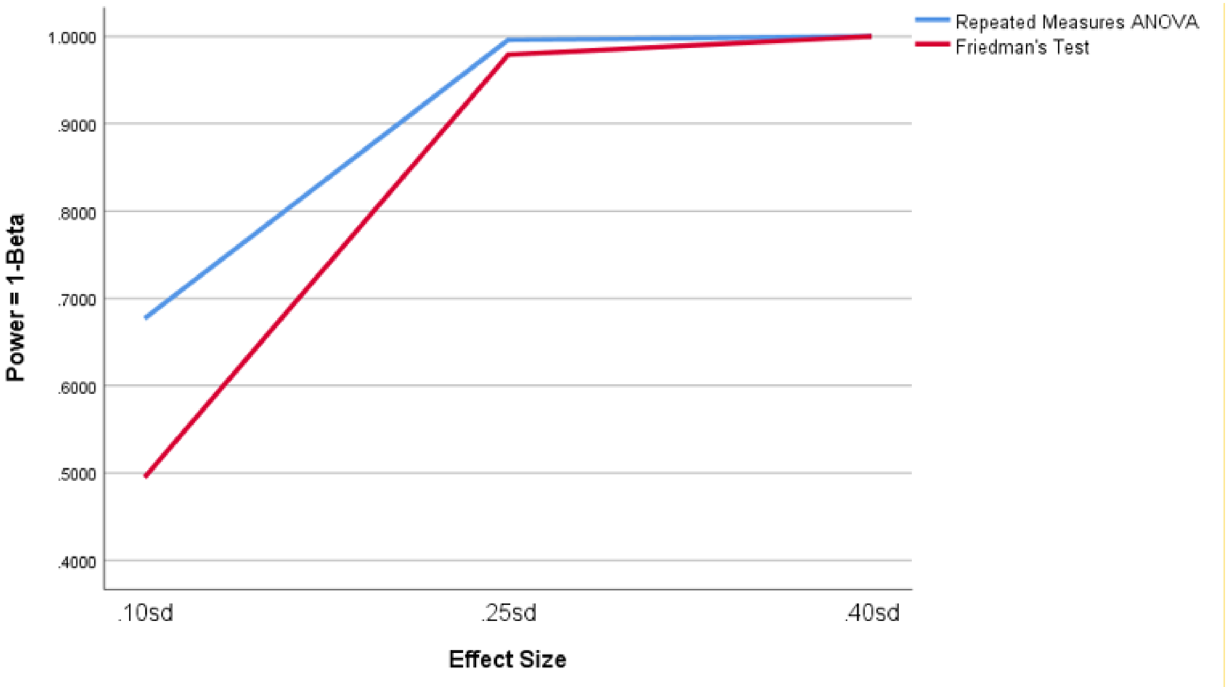


Figure 22. Comparative Power rate for the RM & F_r for n=18,k=7.

Sample $n_1 = n_2 = n_3 = 25$

This group consists of twenty-five equal samples in three different groups, (**n_1=n_2=n_3=25).** The powers were .504 and .189 at the 0.1σ shift for the Repeated Measures ANOVA (RMA) and the Friedman's test. At shift 0.25σ, there was a very sharp increase in the power curves, which brought the powers for both tests to .944 for RMA and .605 for Friedman's test. At the 0.4σ shift, Friedman's test increased drastically with about .339 in power to almost the same power as RMA.

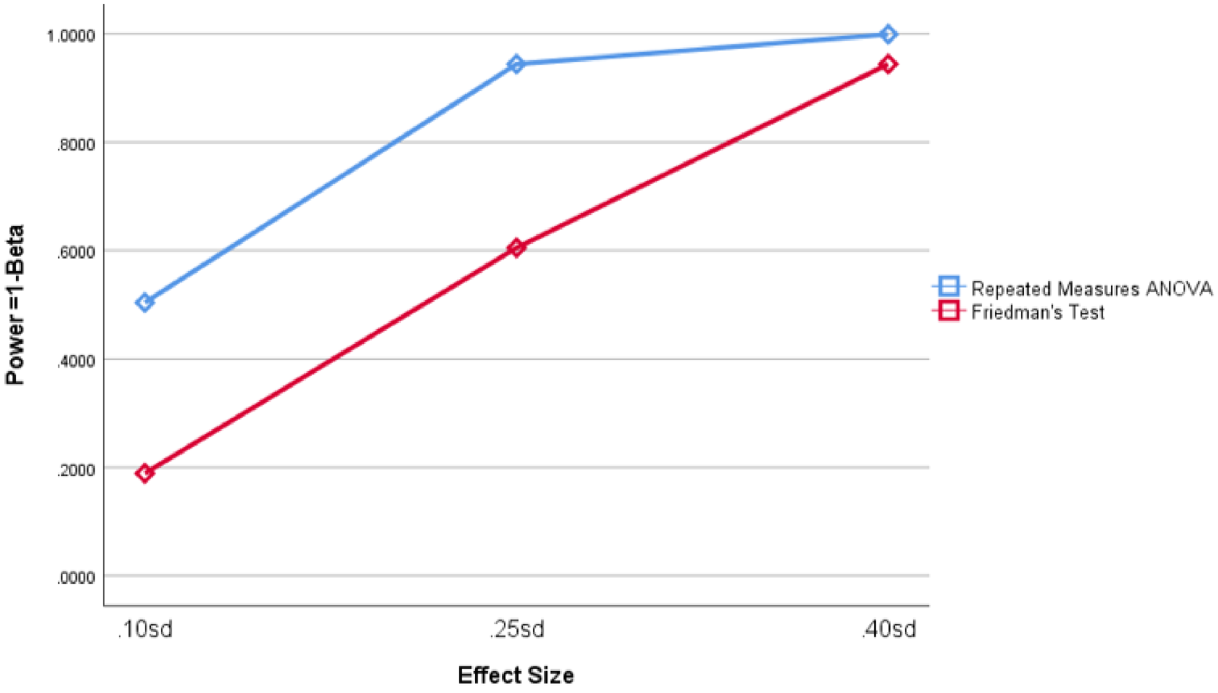


Figure 23. Comparative Power rate for the RM & F_r for n=25,k=3.

Sample $\boldsymbol{n_1 = n_2 = n_3 = n_4 = n_5 = 25}$

This group of random samples explored was twenty-five equal number in each group, and the number of measures was five, (Sample n_1=n_2=n_3=n_4=n_5=25). In this group, the effect size of .1σ shift demonstrated the power of .724 for RMA, and .395 for the Freidman's test. For effect size of .25σ, the powers were .997 for RMA and .944 for Friedman's test, and for 0.4σ, the power was 1.000 for both RMA and for Friedman's test. Conversely, the RMA demonstrated higher power rates over the Friedman's test for the first two effect sizes.

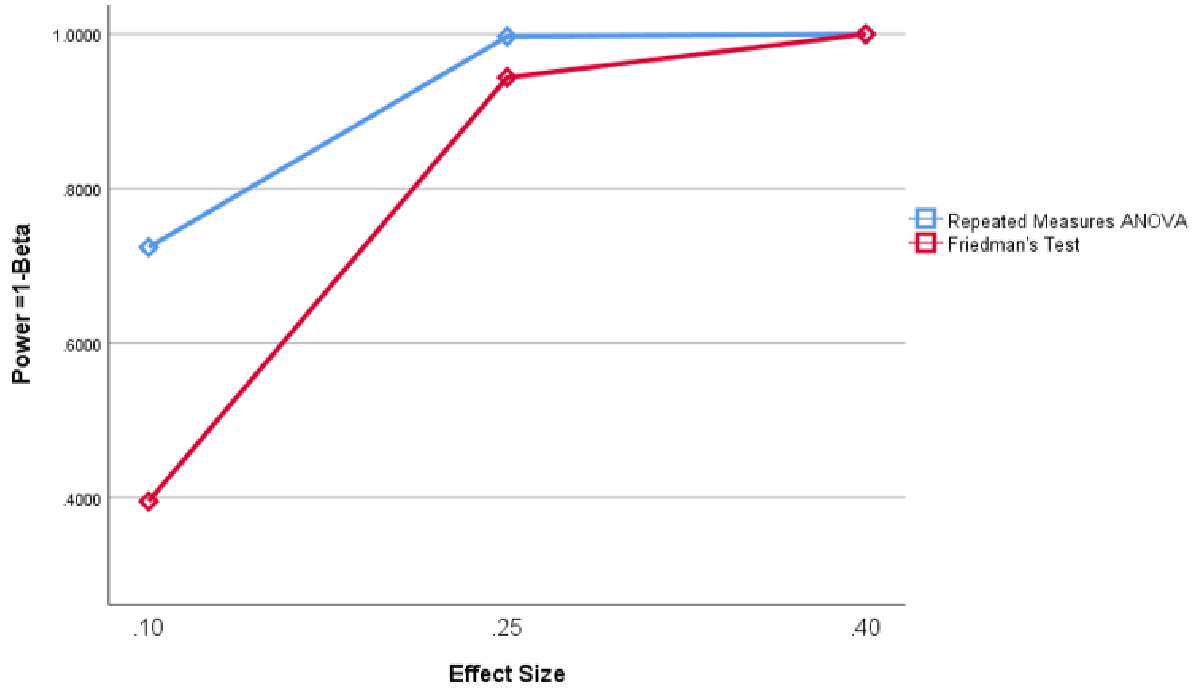


Figure 24. Comparative Power rate for the RM & F_r for n=25,k=5.

Sample $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7 = 25$

This is twenty-five equal samples with seven repeated measures. It gave the power of .852 for RMA, and .605 for the Freidman's test at the .1σ shift. For effect size of .25σ, the powers were almost the same for the two tests,1.000 for RMA and .996 and .979 for Friedman's test. And at 0.4σ, the power was 1.000 for both RMA, and Friedman's test. The power discrepancy very was large at .1σ shift and decreased to almost negligible figure at .25σ shift. The RMA and Friedman's test rose to 1.000 at the 0.4σ shift.

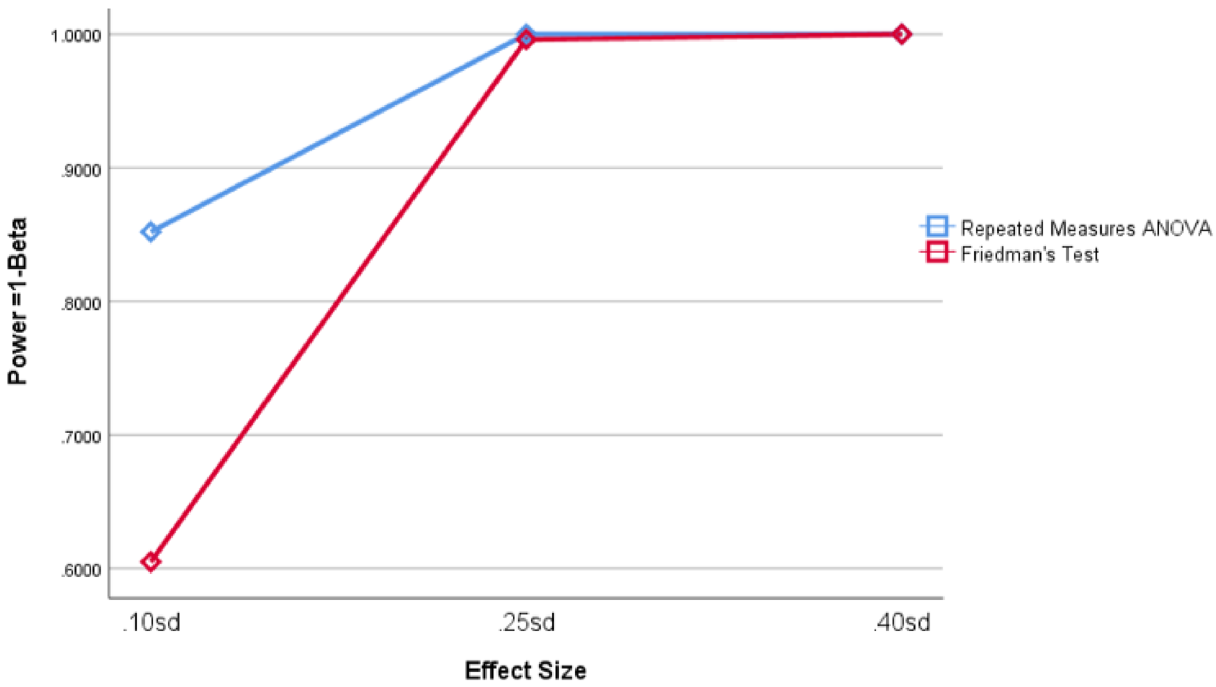


Figure 25. Comparative Power rate for the RM & F_r for n=25,k=7.

Sample $n_1 = n_2 = n_3 = 33$

This is a sample of thirty-three equal observations in three different groups, (n_1=n_2=n_3=33). The powers generated were .641 and .219 at the 0.1σ shift for the Repeated Measures ANOVA (RMA) and the Friedman's test. At shift 0.25σ, there was a very sharp increase also in the power curves, which brought the powers for both tests to .987 for RMA and .705 for Friedman's test. At the 0.4σ shift, Friedman's test had increased significantly by about .275 in power trailing the Repeated Measures ANOVA.

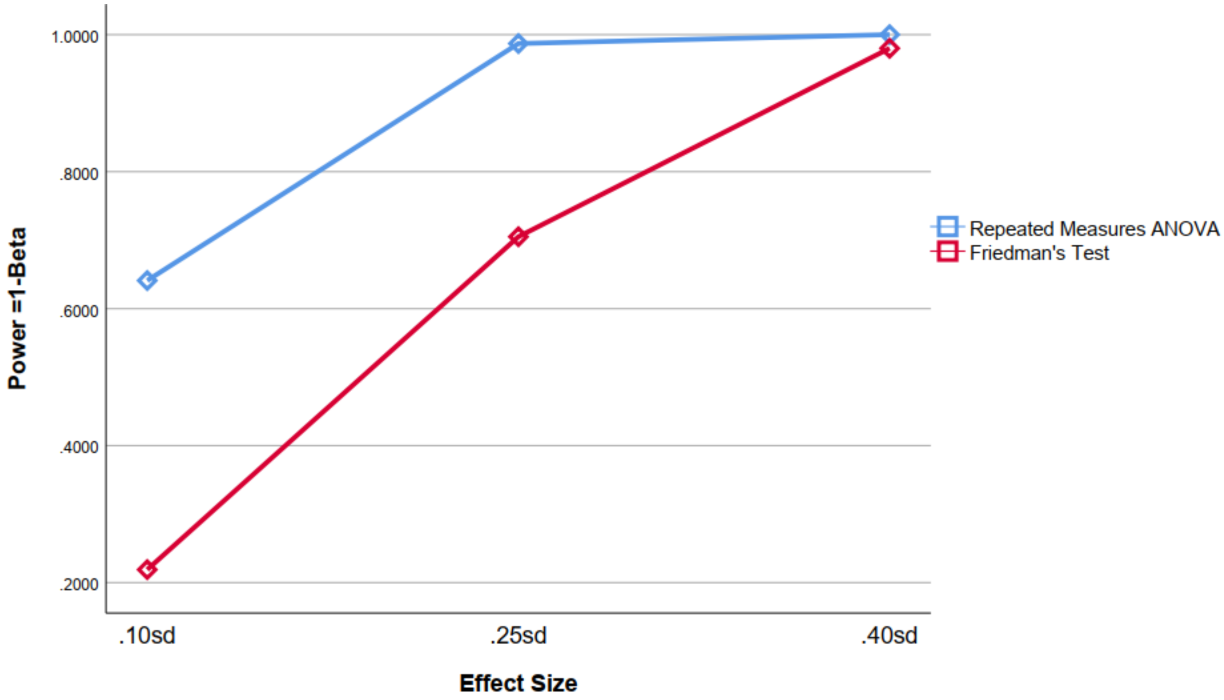


Figure 26. Comparative Power rate for the RM & F_r for n=33,k=3.

Sample $n_1 = n_2 = n_3 = n_4 = n_5 = 33$

The number of random samples in this group was also thirty-three but with increased number of repeated measures, in which case is five, (Sample n_1=n_2=n_3=n_4=n_5=33). In this group, the effect size of .1σ shift demonstrated the power of .858 for RMA, and .471 for the Freidman's test. For effect size of .25σ, the powers were exactly 1.000 for RMA and .980 for Friedman's test, and for 0.4σ, the power were both 1.000 for the two tests. Only at 0.1σ shift, RMA demonstrated higher power rate over the Friedman's test, at the .25σ and 0.4σ shift, the RMA and the Freidman's test were the same in power.

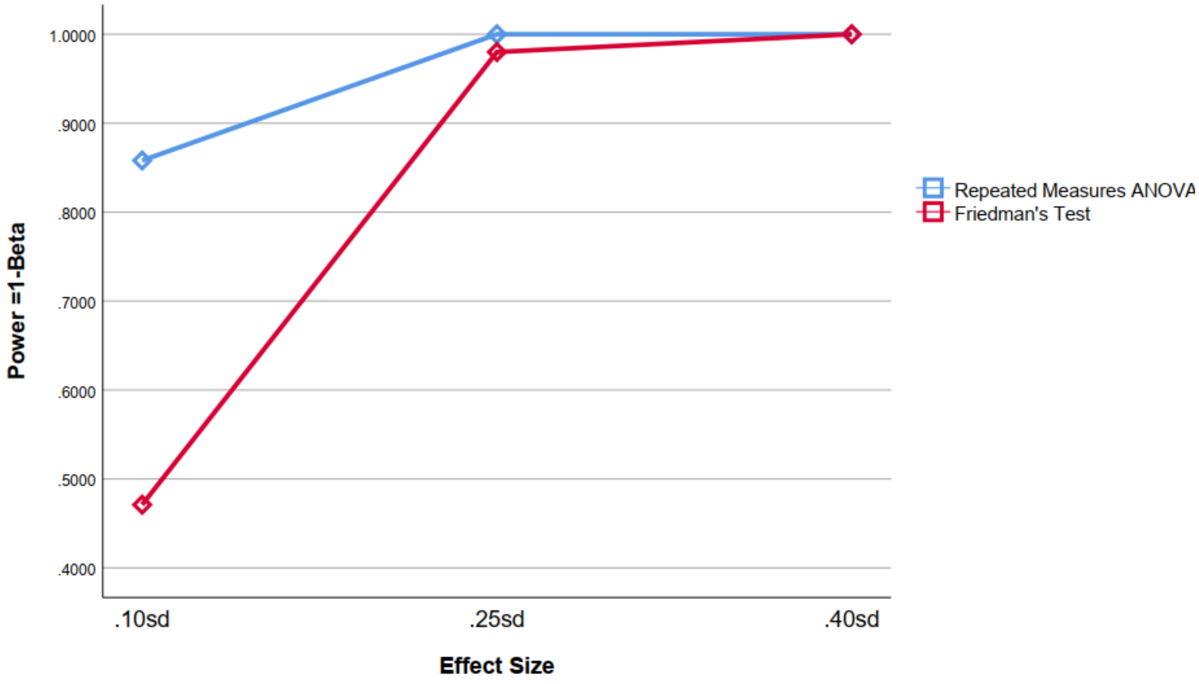


Figure 27. Comparative Power rate for the RM & F_r for n=33,k=5.

Sample $n_1 = n_2 = n_3 = n_4 = n_5 = n_6 = n_7 = 33$

This is the last random sample selected, and it is thirty-three observations in seven different groups, (Sample $n_1=n_2=n_3=n_4=n_5=n_6=n7=33$). In this group, the effect size of .1σ shift yielded the power of .948 for RMA, and .705 for the Freidman's test. At the effect size of .25σ and 0.4σ, the powers had equaled 1.000 for both tests. None of the tests showed any power advantage over the other.

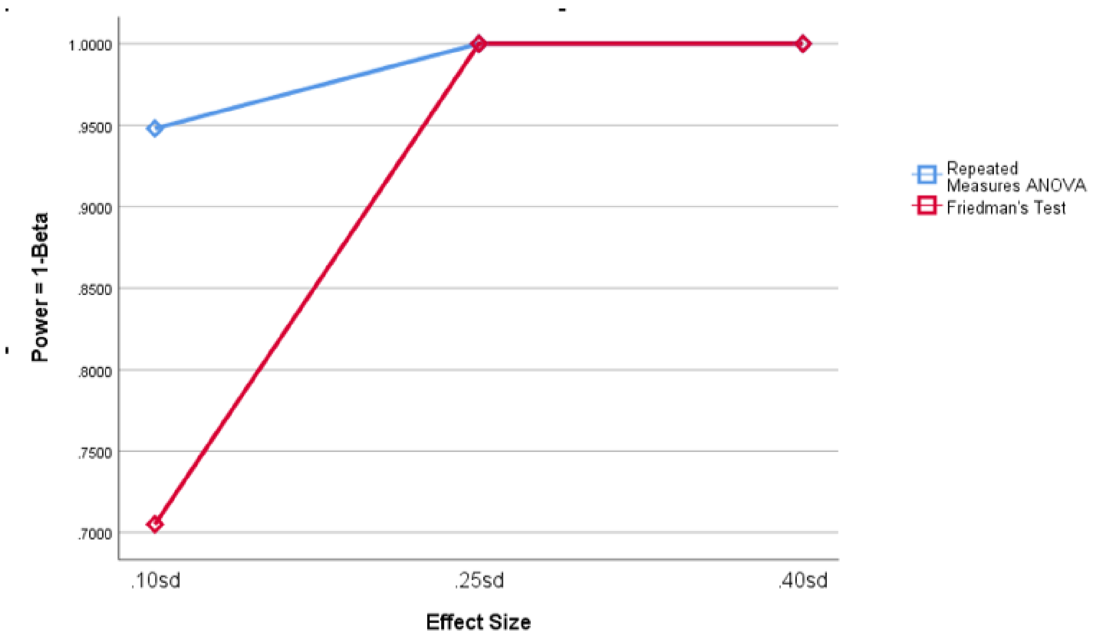


Figure 28. Comparative Power rate for the RM & F_r for n=33,k=7.

Comparative power rates and the robustness of the Repeated measures (RM)ANOVA and the Friedman's tests (FR), under various sample groups, and differing numbers of the repeated measures given three different rates of rejections, (0.01, 0.05, 0.1), ES-(Effect Size).

Table 8. The power rates for n=5, k=3.

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.1	0.099	0.10	0.024	0.026	0.10	0.179	0.173
0.25	0.211	0.219	0.25	0.063	0.081	0.25	0.335	0.329
0.40	0.384	0.396	0.40	0.141	0.190	0.40	0.541	0.524

Table 9. The power rates for n=8, k=3.

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.156	0.115	0.10	0.045	0.032	0.10	0.256	0.197
0.25	0.394	0.294	0.25	0.162	0.121	0.25	0.538	0.416
0.40	0.687	0.578	0.40	0.394	0.340	0.40	0.808	0.698

Table 10. The power rates for n=12, k=3.

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.238	0.135	0.10	0.084	0.040	0.10	0.357	0.224
0.25	0.613	0.381	0.25	0.341	0.177	0.25	0.740	0.511
0.40	0.902	0.730	0.40	0.712	0.501	0.40	0.952	0.825

Table 11. The power rates for n=18, k=3.

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.365	0.161	0.10	0.158	0.051	0.10	0.497	0.259
0.25	0.830	0.495	0.25	0.607	0.263	0.25	0.903	0.625
0.40	0.988	0.866	0.40	0.938	0.693	0.40	0.995	0.924

Table 12. The power rates for n=25, k=3.

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.504	0.189	0.10	0.262	0.063	0.10	0.635	0.296
0.25	0.944	0.605	0.25	0.822	0.361	0.25	0.973	0.724
0.40	0.999	0.944	0.40	0.993	0.838	0.40	1.000	0.972

Table 13. The power rates for n=33, k=3.

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.641	0.219	0.10	0.390	0.077	0.10	0.755	0.333
0.25	0.987	0.705	0.25	0.940	0.467	0.25	0.995	0.807
0.40	1.000	0.980	0.40	1.000	0.927	0.40	1.000	0.991

Table 11. The power rates for n=5, k=5

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.128	0.156	0.10	0.034	0.050	0.10	0.218	0.250

0.25	0.322	0.421	0.25	0.120	0.208	0.25	0.463	0.549
0.40	0.605	0.747	0.40	0.314	0.525	0.40	0.742	0.837

Table 12. The power rates for n=8, k=5

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.218	0.199	0.10	0.074	0.070	0.10	0.334	0.306
0.25	0.599	0.578	0.25	0.329	0.340	0.25	0.728	0.698
0.40	0.907	0.907	0.40	0.726	0.765	0.40	0.955	0.945

Table 13. The power rates for n=12, k=5

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.351	0.250	0.10	0.149	0.096	0.10	0.483	0.368
0.25	0.842	0.730	0.25	0.630	0.501	0.25	0.912	0.825
0.40	0.992	0.978	0.40	0.958	0.921	0.40	0.997	0.990

Table 14. The power rates for n=18, k=5

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.543	0.321	0.10	0.297	0.136	0.10	0.672	0.448
0.25	0.972	0.866	0.25	0.896	0.693	0.25	0.988	0.924
0.40	1.000	0.998	0.40	0.999	0.987	0.40	1.000	0.999

Table 15. The power rates for n=25, k=5

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.724	0.395	0.10	0.484	0.184	0.10	0.822	0.528
0.25	0.997	0.944	0.25	0.984	0.838	0.25	0.999	0.972
0.40	1.000	1.000	0.40	1.000	0.999	0.40	1.000	1.000

Table 16. The power rates for n=33, k=5.

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.858	0.471	0.10	0.672	0.241	0.10	0.919	0.604
0.25	1.000	0.980	0.25	0.999	0.927	0.25	1.000	0.991
0.40	1.000	1.000	0.40	1.000	1.000	0.40	1.000	1.000

Table 17. The power rates for n=5, k=7.

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.152	0.219	0.10	0.044	0.081	0.10	0.251	0.329
0.25	0.420	0.605	0.25	0.183	0.366	0.25	0.565	0.721
0.40	0.756	0.912	0.40	0.484	0.774	0.40	0.858	0.952

Table 18. The power rates for n=8, k=7..

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.273	0.294	0.10	0.102	0.121	0.10	0.399	0.416
0.25	0.742	0.788	0.25	0.488	0.578	0.25	0.842	0.868
0.40	0.975	0.987	0.40	0.896	0.948	0.40	0.990	0.994

Table 19. The power rates for n=12, k=7.

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.448	0.381	0.10	0.217	0.177	0.10	0.583	0.511
0.25	0.996	0.979	0.25	0.978	0.924	0.25	0.999	0.990
0.40	1.000	0.999	0.40	0.996	0.994	0.40	1.000	1.000

Table 20. The power rates for n=18, k=7.

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.677	0.495	0.10	0.430	0.263	0.10	0.786	0.625

0.25	0.996	0.979	0.25	0.978	0.924	0.25	0.999	0.990
0.40	1.000	1.000	0.40	1.000	1.000	0.40	1.000	1.000

Table 21. The power rates for n=25, k=7.

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.852	0.605	0.10	0.662	0.361	0.10	0.915	0.724
0.25	1.000	0.996	0.25	0.999	0.981	0.25	1.000	0.999
0.40	1.000	1.000	0.40	1.000	1.000	0.40	1.000	1.000

Table 22. The power rates for n=33, k=7.

α =0.05			α =0.01			α =0.1		
ES	RM	FR	ES	RM	FR	ES	RM	FR
0.10	0.948	0.705	0.10	0.842	0.467	0.10	0.974	0.807
0.25	1.000	1.000	0.25	0.997	1.000	0.25	1.000	1.000
0.40	1.000	1.000	0.40	1.000	1.000	0.40	1.000	1.000

Power Discrepancies for Group Measures of 3

Table 23. Power differences for all Samples, when K=3.

Statistic/Sample size	Robustness	0.1	0.25	0.4
5,5,5				
RM	0.245	0.1	0.211	0.384
Fr	0.091	0.099	0.219	0.396
Power Differences	NA	0.001	-0.008	-0.012
8,8,8				
RM	0.015	0.156	0.394	0.687
Fr	0.008	0.115	0.294	0.578
Power Differences	NA	0.041	0.1	0.109
12,12,12				
RM	0.007	0.238	0.613	0.902
Fr	0.028	0.135	0.381	0.730
Power Differences	NA	0.103	0.232	0.172

18,18,18				
RM	0.046	0.365	0.830	0.988
Fr	0.000	0.161	0.495	0.866
Power Differences	NA	0.204	0.335	0.122
25,25,25				
RM	0.049	0.504	0.944	0.999
Fr	0.000	0.189	0.605	0.944
Power Differences	NA	0.315	0.339	0.055
33,33,33				
RM	0.006	0.641	0.987	1.000
Fr	0.000	0.219	0.705	0.980
Power Differences	NA	0.422	0.282	0.02

Power Discrepancies for Group Measures of 5

Table 24. Power differences for all Samples, when K=5.

Statistic/Sample size	Robustness	0.1	0.25	0.4
5,5,5,5,5				
RM	0.740	0.128	0.322	0.605
Fr	0.308	0.156	0.421	0.747
Power Differences	NA	-0.028	-0.099	-0.142
8,8,8,8,8				
RM	0.184	0.218	0.599	0.907
Fr	0.004	0.199	0.578	0.907
Power Differences	NA	0.019	0.021	0
12,12,12,12,12				
RM	0.111	0.351	0.842	0.992
Fr	0.015	0.250	0.730	0.978
Power Differences	NA	0.101	0.112	0.014
18,18,18,18,18				
RM	0.007	0.543	0.972	1.000
Fr	0.000	0.321	0.866	0.998
Power Differences	NA	0.222	0.106	0.002
25,25,25,25,25				
RM	0.055	0.724	0.997	1.000
Fr	0.000	0.395	0.944	1.000
Power Differences	NA	0.329	0.053	0

33,33,33,33,33				
RM	0.013	0.858	1.000	1.000
Fr	0.000	0.471	0.980	1.000
Power Differences	NA	0.387	0.02	0

Power Discrepancies for Group Measures of 7

Table 25 Power differences for all Samples, when K=7.

Statistic/Sample size	Robustness	0.1	0.25	0.4
5,5,5,5,5,5,5				
RM	0.557	0.152	0.420	0.756
Fr	0.515	0.219	0.605	0.912
Power Differences	NA	-0.067	-0.185	-0.156
8,8,8,8,8,8,8				
RM	0.044	0.273	0.742	0.975
Fr	0.001	0.294	0.788	0.987
Power Differences	NA	-0.021	-0.046	-0.012
12,12,12,12,12,12,12				
RM	0.072	0.448	0.996	1.000
Fr	0.010	0.381	0.979	0.999
Power Differences	NA	0.067	0.017	0.001
18,18,18,18,18,18,18				
RM	0.001	0.677	0.996	1.000
Fr	0.001	0.495	0.979	1.000
Power Differences	NA	0.182	0.017	0
25,25,25,25,25,25,25				
RM	0.008	0.852	1.000	1.000
Fr	0.000	0.605	0.996	1.000
Power Differences	NA	0.247	0.004	0
33,33,33,33,33,33,33				
RM	0.000	0.948	1.000	1.000
Fr	0.000	0.705	1.000	1.000
Power Differences	NA	0.243	0	0

CHAPTER FIVE DISCUSSION

CONCLUSIONS AND IMPLICATIONS

Overview of the Study

When researchers are faced with the issue of choice about which statistical procedures to use for analysis, priority should be given to the "consideration of power or Type II error properties" (Brownie & Boos, 1994). In general, the power of a test is related to the efficiency of a test, which is the minimum requirements (sample size) needed for a test to demonstrate its power level- the ability of a test to detect a true effect that is present as stated in the alternative hypothesis. The two indices that have been prevalent in defining the power criterion of a test, or the efficiency of a test when it is compared to its counterparts are Asymptotic Relative Efficiency (ARE) and Relative Efficiency (RE). Relative Efficiency is the index that compares the number of samples required by a test to generate the desired power level against the sample size required of an alternative test to reach the same power level. Before the RE index can be used effectively, the comparison must hold under the same conditions: the same nominal alpha and the same hypothesis. While the ARE or Pitman efficiency (Pitman, 1948), "is a large sample index that compares the RE of competing statistical tests when sample a of Test A and sample b of Test B are infinitely large, and the treatment effect is infinitesimally small." (Sawilowsky, 1990, p. 93; Hodges and Lehmann, 1955; see also Lehmann E. L., 1975, and Noether, 1955 for further explanation).

The first part of this interpretation section contains the results from the Type I error rate findings. Then, the results of the power comparison for differing sample sizes under three different group measures were explored and explained in detail.

Type I Error Rate

Table 2 in the previous chapter, displayed the results of the rates of rejection under the null condition for both the one-way repeated measures and Friedman's test. In the k=3 condition of treatments, the comparison was performed using differing sample sizes and different combinations of measures. The results for both tests yielded Type I error rates above 5%, for the sample size of 5, across all three conditions. Although the result showed that Friedman's test was liberal when the number of samples and groups was very small, the rate of rejection was closer to its nominal alpha. As the number of samples gets larger and the number of measures fixed, the p-values were conservative. This is comparable to the findings of Brownie and Boos, (1994), it is reassuring to know that the Friedman T with χ^2_{n-1} percentiles will not be liberal if n is large for the situation where the *k* is fixed and n→∞" (p. 547). The shape of the distribution displayed for this sample size was uniform but has a long peak, (leptokurtic). Conclusively, Friedman's test performed better than the parametric repeated measures ANOVA in this case.

Both tests demonstrated their error rates below the conservative criterion of Bradley's robustness in all the sample sizes except the smallest sample condition. The smallest sample condition error rates for the two tests tend to be liberal

When the number of treatments was kept at five, (k=5), the ANOVA's error rates were beyond the bounds of Bradley's liberal criterion of 0.5 α < α < 1.5 α in all the samples, but for samples of 18 & 33. Whereas, Friedman's test was able to control for its rejection rates below the nominal alpha. Only in the sample of 5, did it behave like the counterpart, repeated-measures ANOVA. "The conservative nature of Friedman's procedure may be appealing in some contexts, if avoiding Type I errors is paramount importance (*p*. 547). Friedman's test demonstrated higher power both for small measures and groups as well as when the groups and measures are large.

For similar sets of samples but under the k=7 condition, both tests trailed each other in the rates display. But as the number of samples increases, there was a decreasing order in the patterns of the p-values displayed, to the point of controlling for its errors below the nominal alpha level. For the rest of the groups and conditions combinations, repeated measures ANOVA generated the p-values below the nominal alpha when the number of the samples were tending toward the central limit theorem, that is the number of observations is increasing, moving from 8 up to 33. As the number of samples tends towards infinity, the p-values for Friedman's test, which are approximately distributed according to the normal F with the degrees of freedom, n-1, (k-1)(n-1), become accurate. This is not so surprising as the work of Brownie and Boos, (1994) gave a detailed breakdown of this scenario. He suggested an "adjustment factor for distributions that are affected by nonnormality, to be approximately $1 + (\beta_2 - 3)/N$, where β_2 is the kurtosis of the error distribution of the data," The conclusion of the study was that the distributions affected by location shift will always generate error rates that are higher than the nominal alpha level, (liberal), also, those distributions that are almost normal will yield lower p-values (conservative). This is what is called the central limit theorem "(CLT)-based asymptotic for both the fixed $\mathrm{t,b} \rightarrow \infty$, and fixed b, t→ ∞ situations" (*p*. 547).

Consequences of the lack of sphericity on the Type I error rates

Table 4 in the previous chapter displayed the sphericity assumption results performed in SPSS 26.01. It shows that when the sample sizes are small, the assumption of circularity was met, i.e., the *p*-values were not significant, (n=5,8, &12). It has been established that it only takes smaller sample sizes to meet the assumption of equal variances, and they tend to have larger variances (the within-block homogeneity) (Hodges and Lehmann, 1960). The assumption of circularity or sphericity is sufficient for one-way repeated measures ANOVA to utilize few samples for greater power but is not a necessity (Huynh and Feldt, 1970). When the result of an F test is significant, there are three solutions to report accurate test results, which involve decreasing the degrees of freedom. The Huynh-Feldt (1976) test (HF), the Greenhouse-Geisser (1958, 1959) test (GG), and the GG conservative test. The choice and basis of the df correction test were detailed in chapter two of this study. The Type I error rates can be highly inflated if the assumption of sphericity does not hold, and the unadjusted F results were reported. In this research study, the GG corrections were reported, this controls for the Type I error rate well and maximizes power. Although, choosing a test statistic based on whether the assumption of sphericity and circularity holds has been kicked against seriously (Muller & Barton, 1989, see also Keselman & Rogan, 1980; Keselman, Algina, & Kowalchuk, 2001).

Comparative Statistical Power

Different sample sizes are grouped based on the same number of treatments. G*Power 3.1.9.4 software was used to analyze the power function for various samples. The values generated from the software were reentered into the SPSS 26.01 and were used to run both the line graphs and the bar graphs. The line graphs for various effect sizes are presented in the result section of this research study, while the bar graphs are referenced in Appendix A.

For sample sizes under the treatment level of three, (*k*=3), Friedman’s test demonstrated power advantages only when the sample size was 5 and under 0.25σ and 0.4σ. For the 0.1σ, the two tests carry the same power. And as the sample sizes increased, there was a proportionate increase in the power levels demonstrated by both tests. For the rest of the sample sizes under this group, Friedman’s test trailed the one-way repeated measures ANOVA in power when the shift was 0.4σ, with the power discrepancies ranging from 0.055 to 0.172 for the one-way RM ANOVA. The power differences demonstrated by Friedman’s test were between -0.008 to -0.012 only when the number of samples was five. However, Friedman’s test displayed almost as much as the same power held by the repeated measures when the shift was 0.4σ. When the shift was 0.1σ and 0.25σ, it showed that the one-way Repeated Measures (RM) ANOVA held the power advantages over the nonparametric counterpart. Both tests behaved in the same manner.

In the k=5 power comparison, the one-way RM ANOVA has a greater power advantage over Friedman’s test only when the shift was 0.1σ. The rates at which the power of the one-way RM ANOVA increases with the increase in sample size doubles the rates at which the power of the Friedman’s test was increasing. Under the effect size of 0.25σ, the power levels for both tests were almost at the same level. whereas by the time the shift was 0.4σ, Friedman’s test displayed the same power in about four sample sizes, except for n=5 for the three shifts. This is comparable to the findings from previous studies (Iman, Hora, and Conover, 1984).

When the number of treatments increased to *k*=7, Friedman's test carried power advantages in 6 out of 18 (33.3%) of the comparisons, with the power differences ranging from -0.012 to -0.185. Overall, in this group, the Friedman's test tallied with the power of RM ANOVA in 6 of 12 remaining comparisons (50%). This was also confirmed in the works of (Brownie and Boos, 1994; Friedman,1937), "the power of Friedman's test is known to increase with *k."* (Iman, Hora, and Conover, 1984, *p*. 680).

Conclusion

Over the years, it has been established that when the underlying assumptions are in place, the parametric F-test should be preferred. However, evidence has shown that some of these assumptions of parametric tests are rarely met especially in real-world circumstances (Blanca, Arnau, López-Montiel, Bono, & Bendayan, 2013; Micceri, 1986 November**;** 1989; Pearson & Please, 1975; Sawilowsky, Blair, & Micceri, 1990; Tan, 1982). Even though these assumptions do not always hold true, researchers have used the parametric *F* tests "indiscriminately based on the belief that this *F* statistic was immune to nonnormality, or that nonnormally distributed data was rare" (Sawilowsky, 2006, p. 208). The outcome of this study has established that the assumption of centrality is very rare in real-world datasets. The results of the graphical representations of the COVID-19 mortality datasets were referenced in Appendix B and C.

Once the parametric *F* tests missed out in the assumption of location shift, other alternating statistics could obtain far greater power under the same levels and conditions of testing as the parametric statistic, to the tune of 0.99, (Sawilowsky, 2006). This is one of the confirmations from this study. Friedman's test displayed a power level of 0.99 to 1 when the sample was even as small as 12. Friedman's test was first proposed by

Friedman (1937) and was classified under the rank transform procedures by Conover and Iman, (1981).

Few studies have been carried out to demonstrate the power of rank transform procedures in various situations (Beasley, 2000; Brownie and Boos, 1994; Bryan, 2009; Gibbons, 1985; Hodges & Lehmann, 1960; Iman, Hora, & Conover, 1984; Kelley & Sawilowsky, 1997; Lehmann, 1975; Sawilowsky, Blair, & Higgins, 1989; Siegel, 1956; Thompson & Ammann, 1989; Toothaker & Chang, 1980; Zimmerman, 1992; Zimmerman & Zumbo, 1993). And some of the following advantages have been evident as the outcomes of these studies. RT procedure has favored data sampled from the distributions that are nonnormal, or even those with the parent distributions that are not well known to the researchers. RT procedures have demonstrated considerable power when the sample size is very small. RT test is insensitive to the shift in the location parameter. RT is almost as powerful as the F test when the underlying assumptions hold true as well as when the assumptions are not in place. RT has greater power in preserving the information in the samples. RT procedures have also favored heavy-tailed distributions.

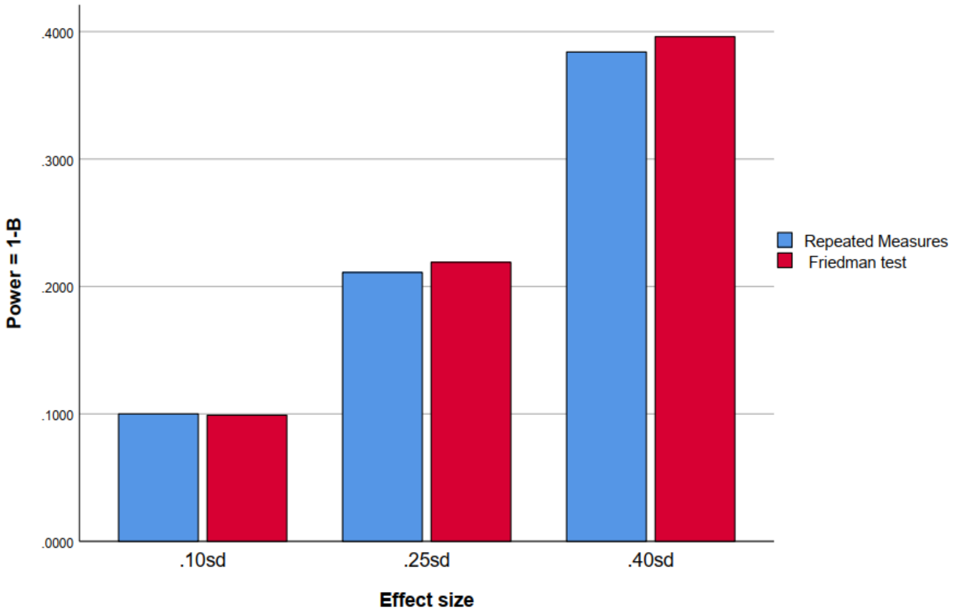
Sawilowsky (1990) did a detailed study on the comparative power of nonparametric ANOVA and confirmed that Friedman's test can be used to analyze the "randomized complete block designs, assuming there are no interactions and only one observation per cell is prevalent" (*p*. 100).

In the situations where distributions are heavily tailed, Friedman's test shows the power level comparable to the one-way RM ANOVA. Also, when the distributions were almost normal, the level of power yielded is very high.

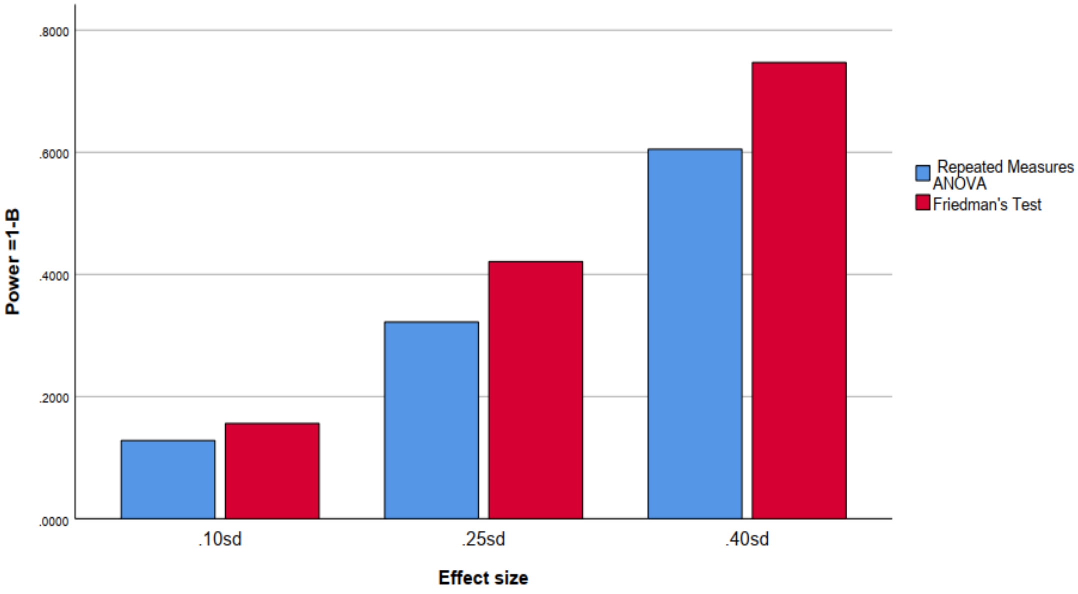
It is therefore concluded that whenever the one-way repeated measures ANOVA failed in the area of shift in location, it is opined that the Friedman's test based on the rank transform procedure can comfortably be the best alternative (Bryan, 2009; Harwell & Serlin, 1994; Iman, Hora, and Conover, 1984).

Finally, it is evident in this research study that the comparison of these tests behaved in similar ways as those carried out previously using the Monte Carlo simulation methods. The prevailing power advantage of the nonparametric tests with regards to the Type I error rates is always evident in the smaller sample sizes (Sawilowsky, 2006). Nonparametric tests require smaller sizes of samples to identify the true effect that is present among group means.

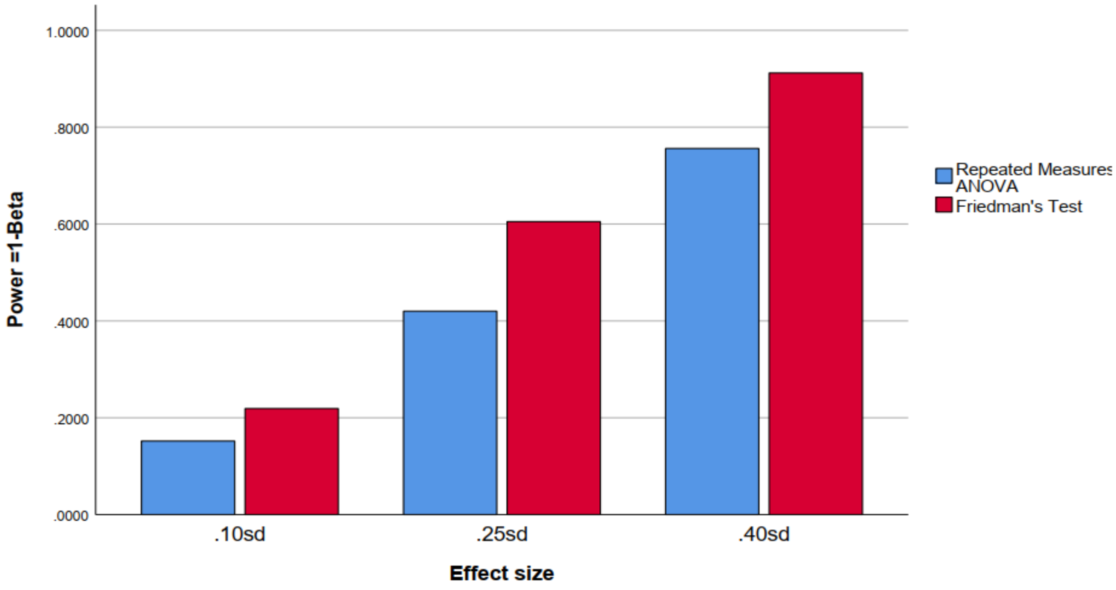
APPENDIX A



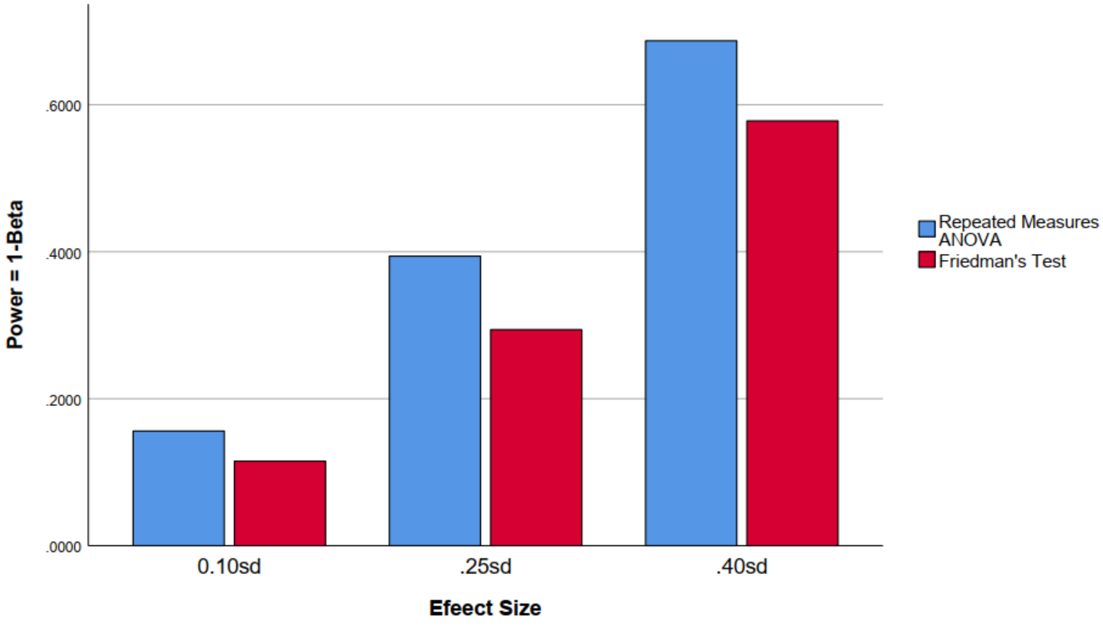
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3)=5 and Alpha =.05



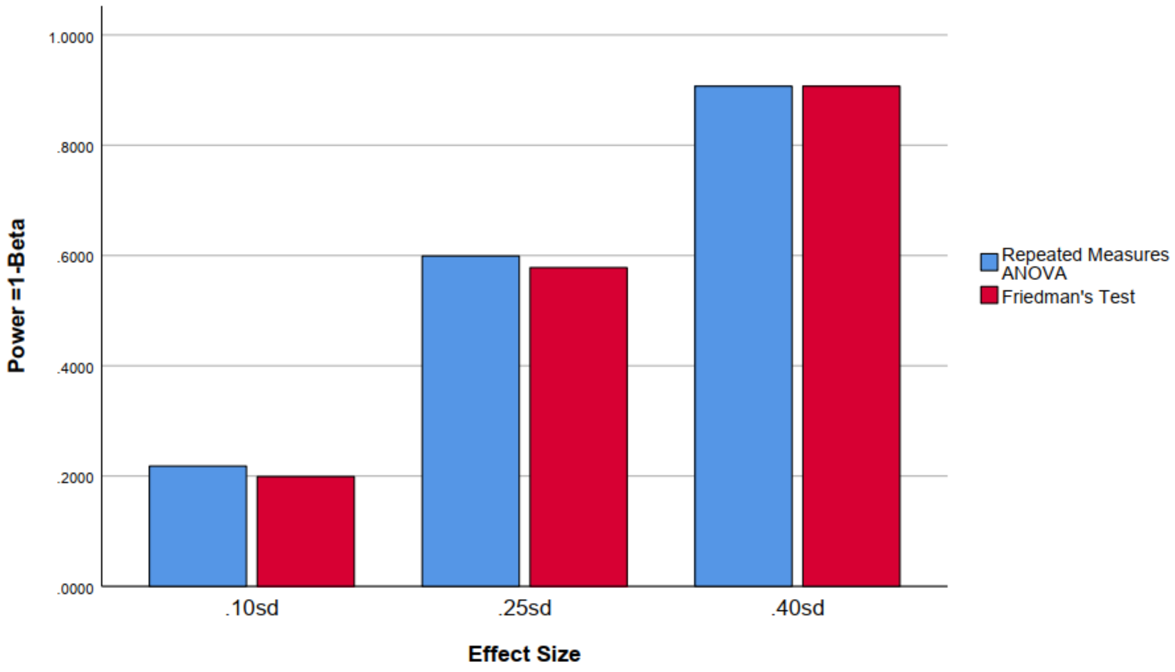
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3,n4,n5)=5 and Alpha =.05 (Bar Graph)



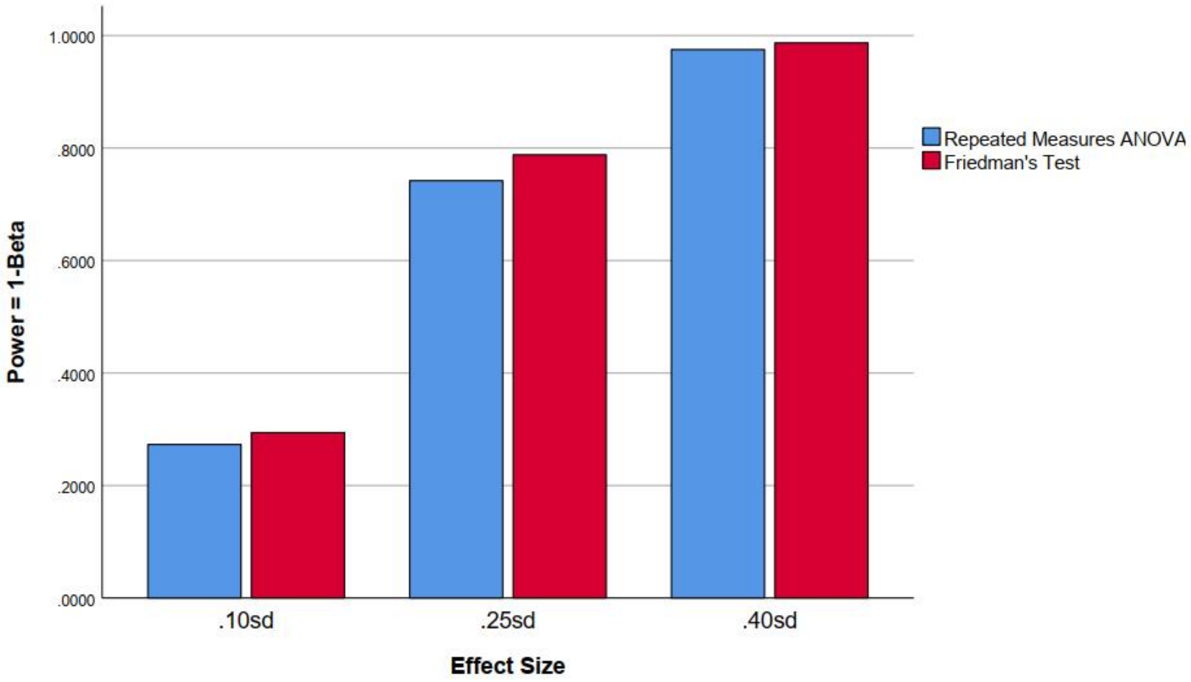
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3,n4,n5,n6,n7)=5 and Alpha =.05 (Bar Graph)



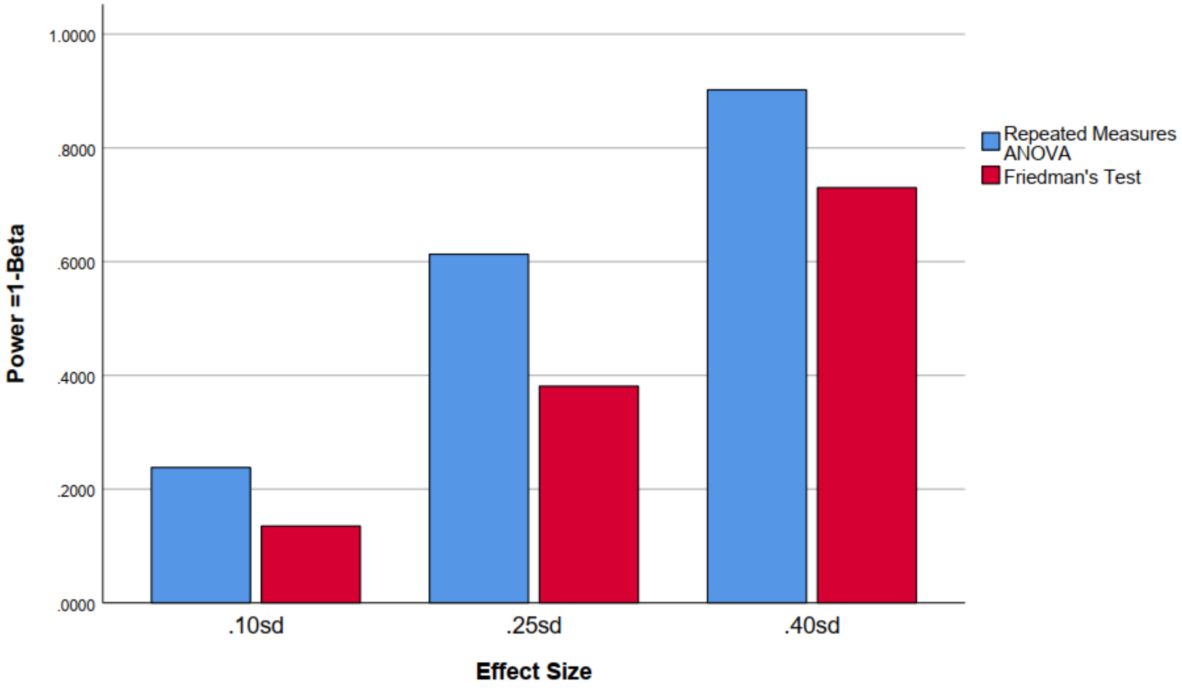
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3)=8 and Alpha =.05 (Bar Graph)



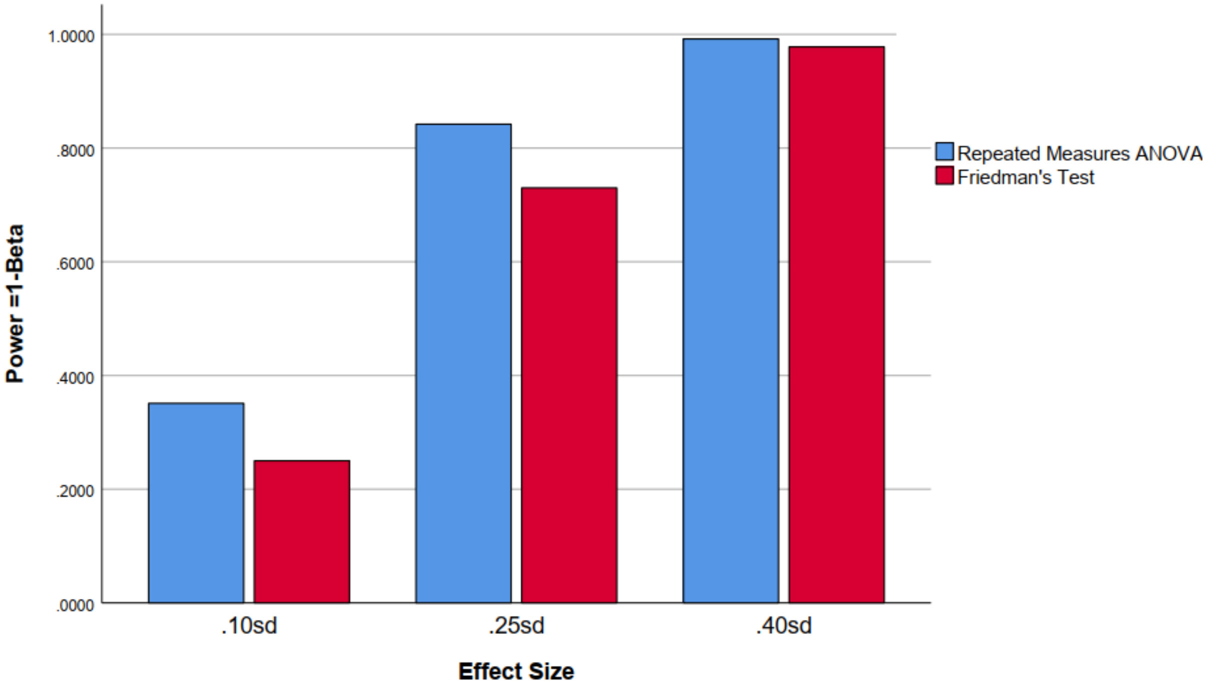
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3,n4,n5)=8 and Alpha =.05 (Bar Graph)



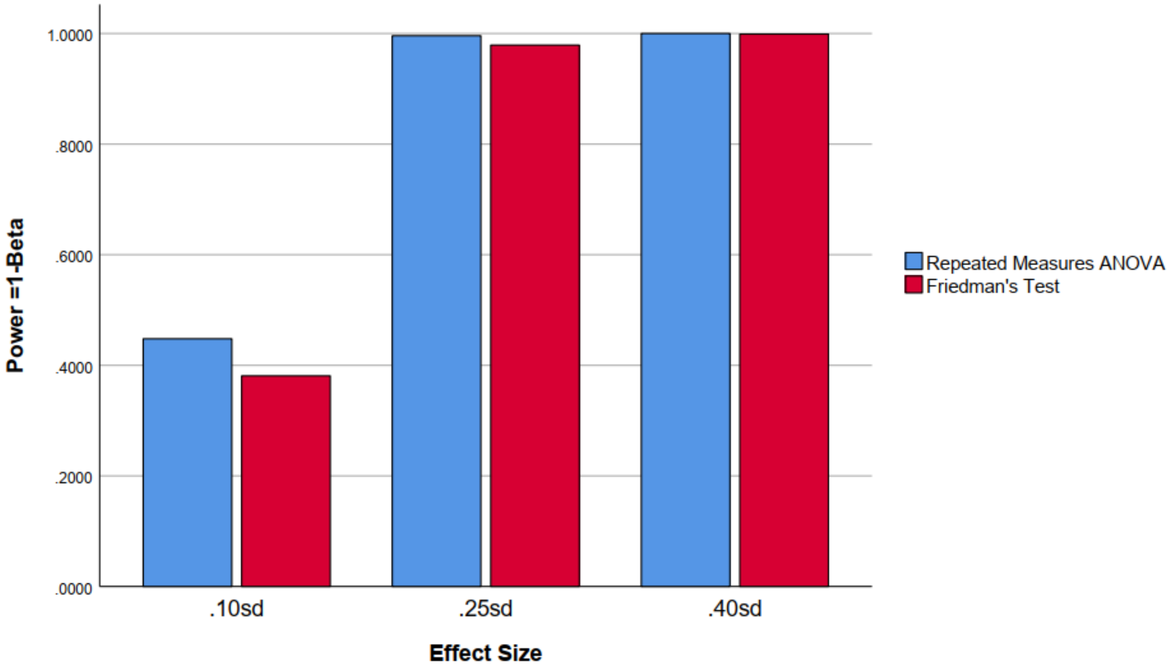
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3,n4,n5,n6,n7)=8 and Alpha =.05 (Bar Graph)



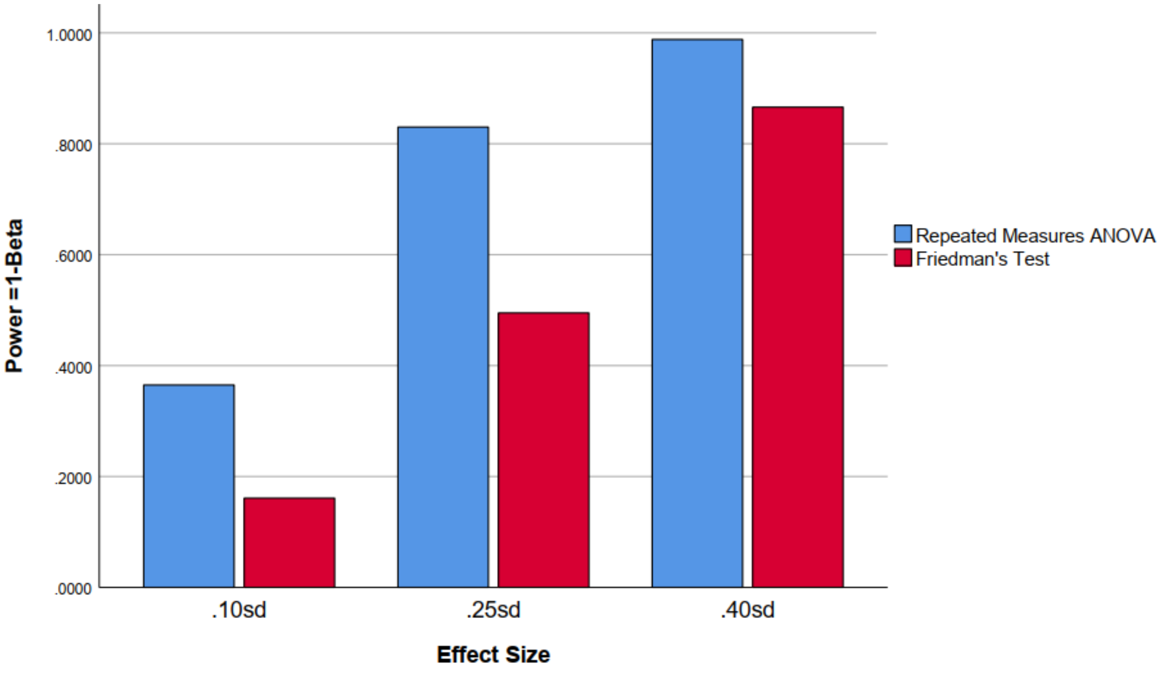
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3)=12 and Alpha =.05 (Bar Graph)



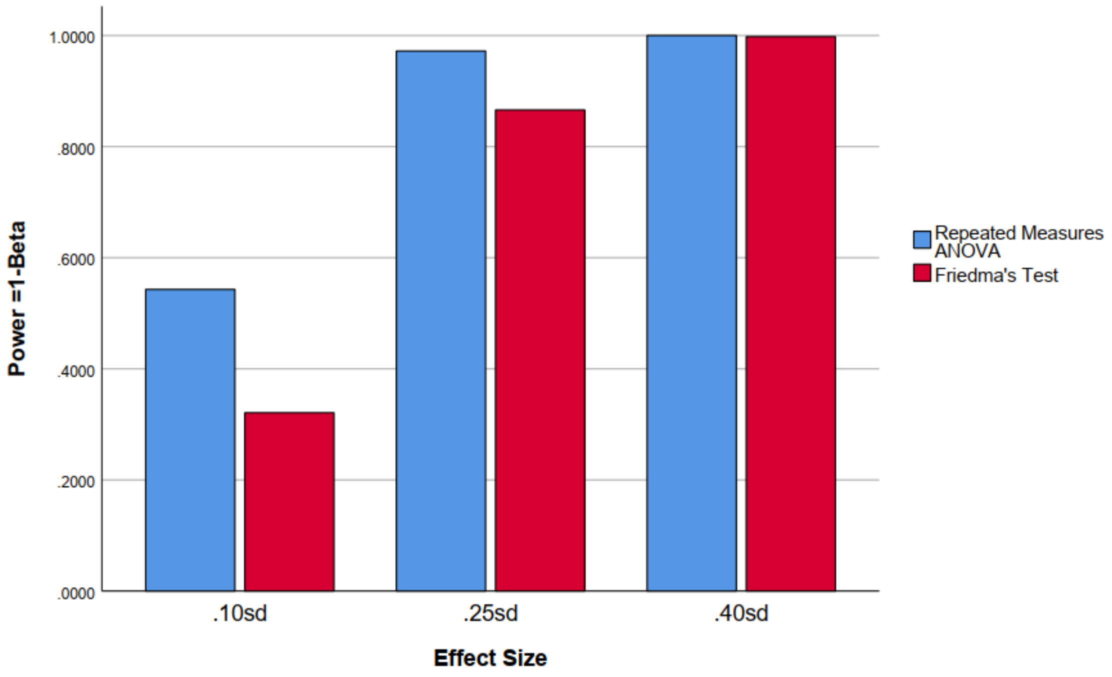
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3,n4,n5)=12 and Alpha =.05 (Bar Graph)



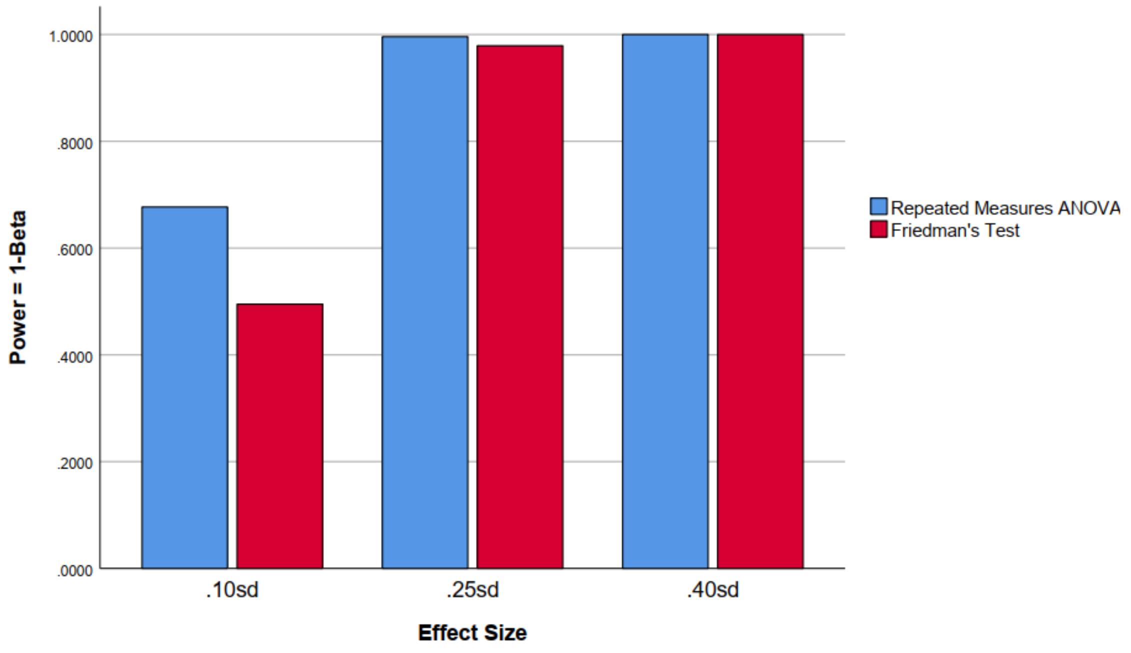
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3,n4,n5,n6,n7)=12 and Alpha =.05 (Bar Graph



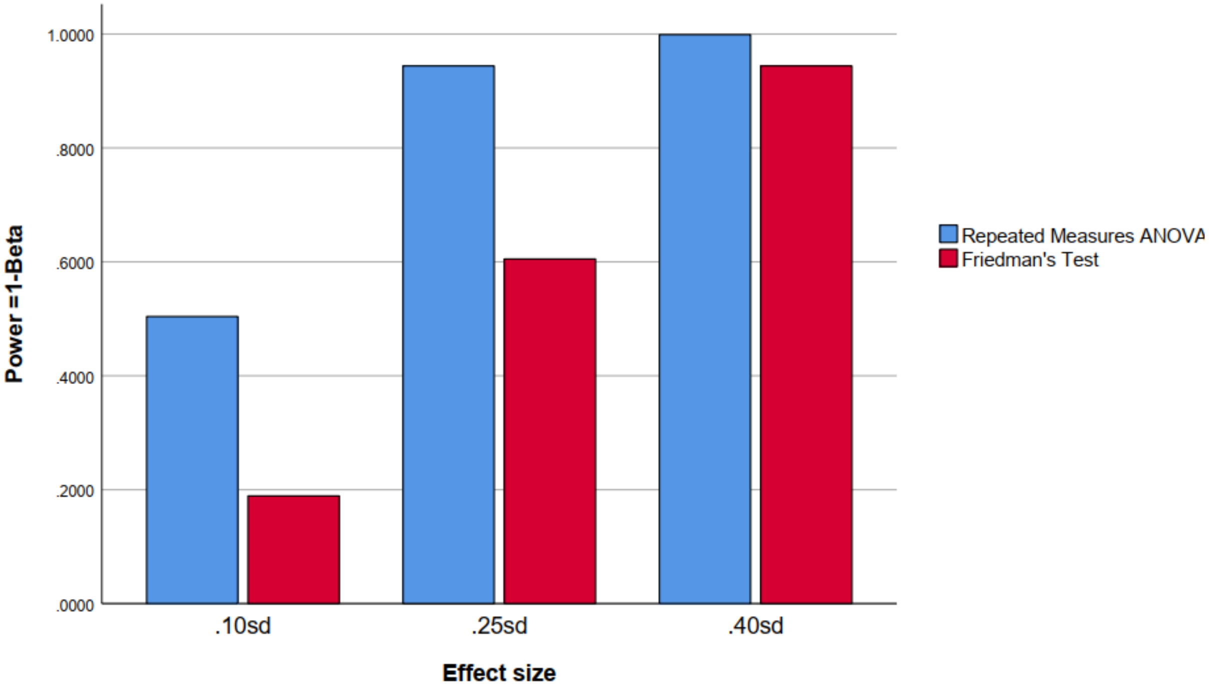
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3)=18 and Alpha =.05 (Bar Graph)



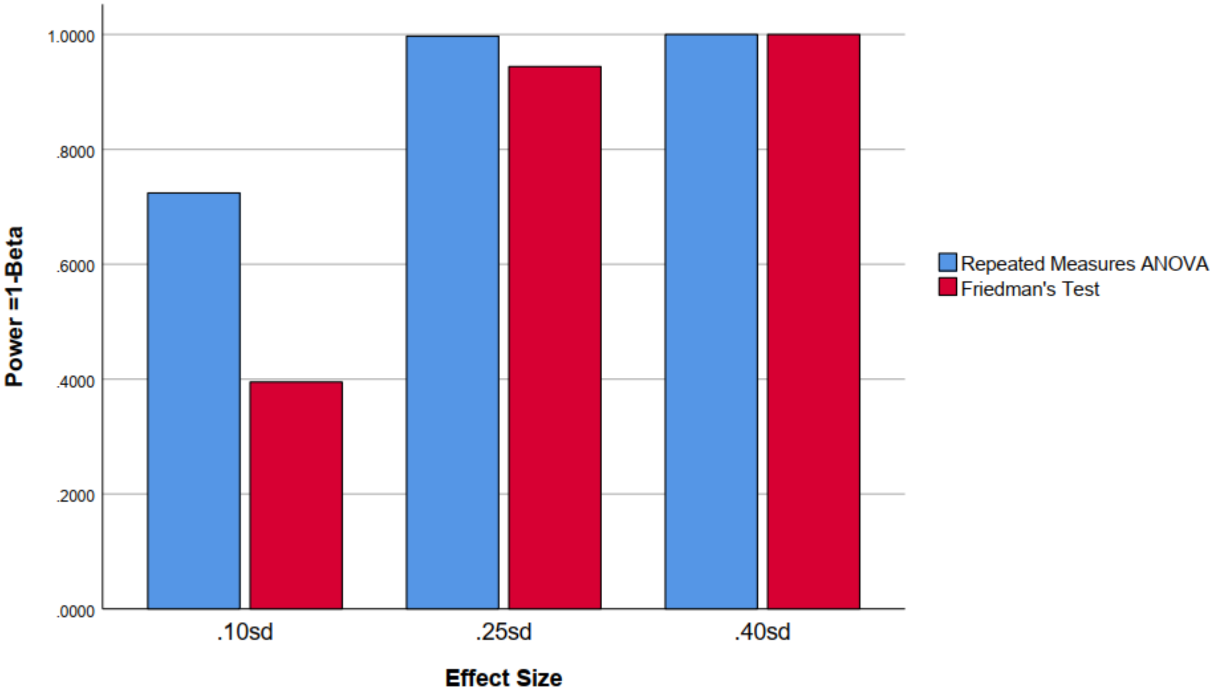
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3,n4,n5)=18 and Alpha =.05 (Bar Graph)



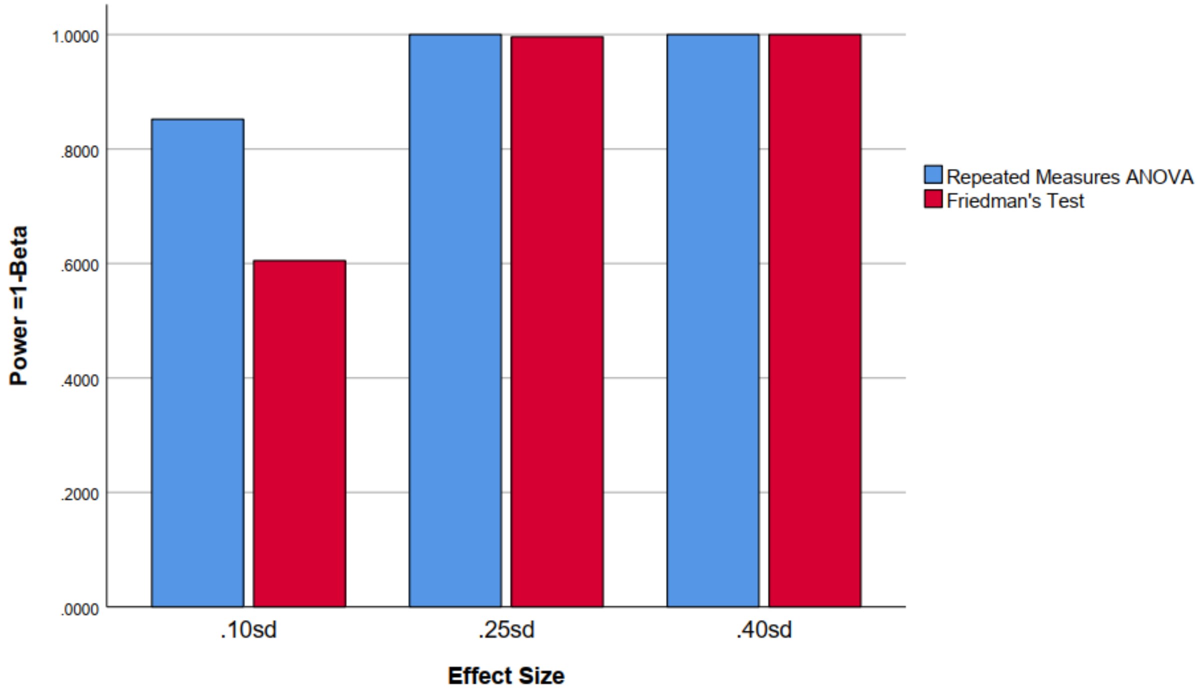
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3,n4,n5,n6,n7)=18 and Alpha =.05 (Bar Graph



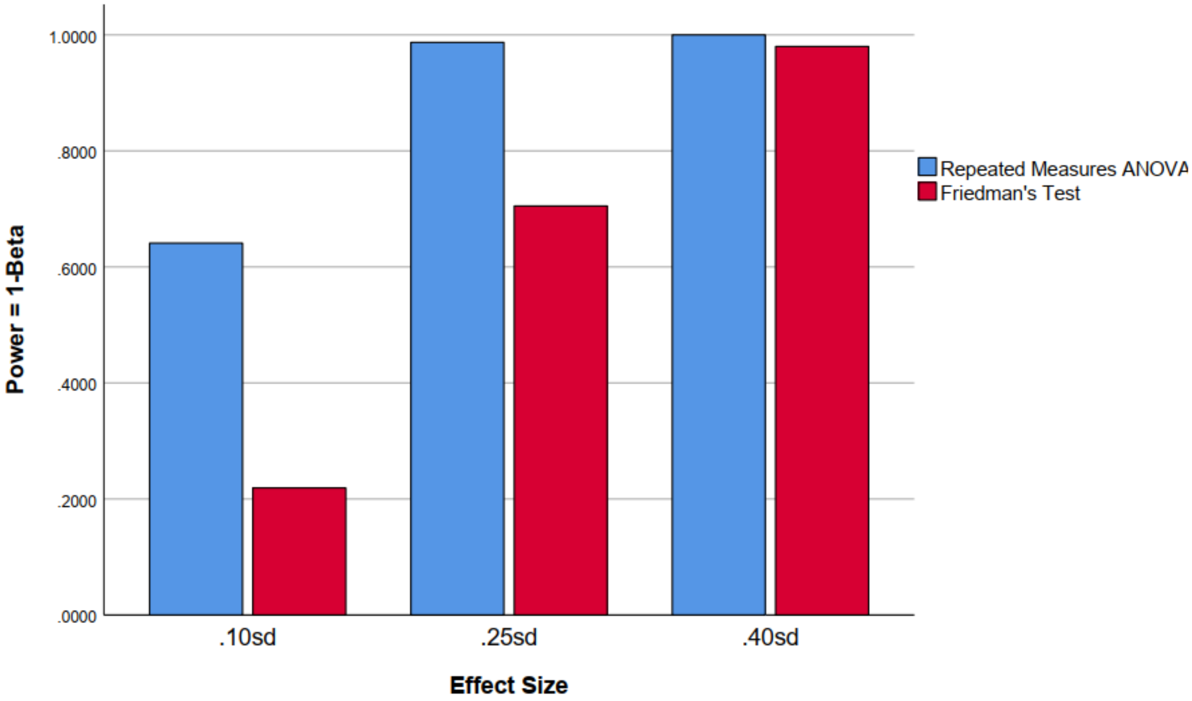
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3)=25 and Alpha =.05 (Bar Graph)



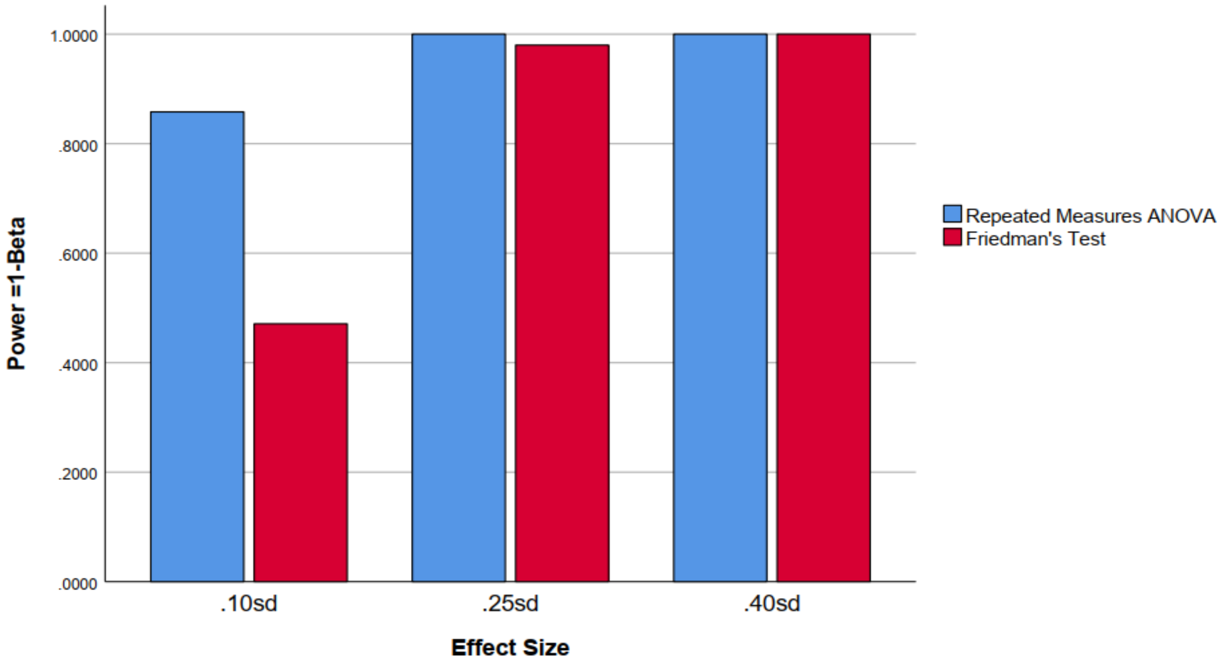
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3,n4,n5)=25 and Alpha =.05 (Bar Graph)



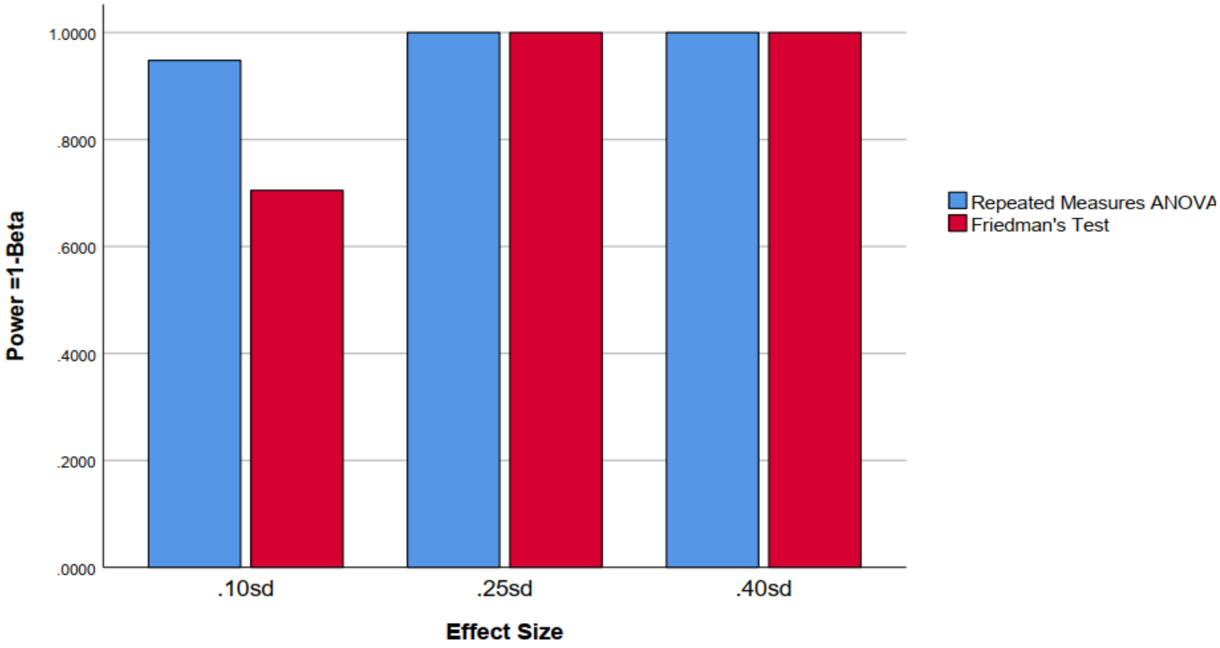
Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3,n4,n5,n6,n7)=25 and Alpha =.05 (Bar Graph



Comparative Power rates for the Repeated Measures ANOVA and Friedman's Test for (n1,n2,n3)=33 and Alpha =.05 (Bar Graph)

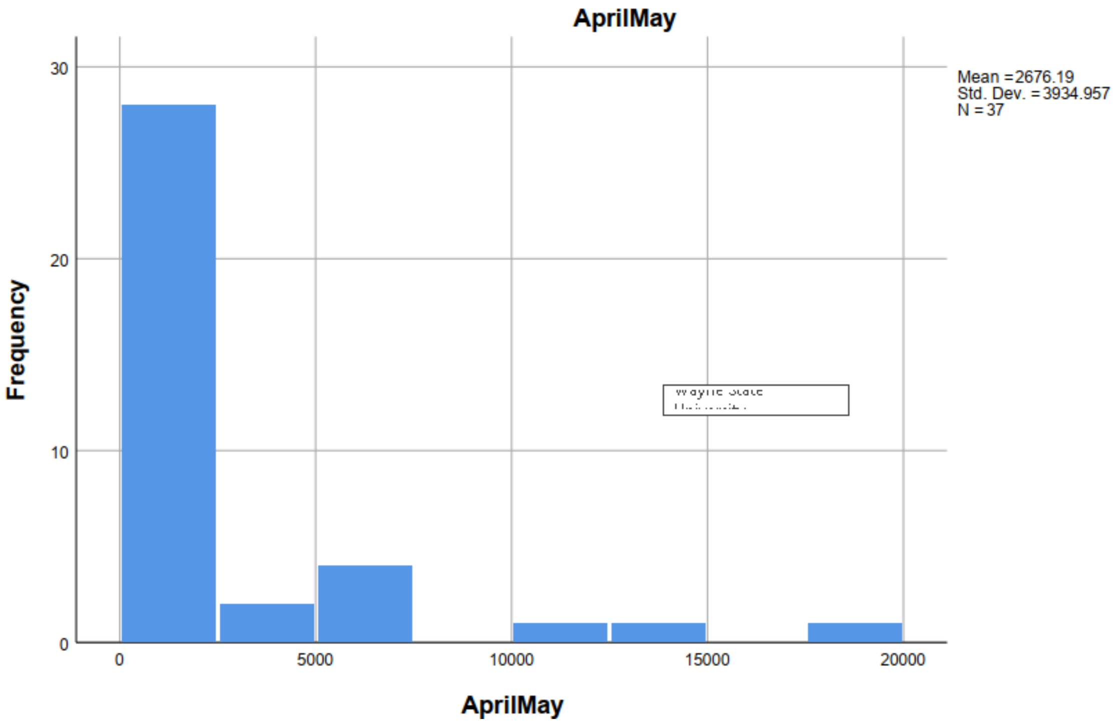


Comparative Power rates for the Repeated Measures ANOVA & Friedman's Test for (n1,n2,n3,n4,n5)=33 and Alpha =.05 (Bar Graph)

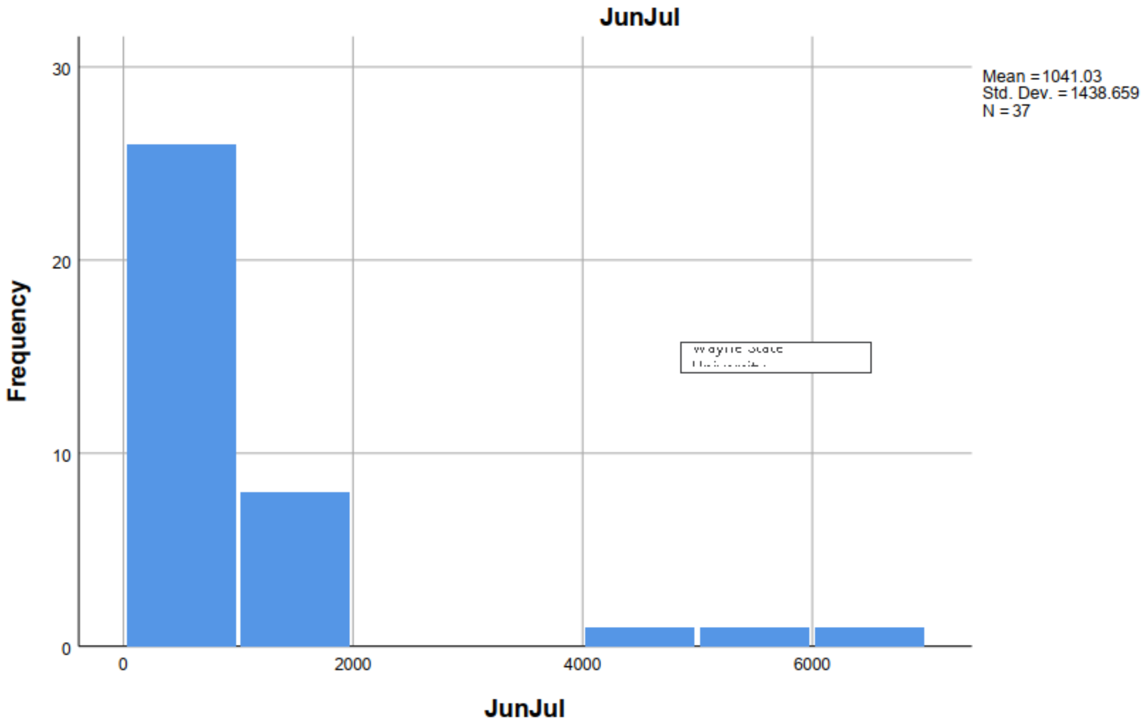


Comparative Power rates for the Repeated Measures ANOVA & Friedman's Test for (n1,n2,n3,n4,n5,n6,n7)=33 and Alpha =.05 (Bar Graph)

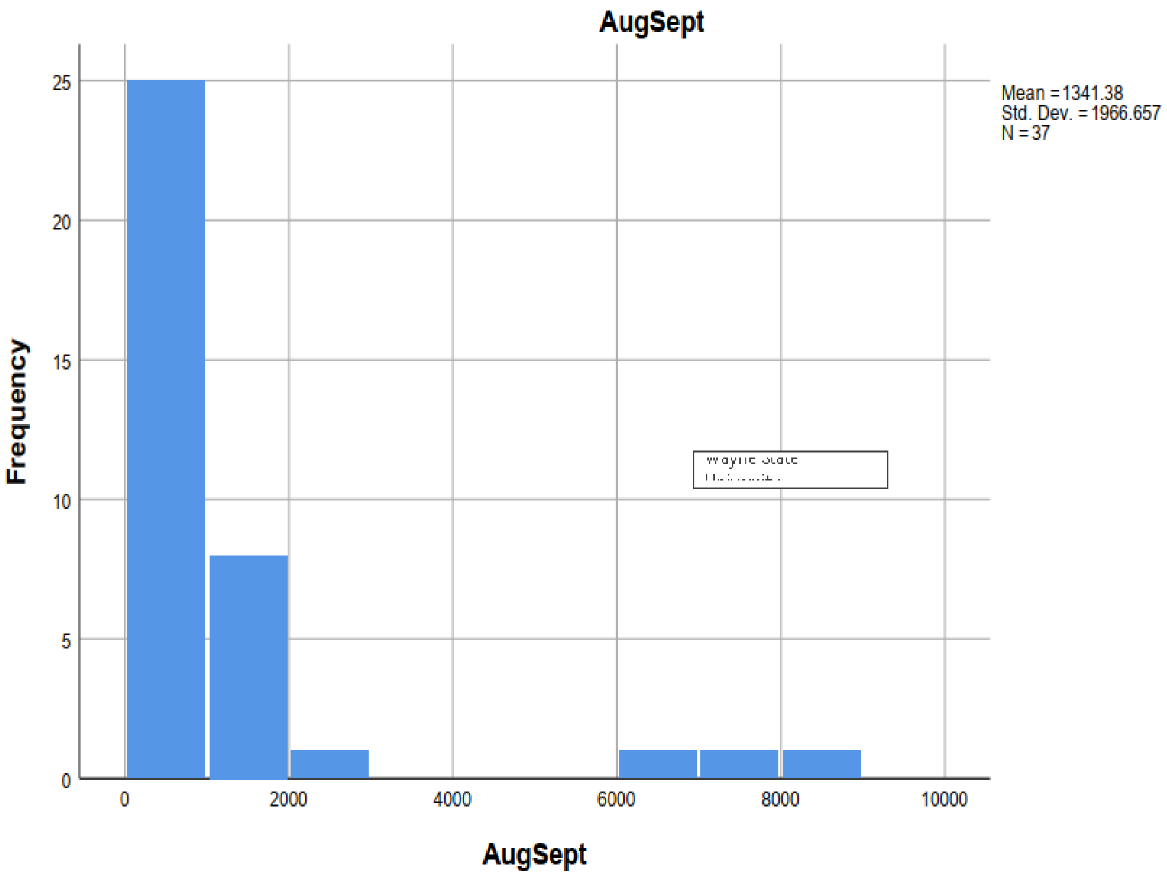
APPENDIX B



Histogram displaying the distribution of the April/May 2020 mortality counts

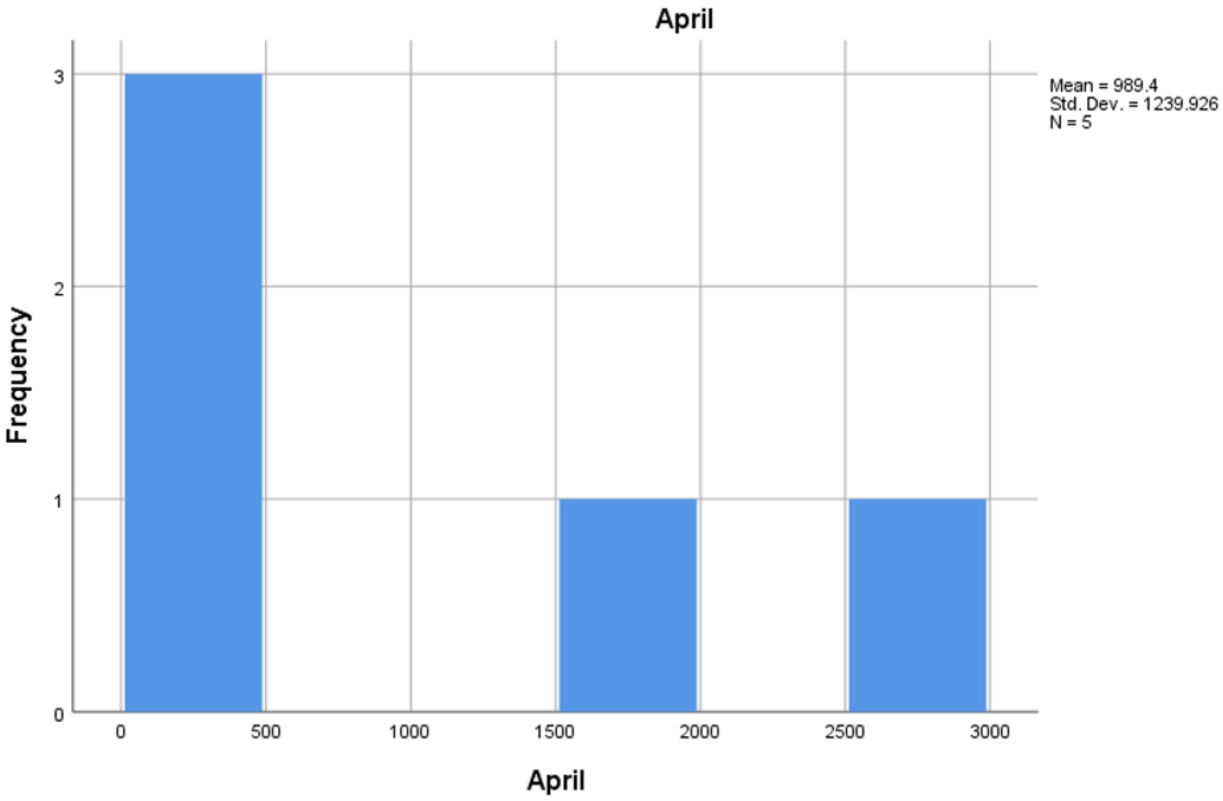


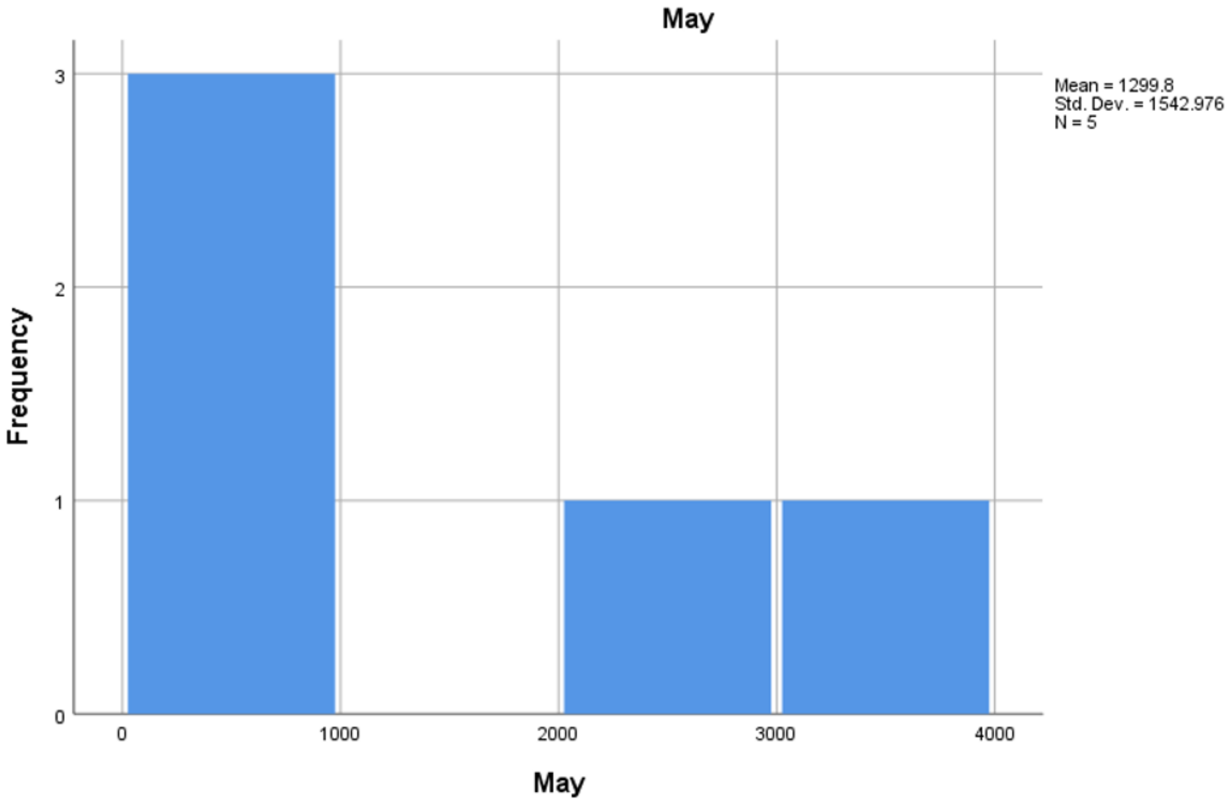
Histogram displaying the distribution of the June/July 2020 mortality counts

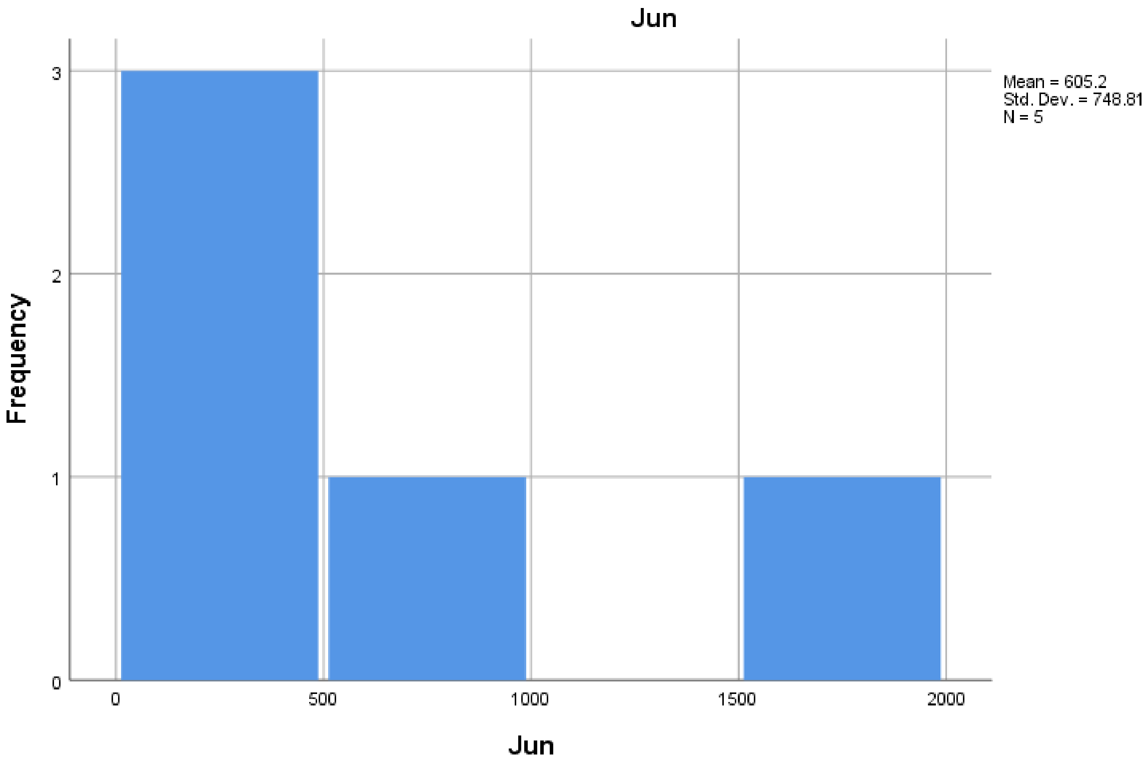


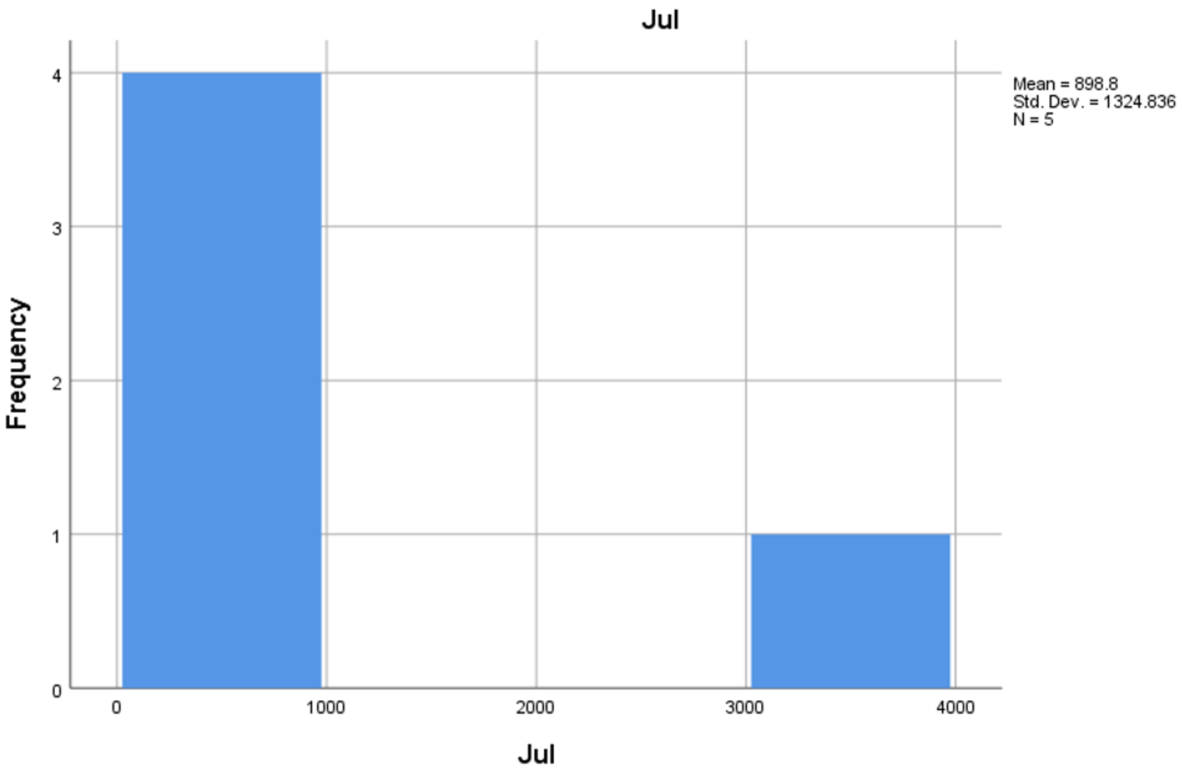
Histogram displaying the distribution of the Aug/Sept 2020 mortality counts

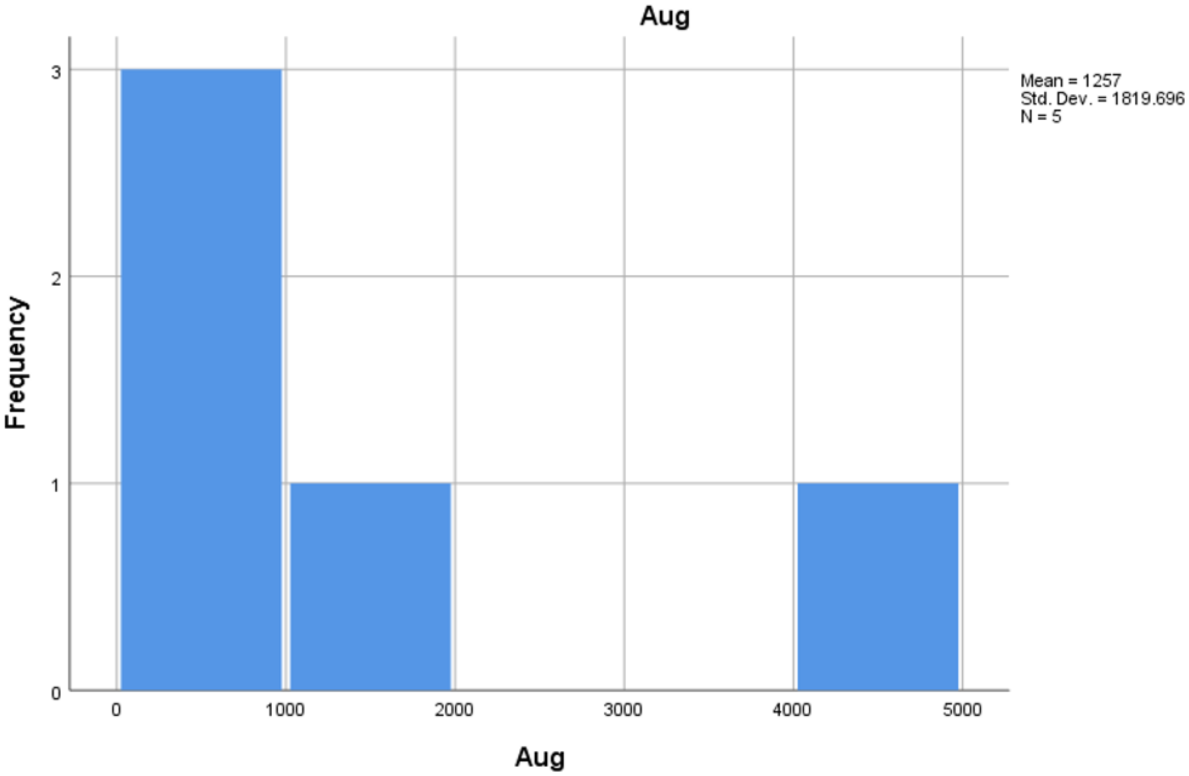
APPENDIX C

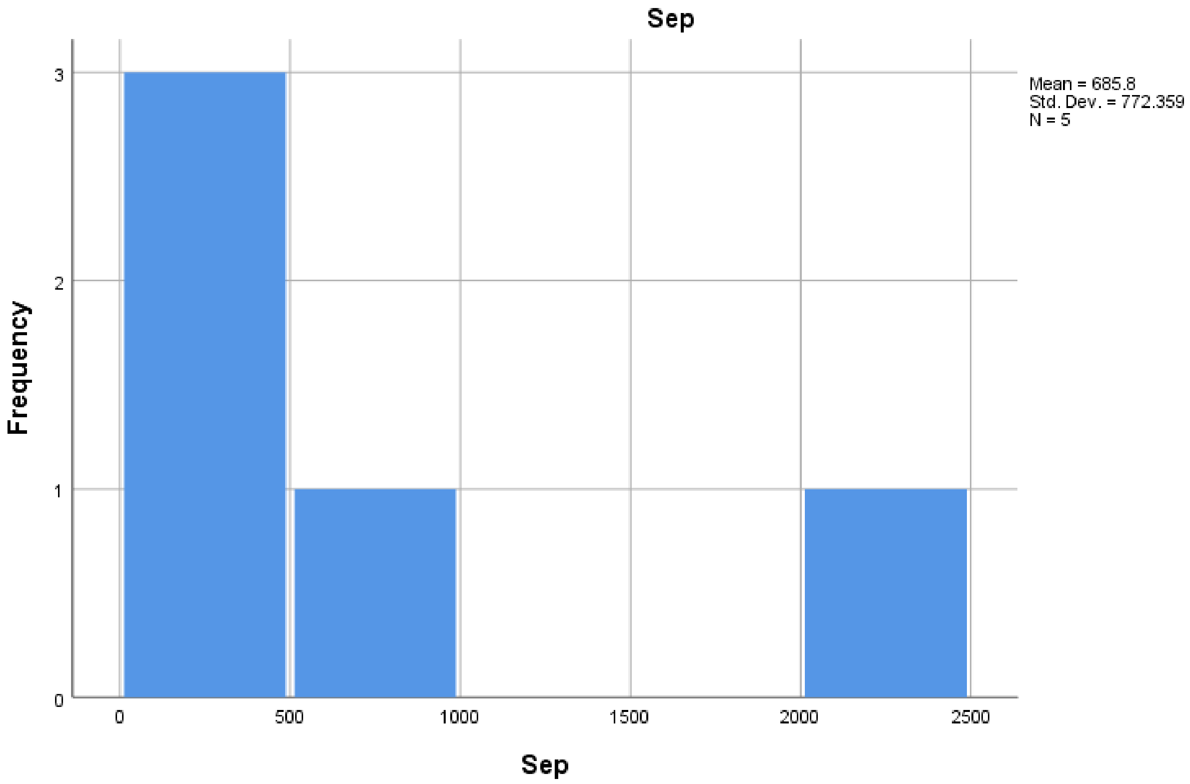

April
Mean = 989.4
Std. Dev. = 1239.926
N = 5
Frequency
0
1
2
3
0
500
1000
1500
2000
2500
3000
April

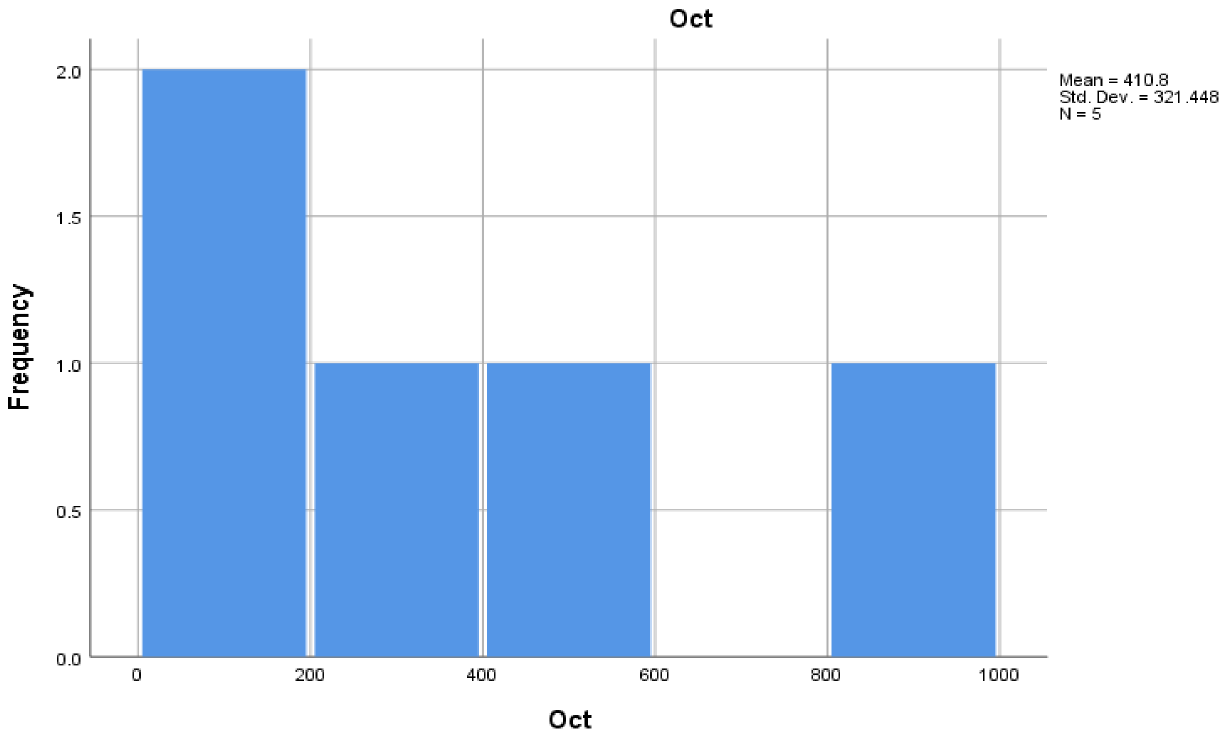

May
Frequency
3
2
1
0
0
1000
2000
3000
4000
May
Mean = 1299.8
Std. Dev. = 1542.976
N = 5


Jun
Frequency
3
2
1
0
0
500
1000
1500
2000
Jun
Mean = 605.2
Std. Dev. = 748.81
N = 5

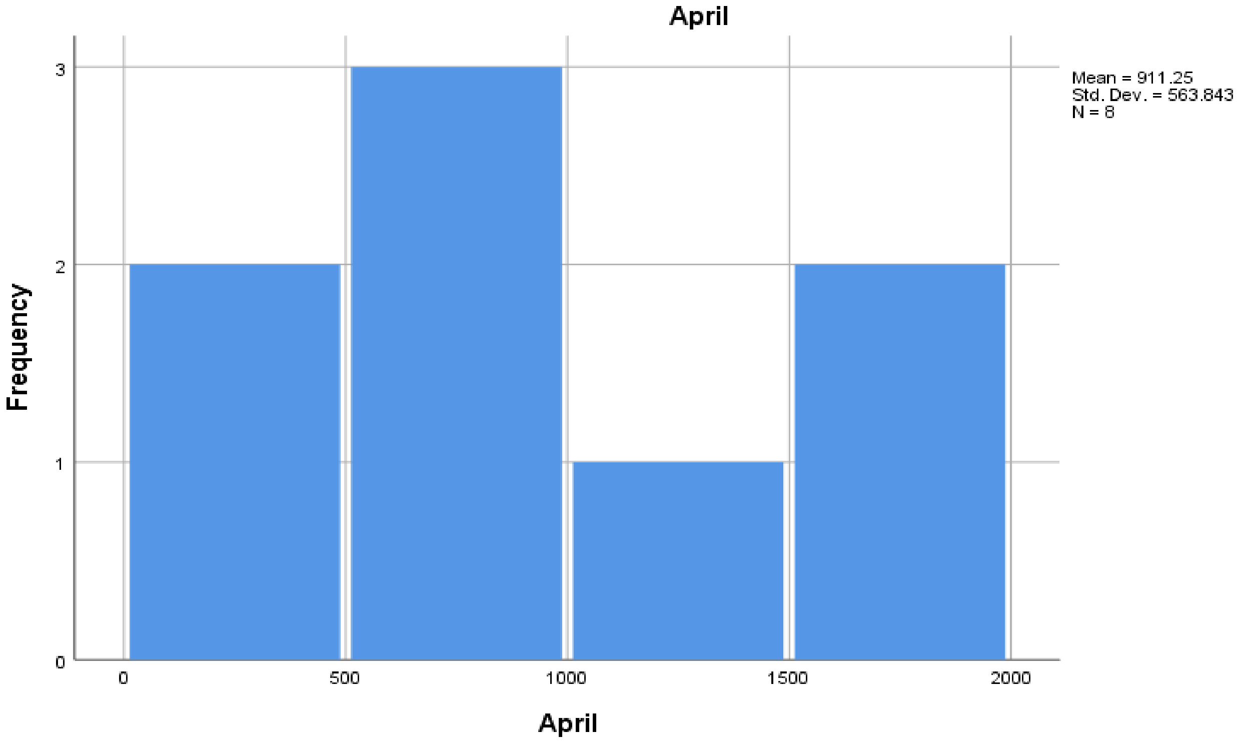

Jul
Mean = 898.8
Std. Dev. = 1324.836
N = 5
Frequency
0
1
2
3
4
0
1000
2000
3000
4000
Jul

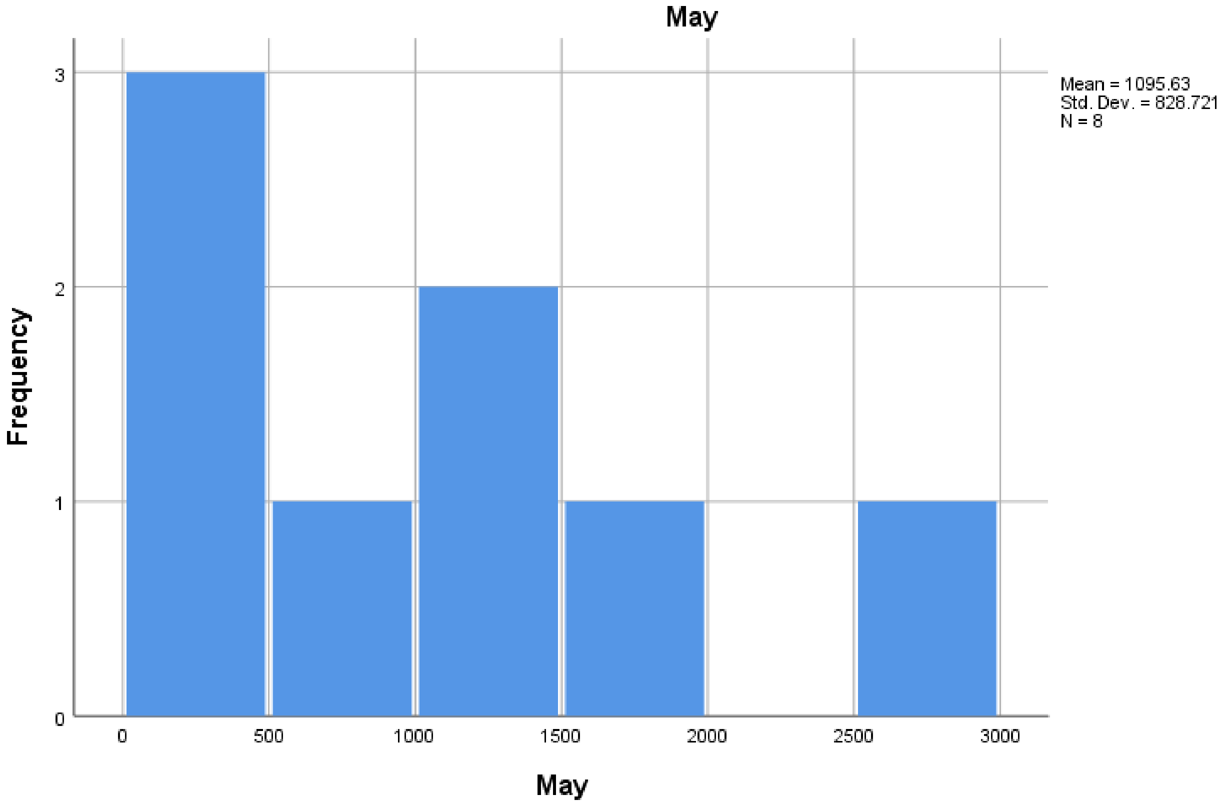

Aug
Mean = 1257
Std. Dev. = 1819.696
N = 5
Frequency
0
1
2
3
0
1000
2000
3000
4000
5000
Aug

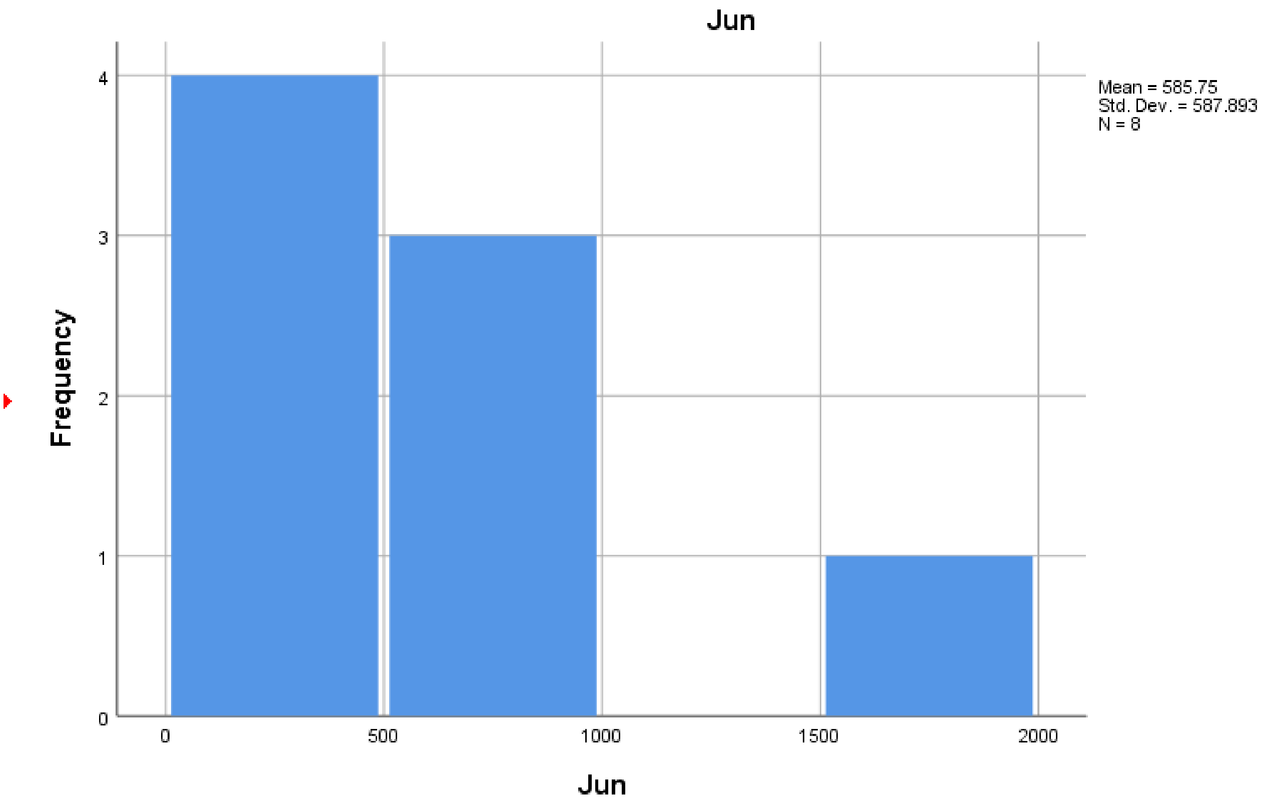

Sep
Frequency
3
2
1
0
0
500
1000
1500
2000
2500
Sep
Mean = 685.8
Std. Dev. = 772.359
N = 5

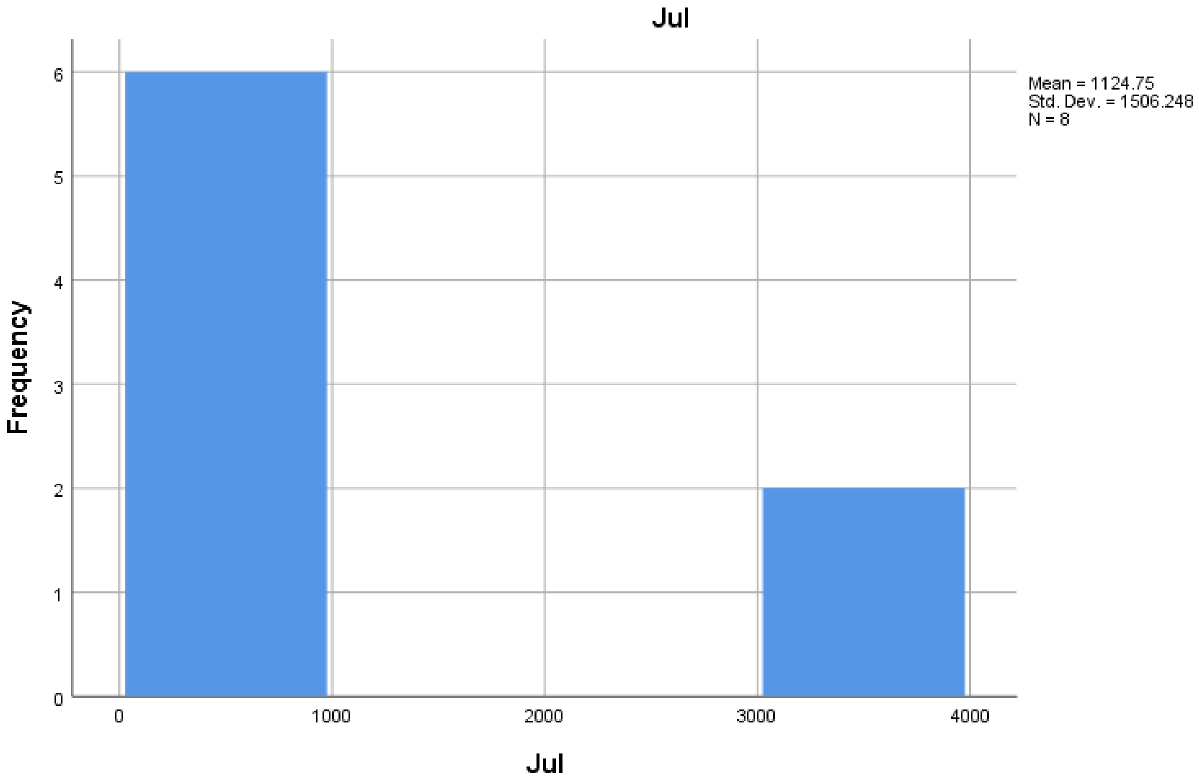

Oct
Frequency
2.0
1.5
1.0
0.5
0.0
0
200
400
600
800
1000
Oct
Mean = 410.8
Std. Dev. = 321.448
N = 5

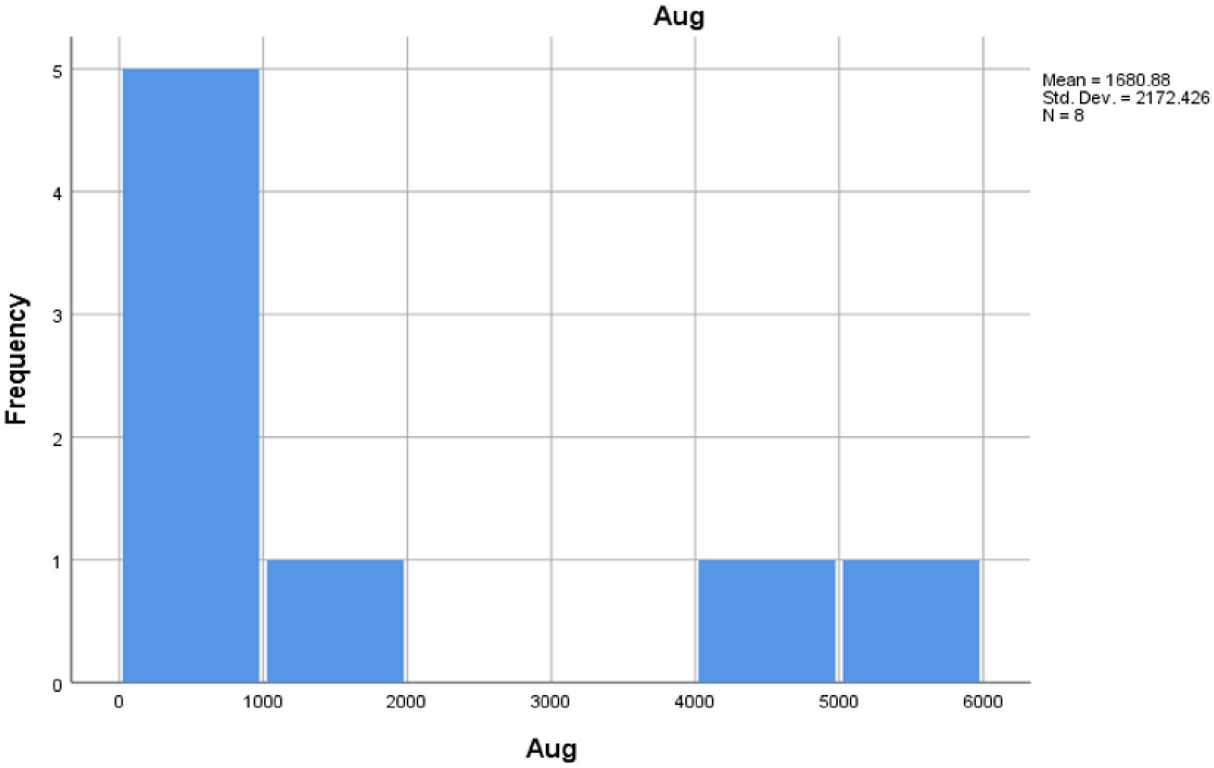
Histogram

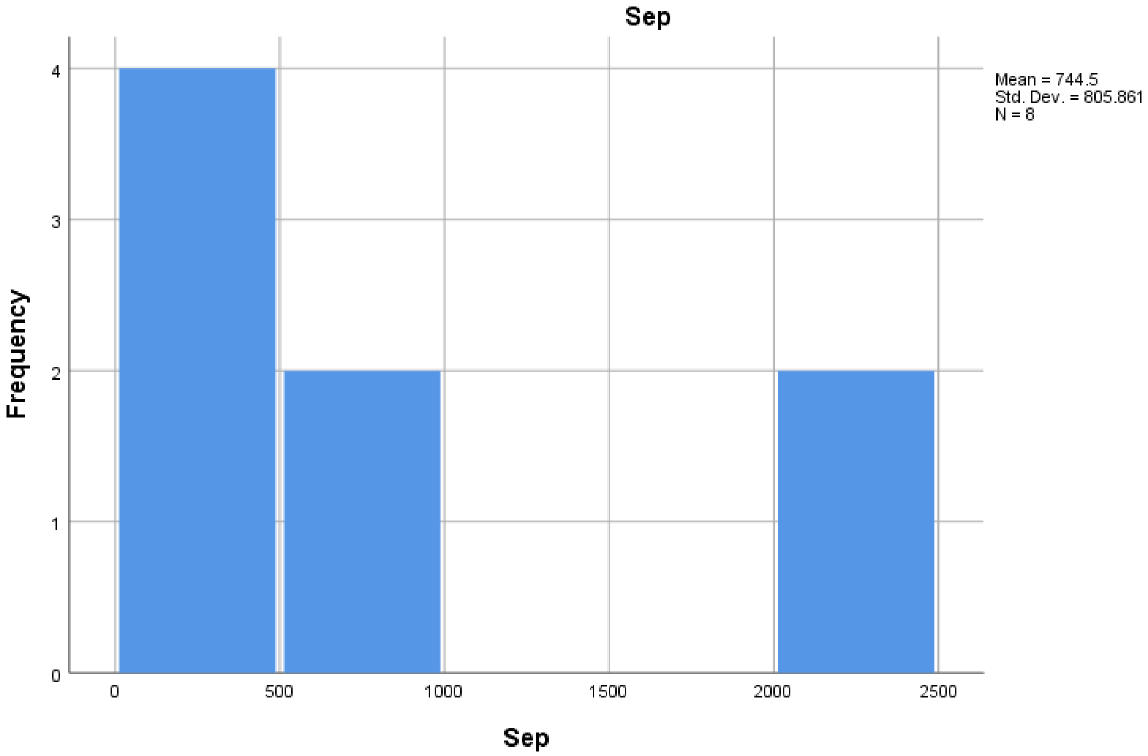

April
Mean = 911.25
Std. Dev. = 563.843
N = 8
Frequency
0
1
2
3
0
500
1000
1500
2000
April

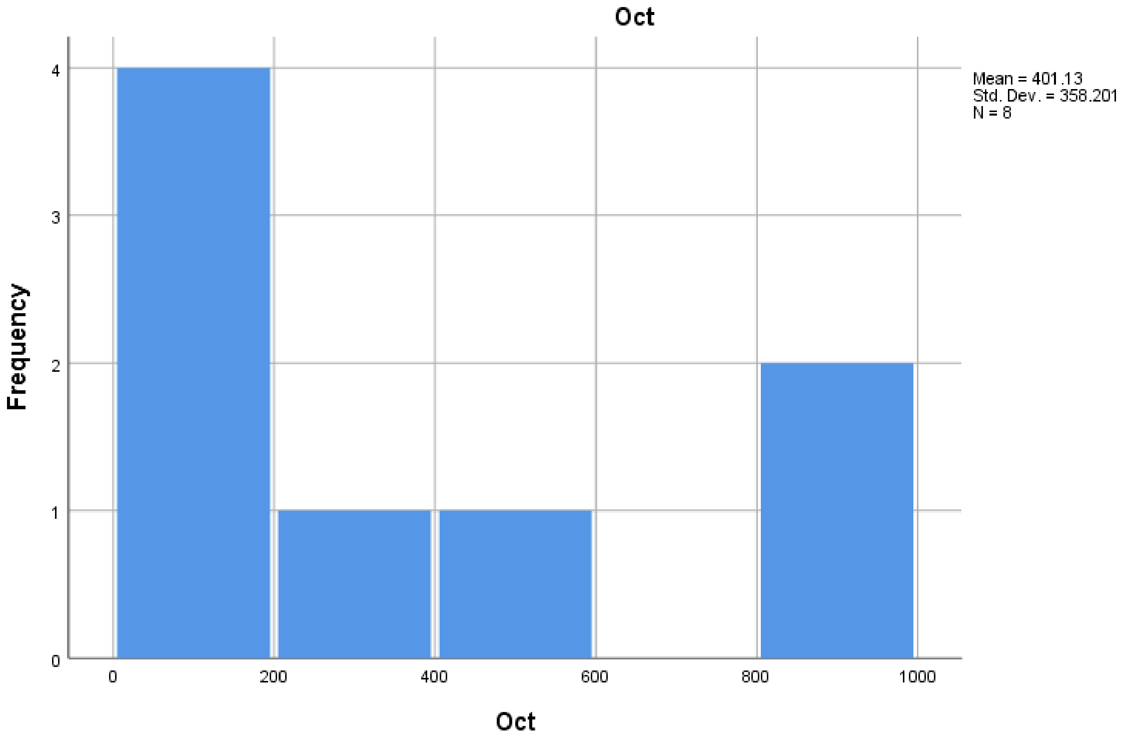

May
Mean = 1095.63
Std. Dev. = 828.721
N = 8
Frequency
3
2
1
0
0
500
1000
1500
2000
2500
3000
May

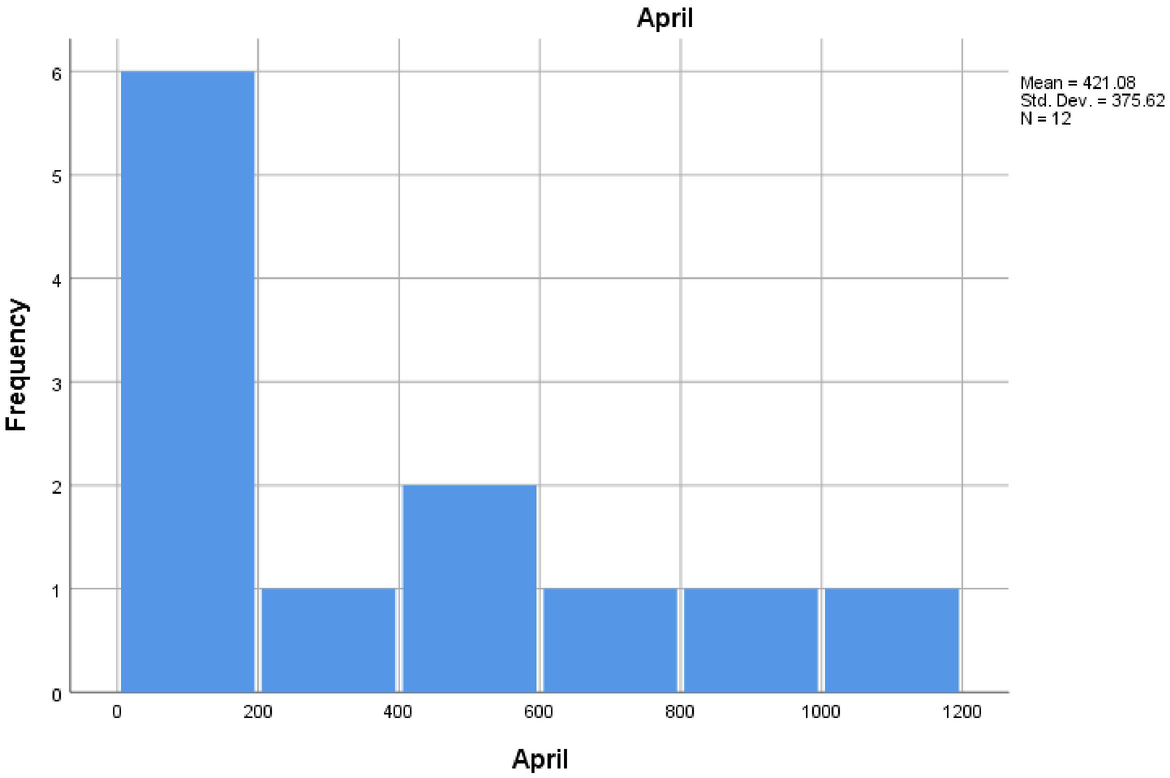

Jun
Mean = 585.75
Std. Dev. = 587.893
N = 8
Frequency
0
1
2
3
4
0
500
1000
1500
2000
Jun

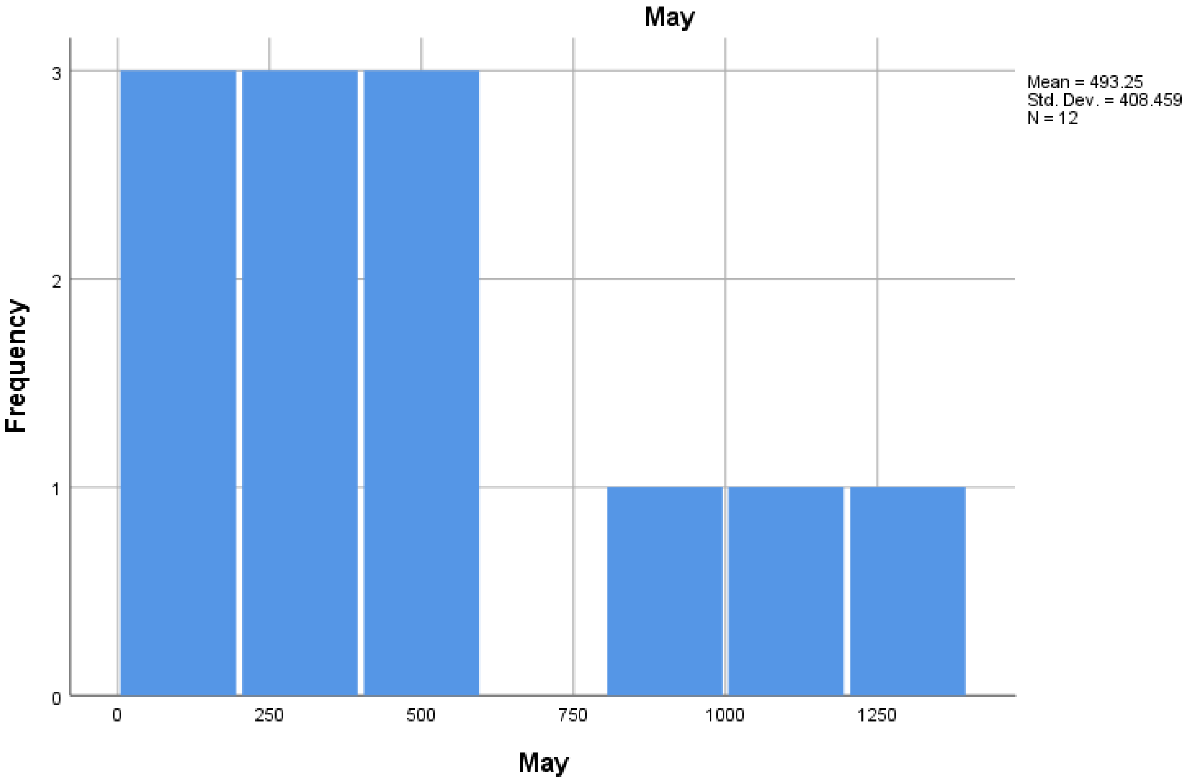

Jul
Frequency
0
1
2
3
4
5
6
0
1000
2000
3000
4000
Jul
Mean = 1124.75
Std. Dev. = 1506.248
N = 8

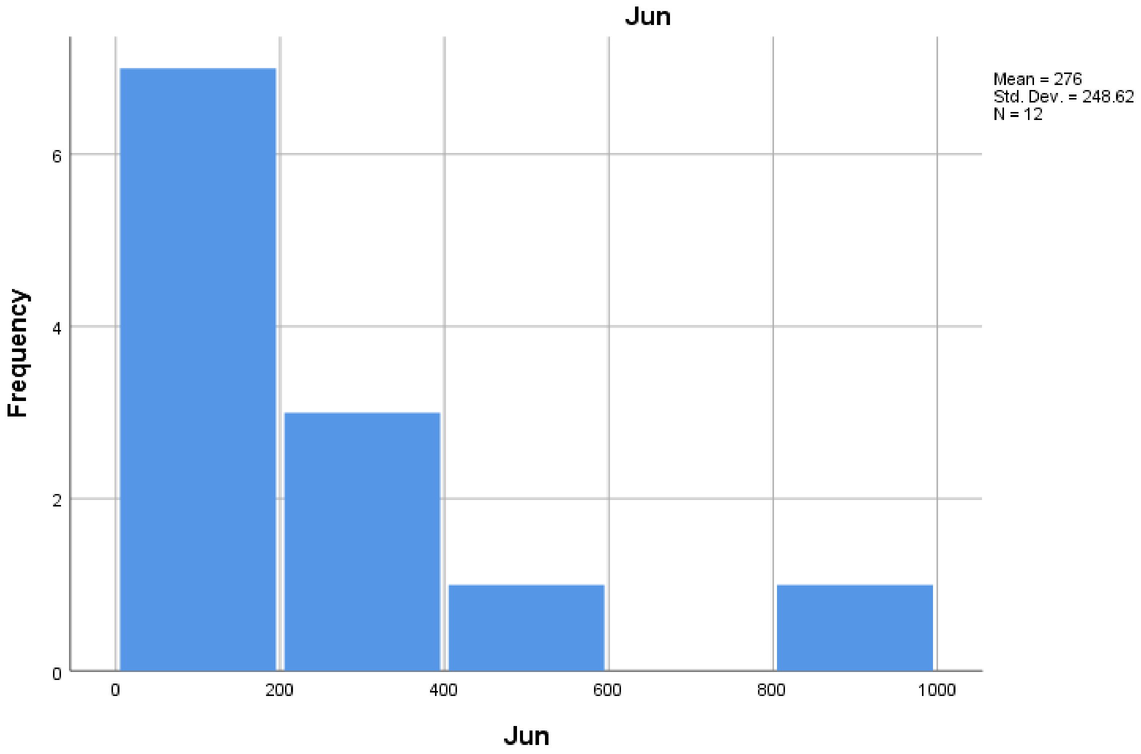

Aug
Frequency
0
1
2
3
4
5
0
1000
2000
3000
4000
5000
6000
Aug
Mean = 1680.88
Std. Dev. = 2172.426
N = 8

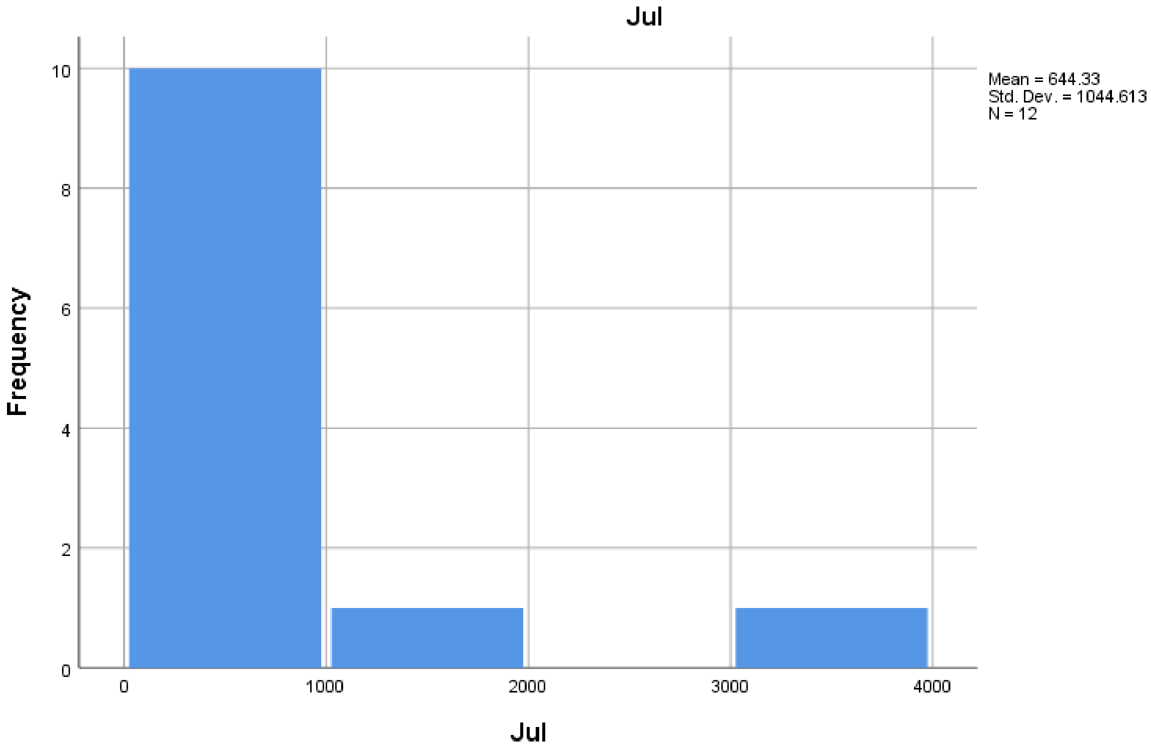

Sep
Frequency
0
1
2
3
4
0
500
1000
1500
2000
2500
Sep
Mean = 744.5
Std. Dev. = 805.861
N = 8

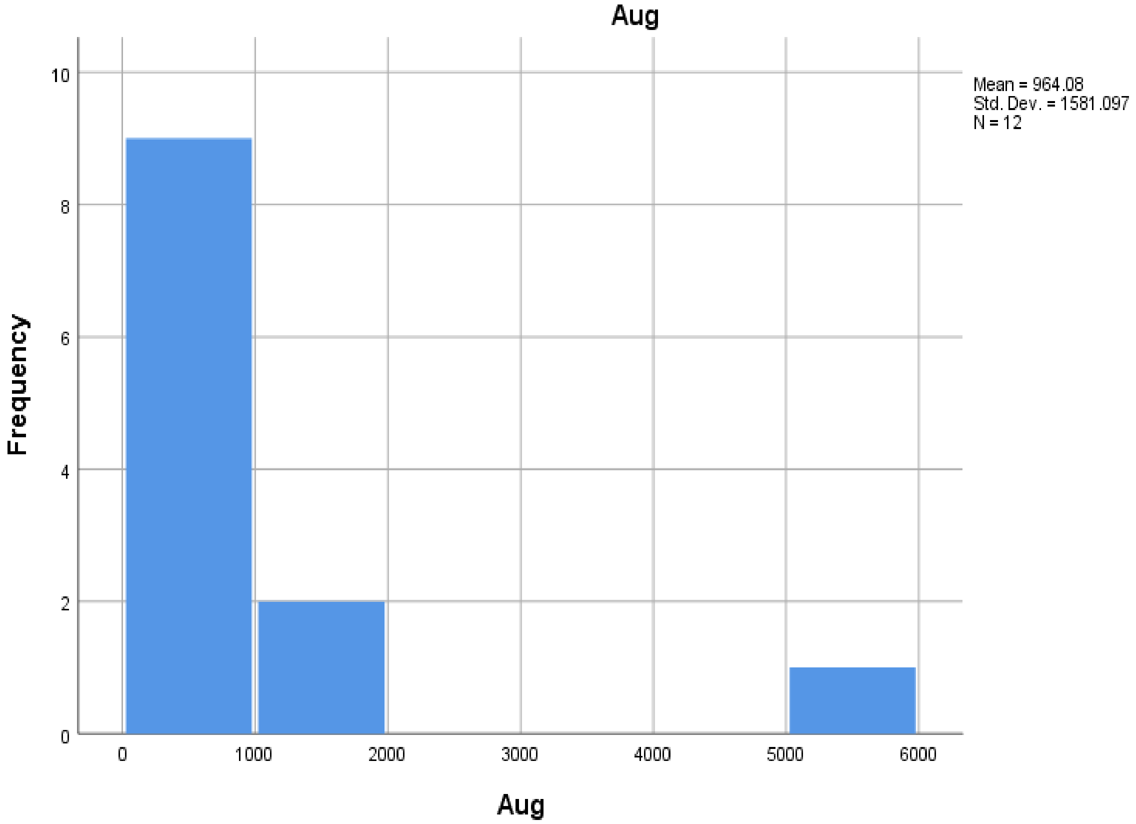

Oct
Frequency
0
1
2
3
4
0
200
400
600
800
1000
Oct
Mean = 401.13
Std. Dev. = 358.201
N = 8

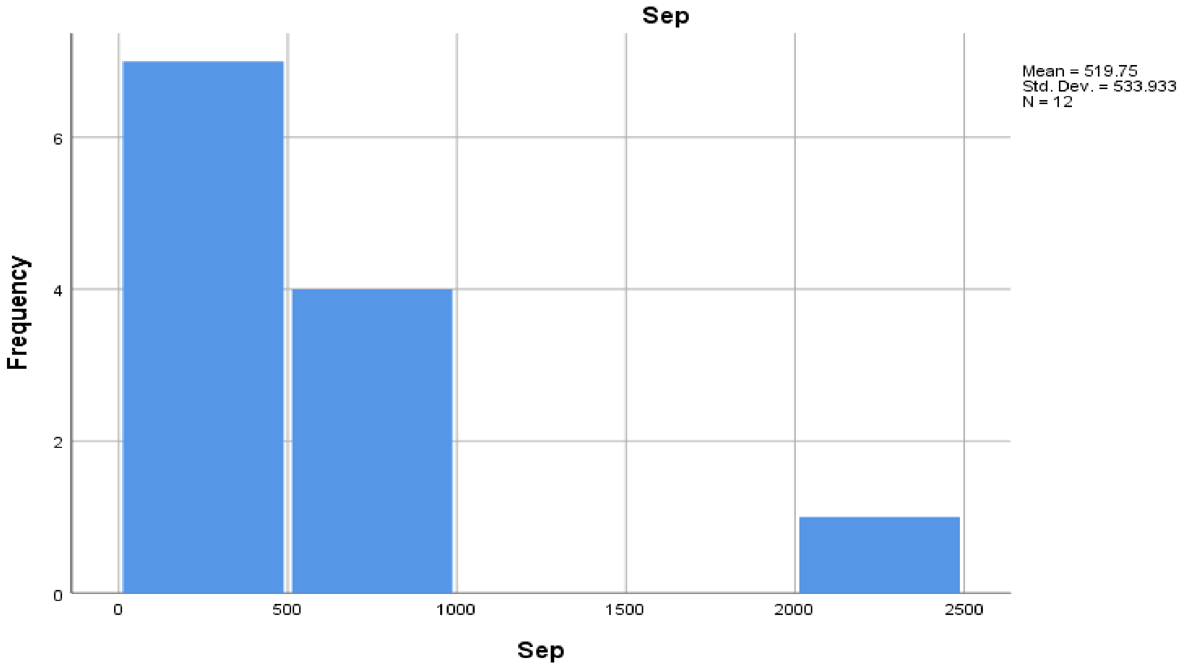

April
Mean = 421.08
Std. Dev. = 375.62
N = 12
Frequency
6
5
4
3
2
1
0
0
200
400
600
800
1000
1200
April

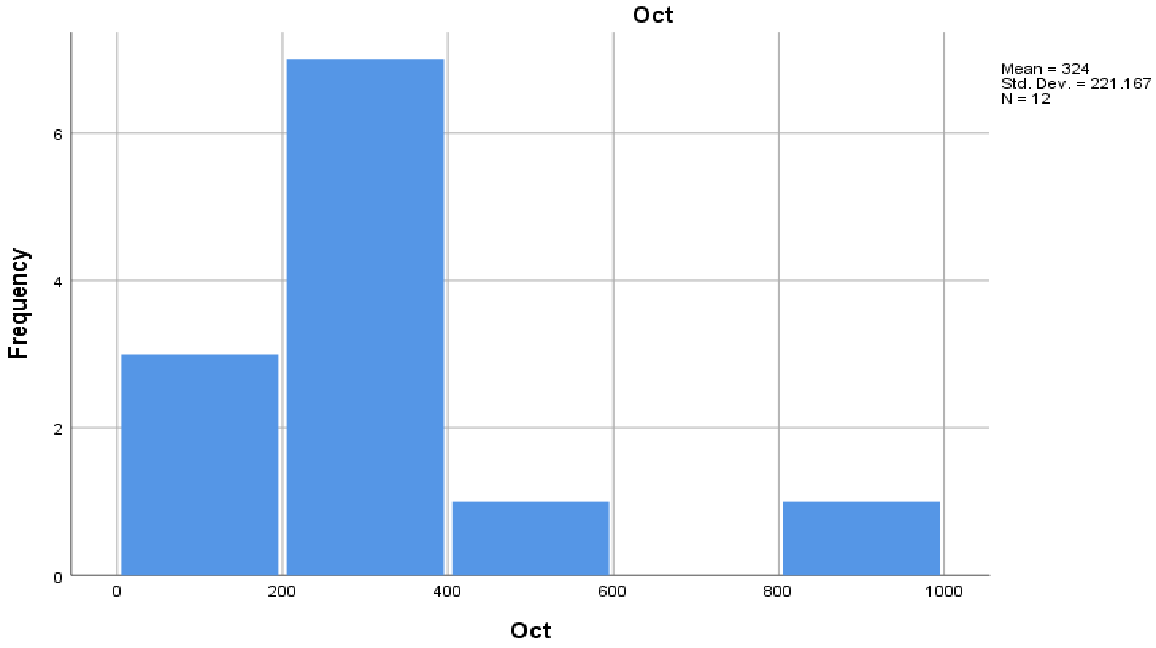

May
Mean = 493.25
Std. Dev. = 408.459
N = 12
Frequency
3
2
1
0
0
250
500
750
1000
1250
May

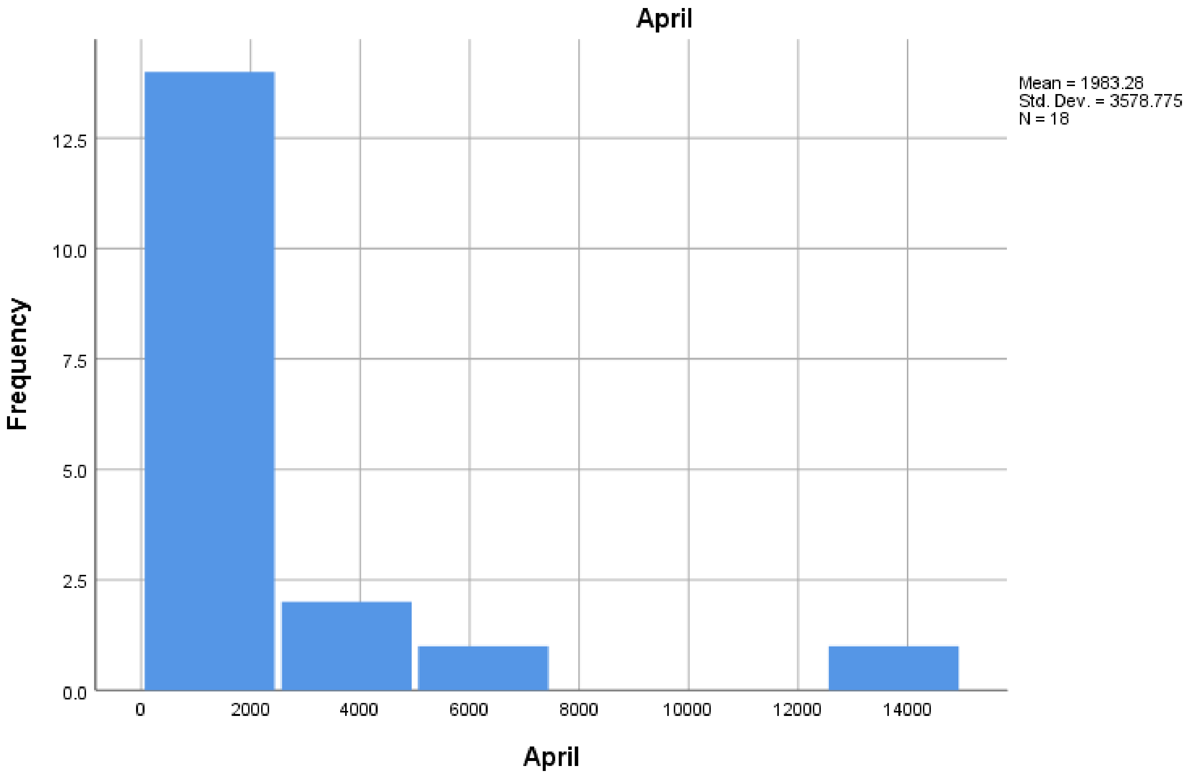

Jun
Frequency
0
2
4
6
0
200
400
600
800
1000
Jun
Mean = 276
Std. Dev. = 248.62
N = 12

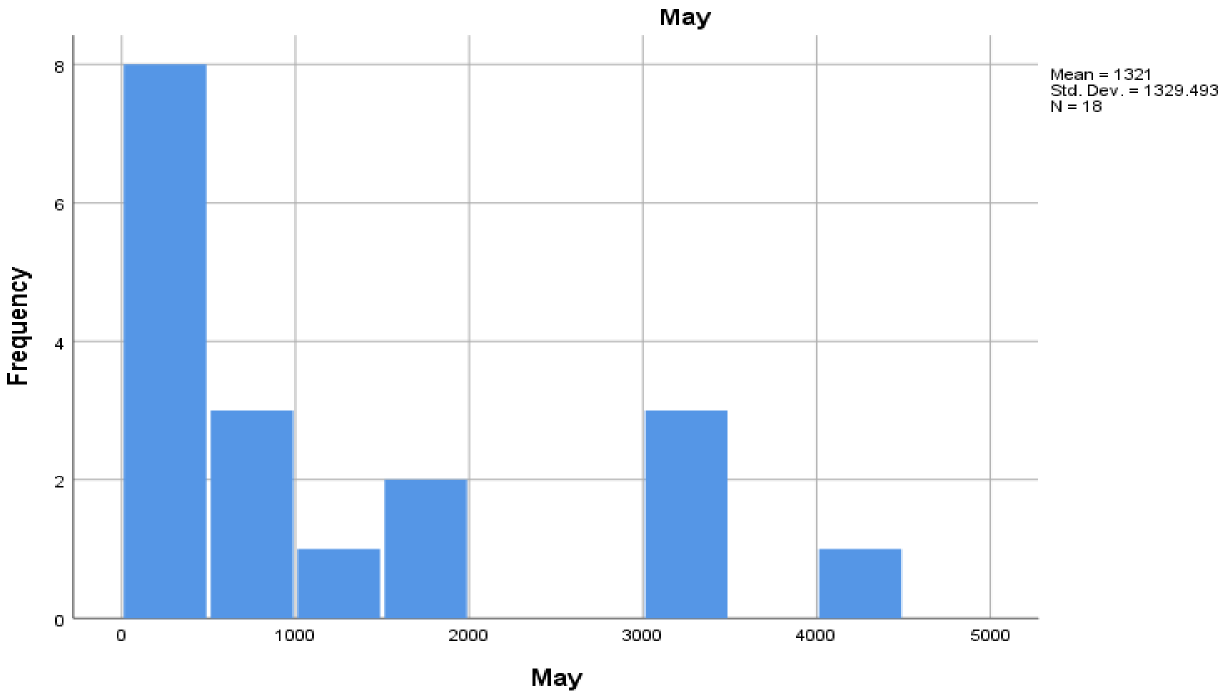

Jul
Frequency
0
2
4
6
8
10
0
1000
2000
3000
4000
Jul
Mean = 644.33
Std. Dev. = 1044.613
N = 12

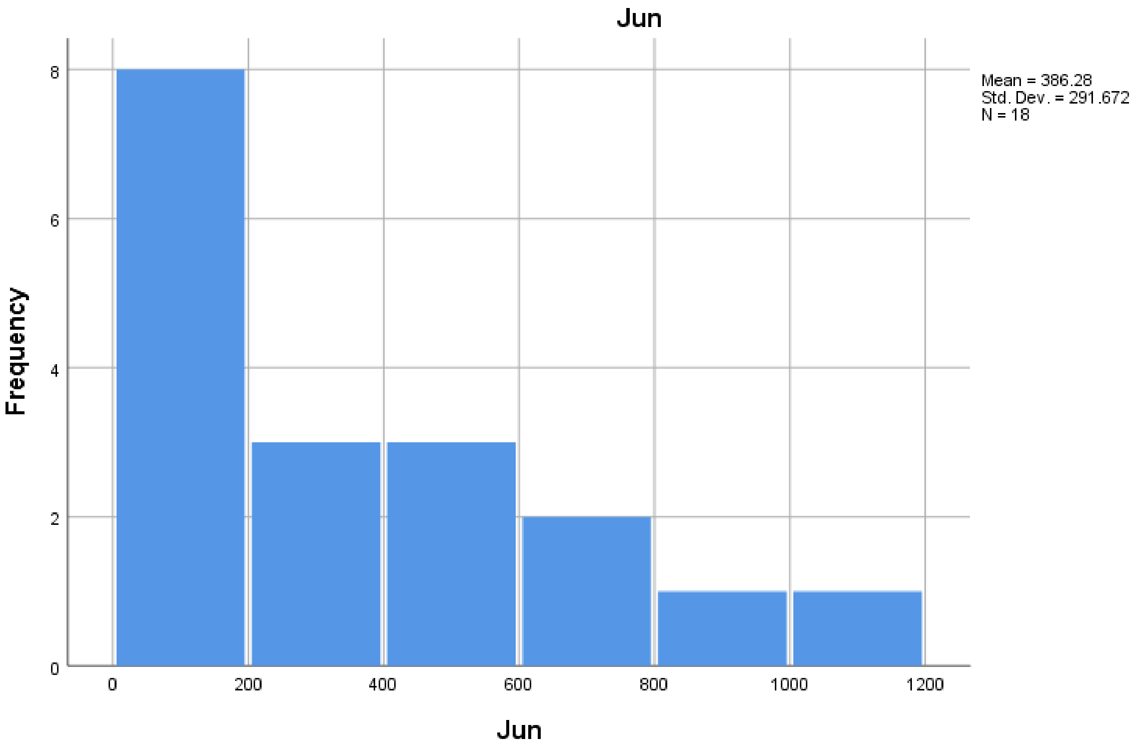

Aug
Frequency
10
8
6
4
2
0
0
1000
2000
3000
4000
5000
6000
Aug
Mean = 964.08
Std. Dev. = 1581.097
N = 12

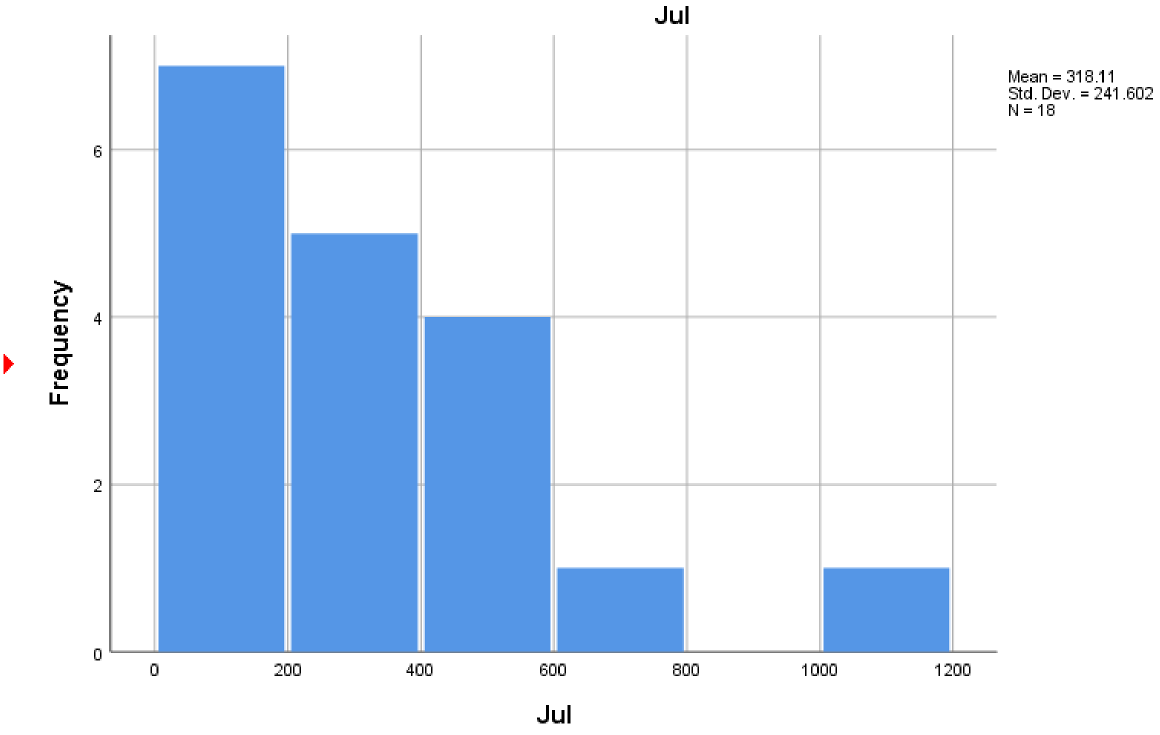

Sep
Frequency
6
4
2
0
0
500
1000
1500
2000
2500
Sep
Mean = 519.75
Std. Dev. = 533.933
N = 12

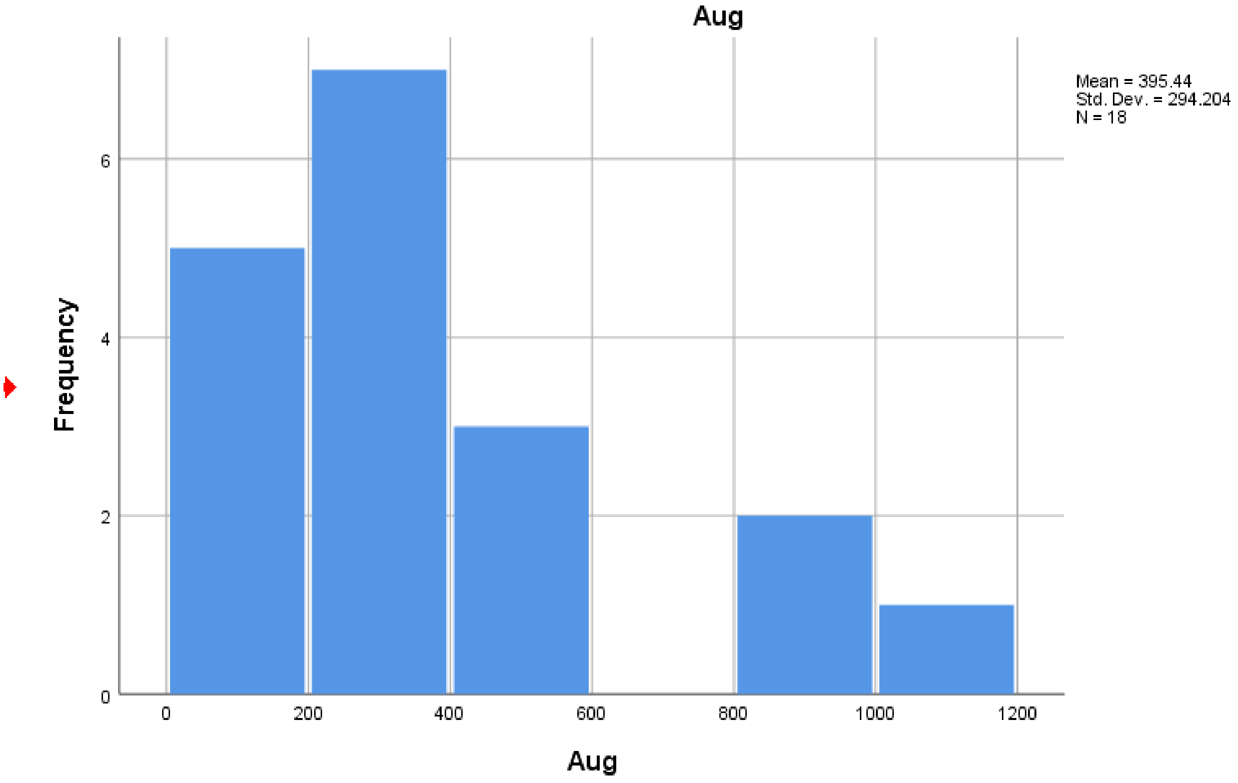

Oct
Mean = 324
Std. Dev. = 221.167
N = 12
Frequency
0
2
4
6
0
200
400
600
800
1000
Oct

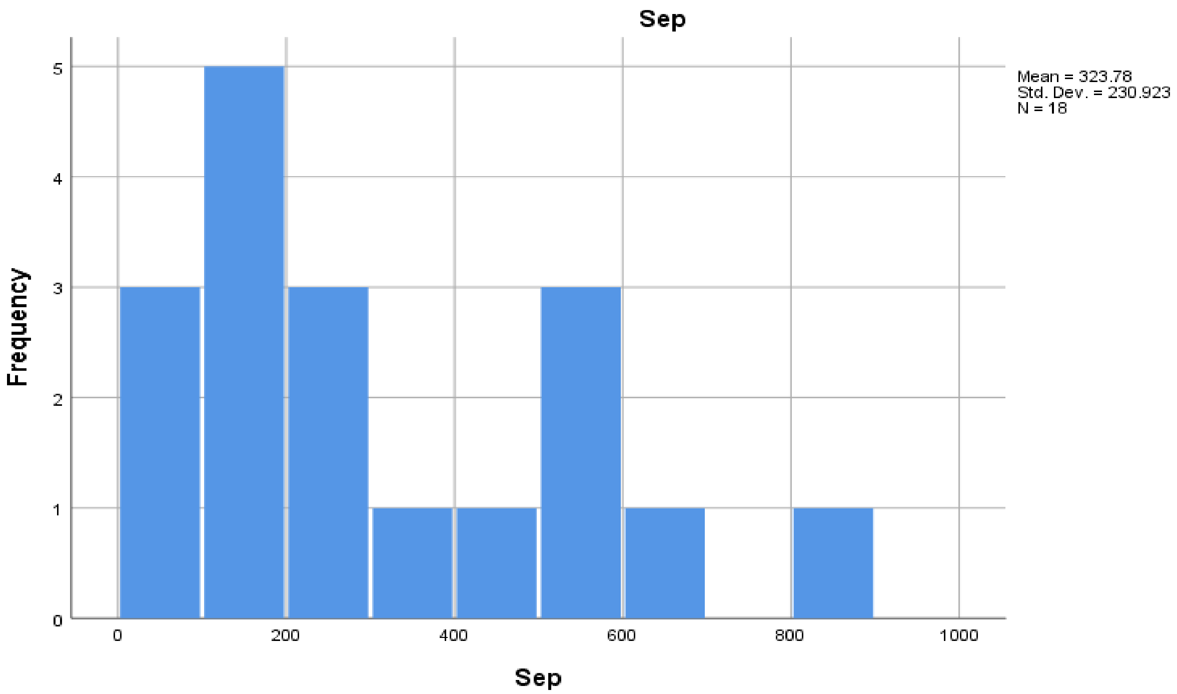

April
Mean = 1983.28
Std. Dev. = 3578.775
N = 18
Frequency
0.0
2.5
5.0
7.5
10.0
12.5
0
2000
4000
6000
8000
10000
12000
14000
April

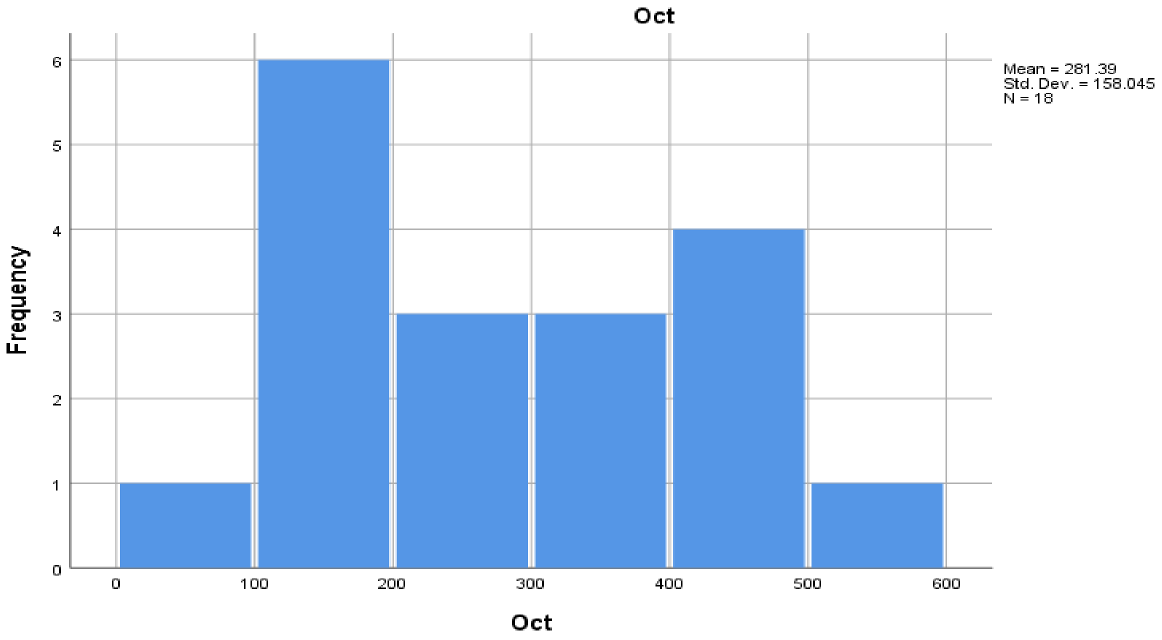

May
Mean = 1321
Std. Dev. = 1329.493
N = 18
Frequency
0
2
4
6
8
0
1000
2000
3000
4000
5000
May

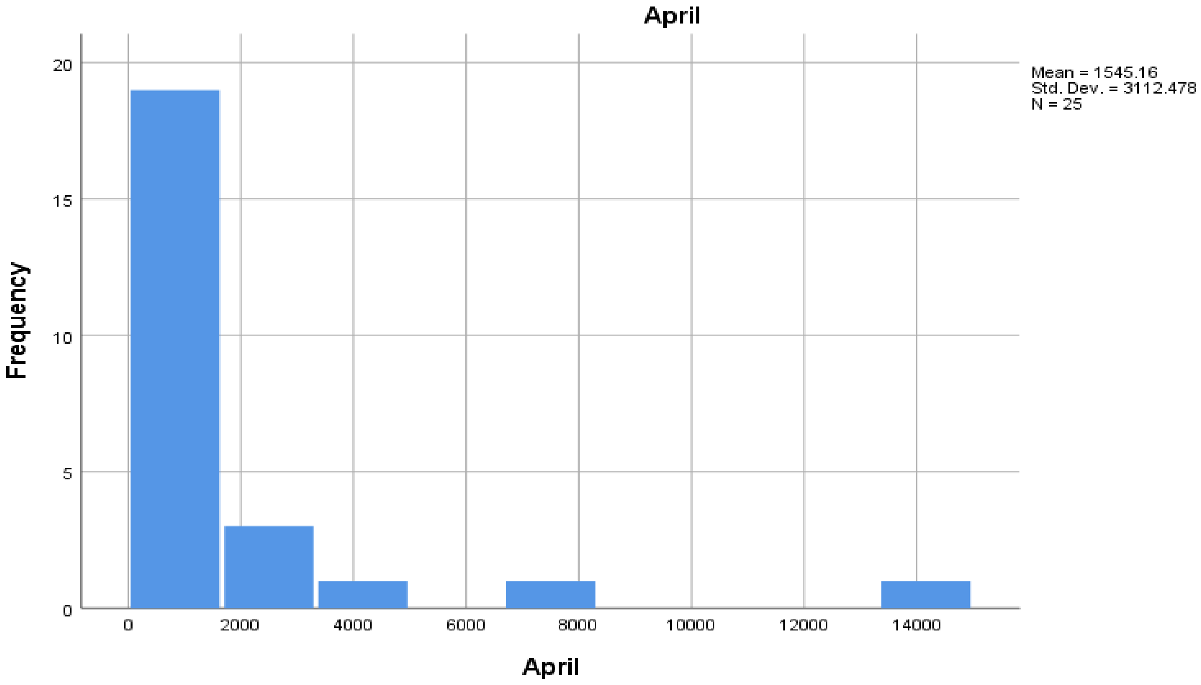

Jun
Mean = 386.28
Std. Dev. = 291.672
N = 18
Frequency
0
2
4
6
8
0
200
400
600
800
1000
1200
Jun

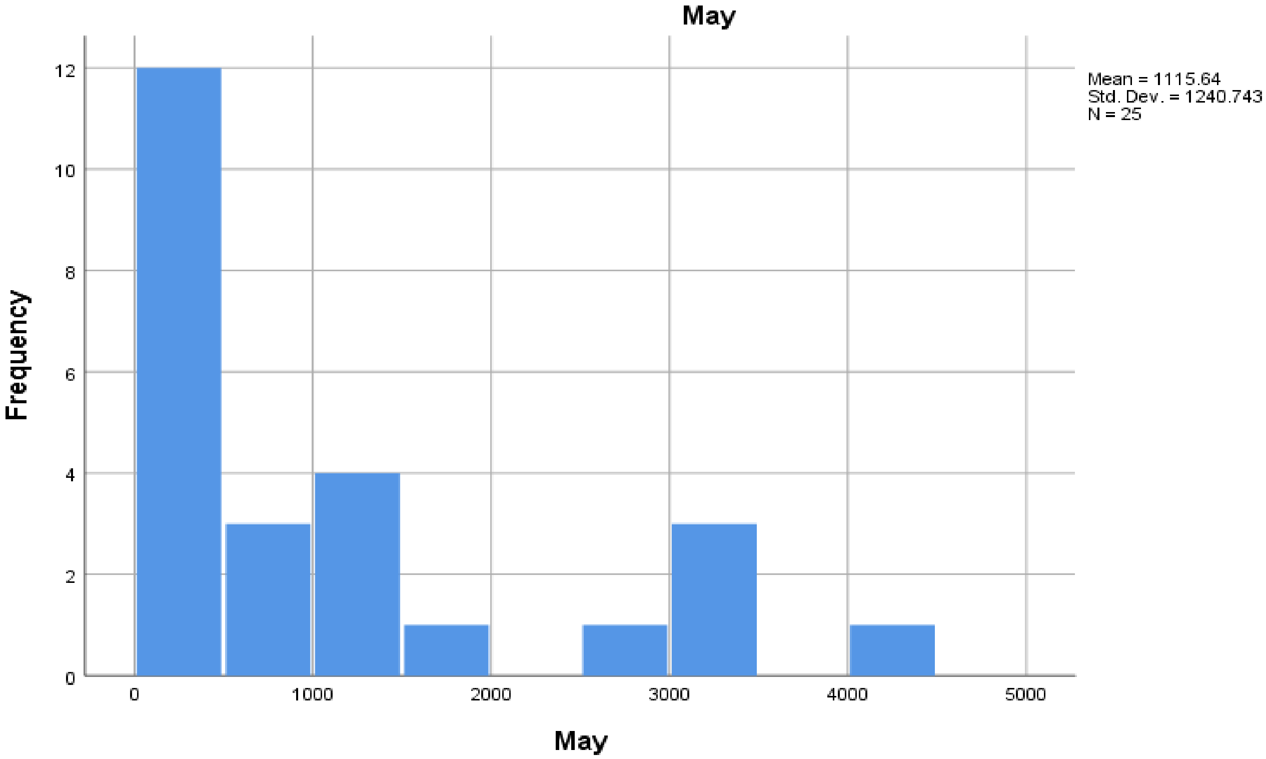

Jul
Mean = 318.11
Std. Dev. = 241.602
N = 18
Frequency
0
2
4
6
0
200
400
600
800
1000
1200
Jul

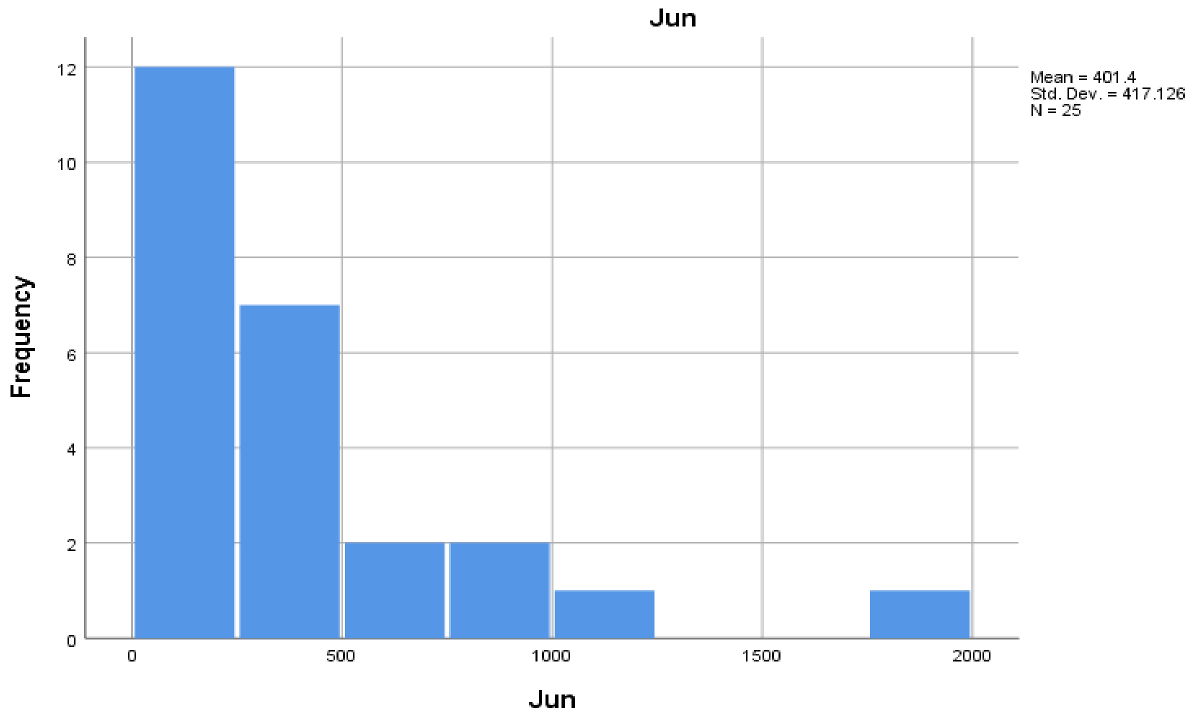

Aug
Mean = 395.44
Std. Dev. = 294.204
N = 18
Frequency
0
2
4
6
0
200
400
600
800
1000
1200
Aug

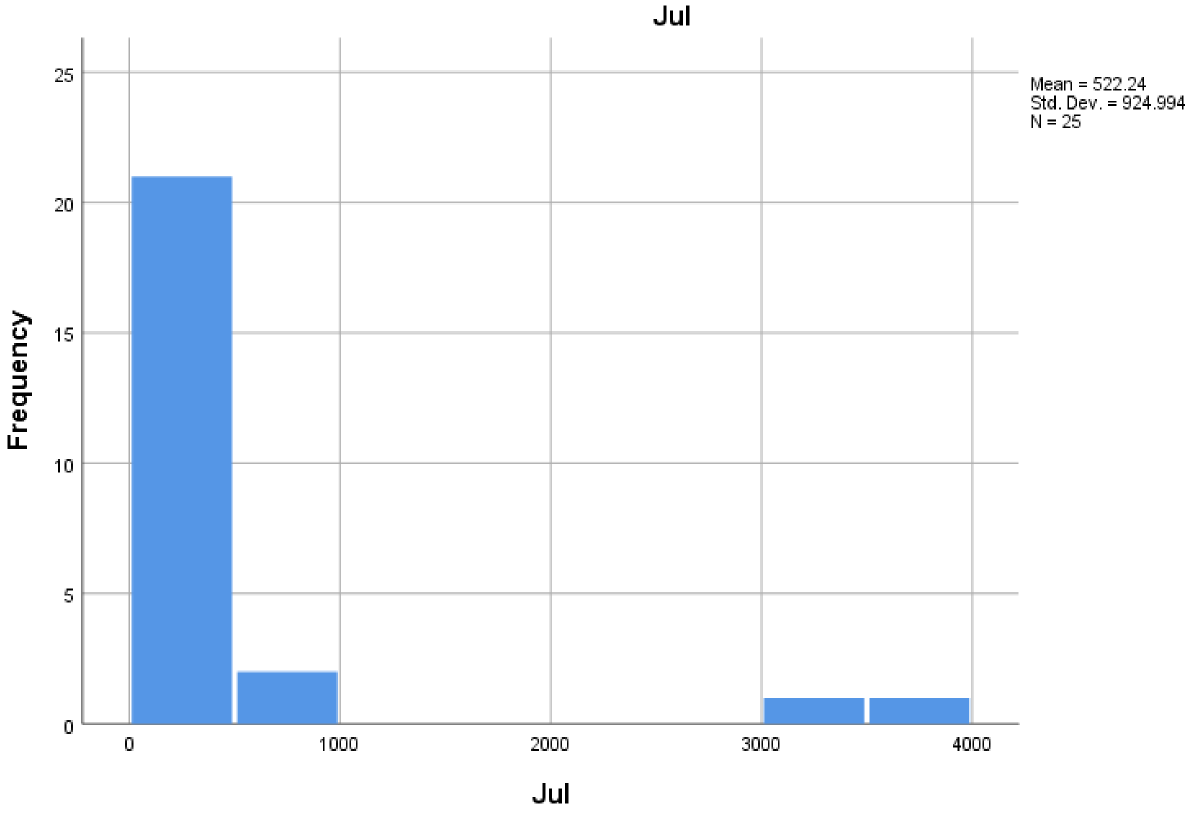

Sep
Mean = 323.78
Std. Dev. = 230.923
N = 18
Frequency
0
1
2
3
4
5
0
200
400
600
800
1000
Sep

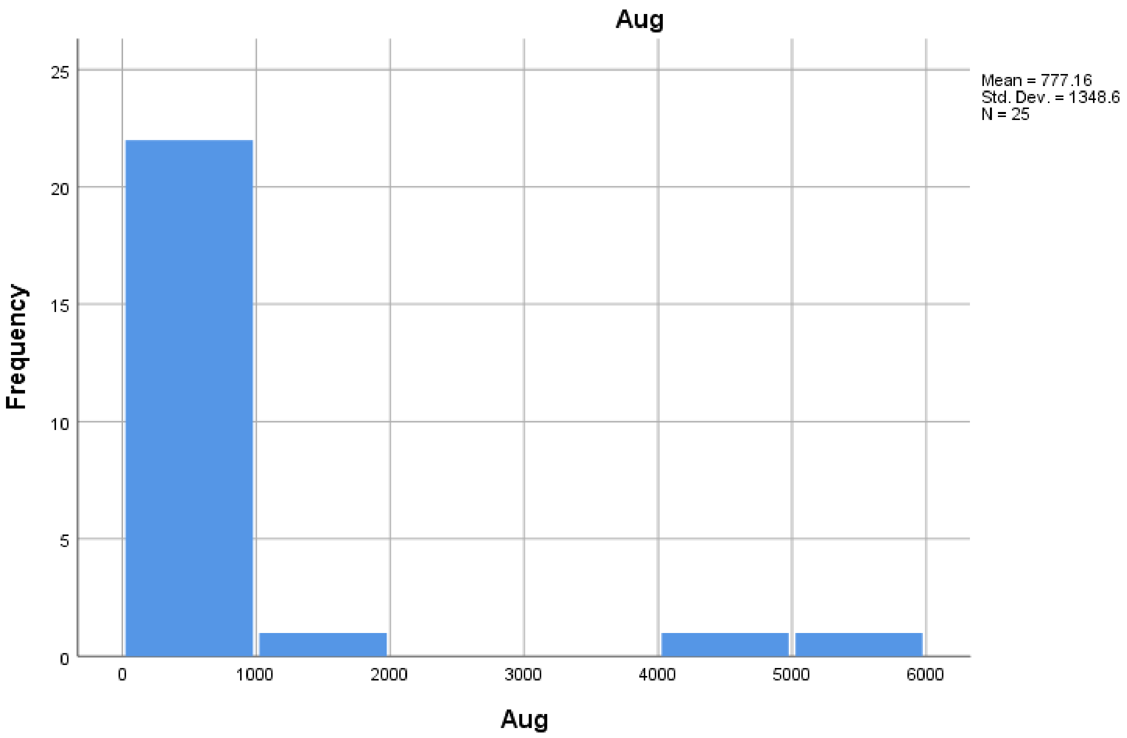

Oct
Mean = 281.39
Std. Dev. = 158.045
N = 18
Frequency
0
1
2
3
4
5
6
0
100
200
300
400
500
600
Oct

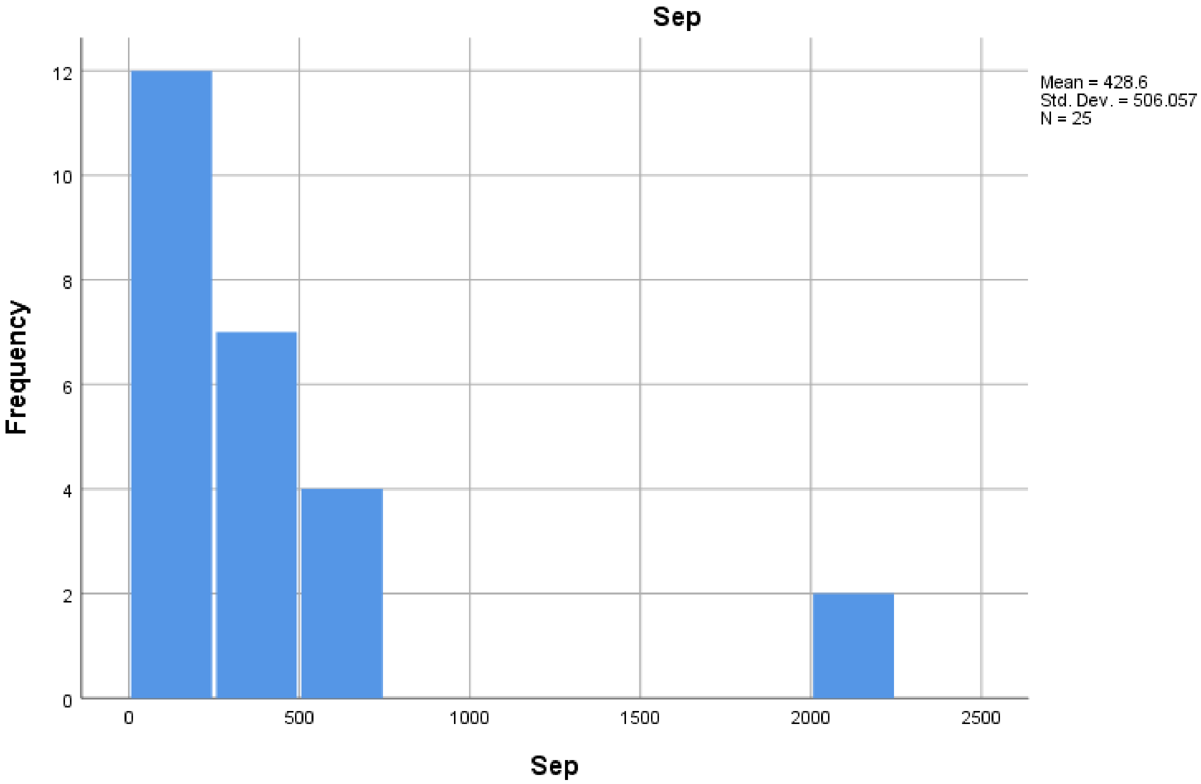

April
Mean = 1545.16
Std. Dev. = 3112.478
N = 25
Frequency
0
5
10
15
20
0
2000
4000
6000
8000
10000
12000
14000
April

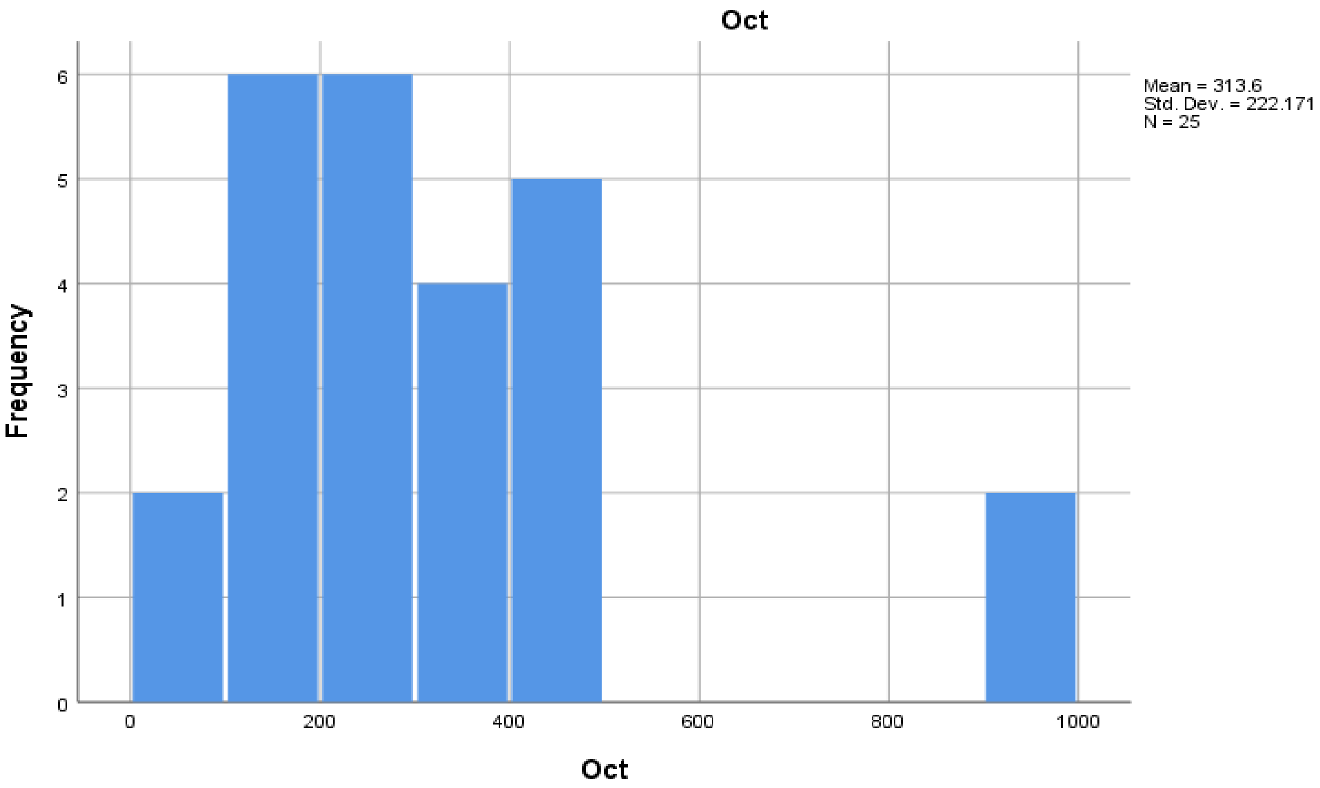

May
Mean = 1115.64
Std. Dev. = 1240.743
N = 25
Frequency
0
2
4
6
8
10
12
0
1000
2000
3000
4000
5000
May

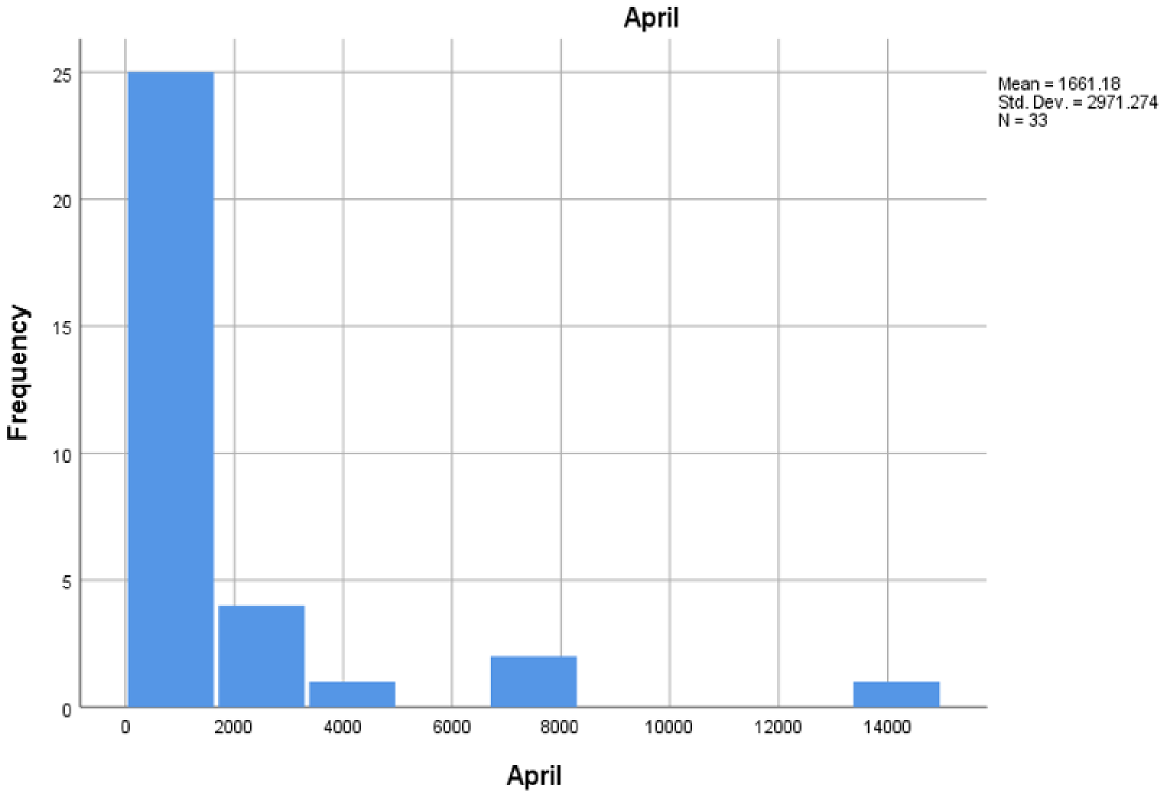

Jun
Frequency
12
10
8
6
4
2
0
0
500
1000
1500
2000
Jun
Mean = 401.4
Std. Dev. = 417.126
N = 25

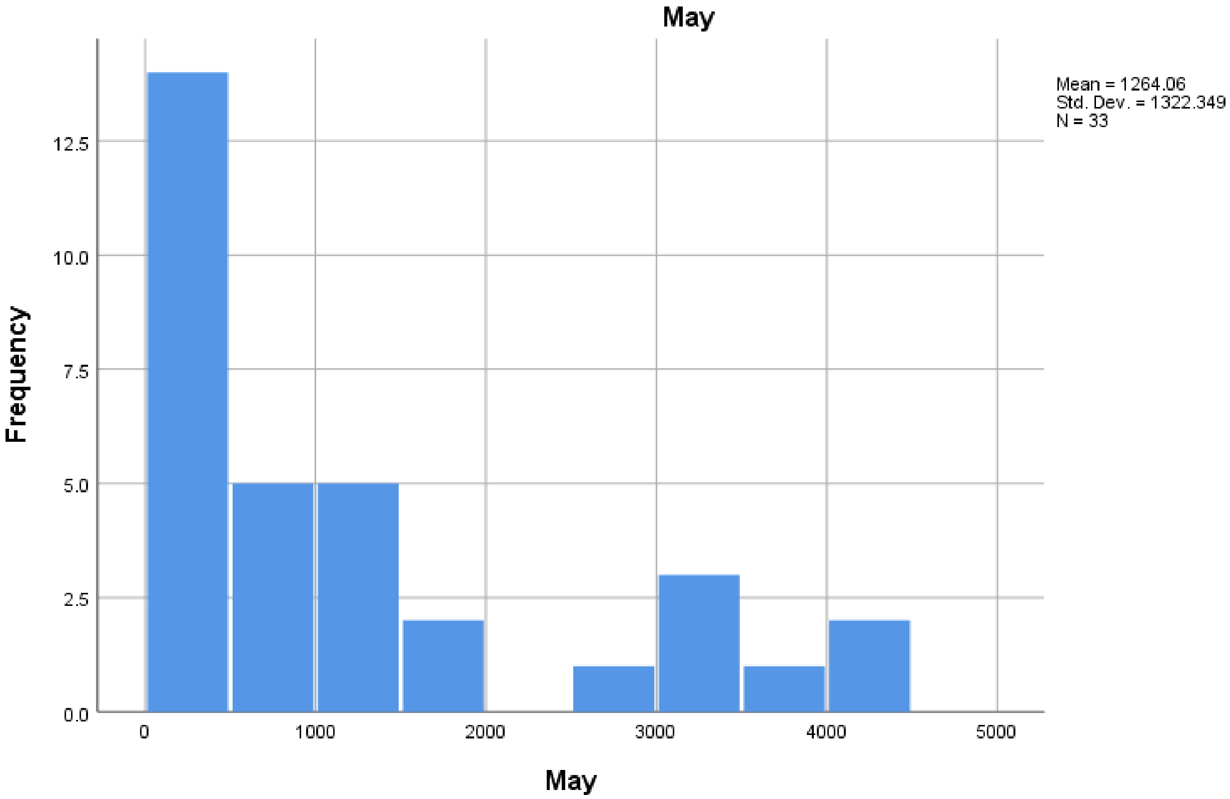

Jul
Frequency
25
20
15
10
5
0
0
1000
2000
3000
4000
Jul
Mean = 522.24
Std. Dev. = 924.994
N = 25

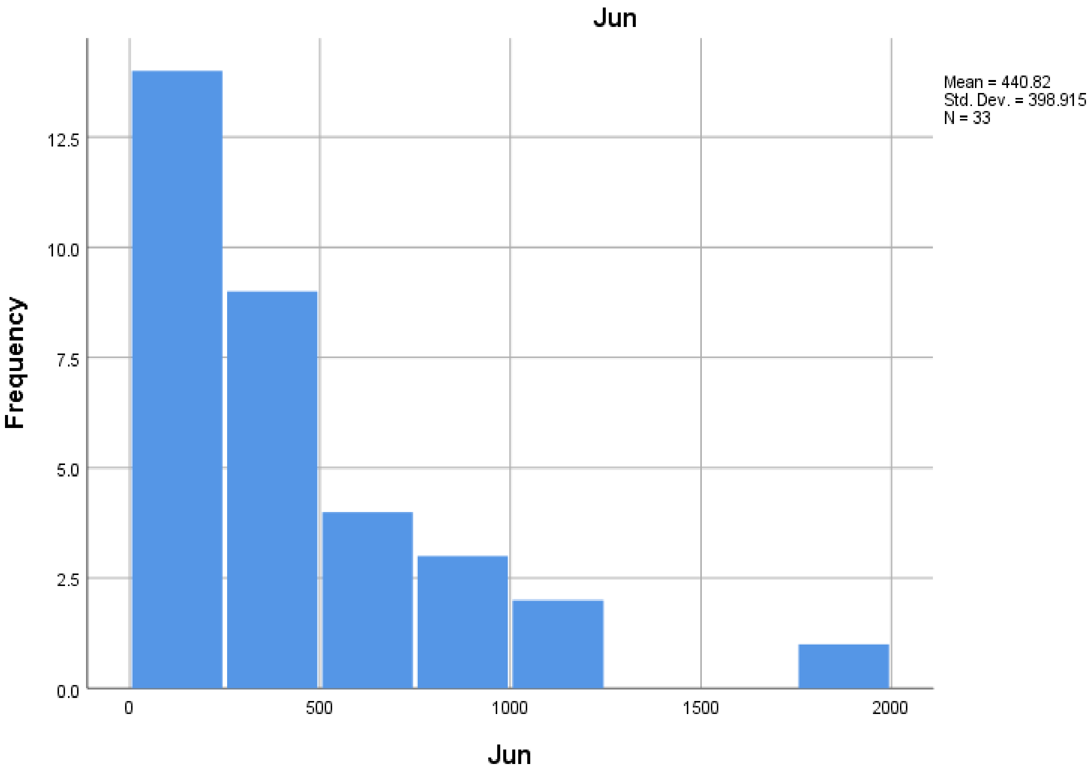

Aug
Mean = 777.16
Std. Dev. = 1348.6
N = 25
Frequency
0
5
10
15
20
25
0
1000
2000
3000
4000
5000
6000
Aug

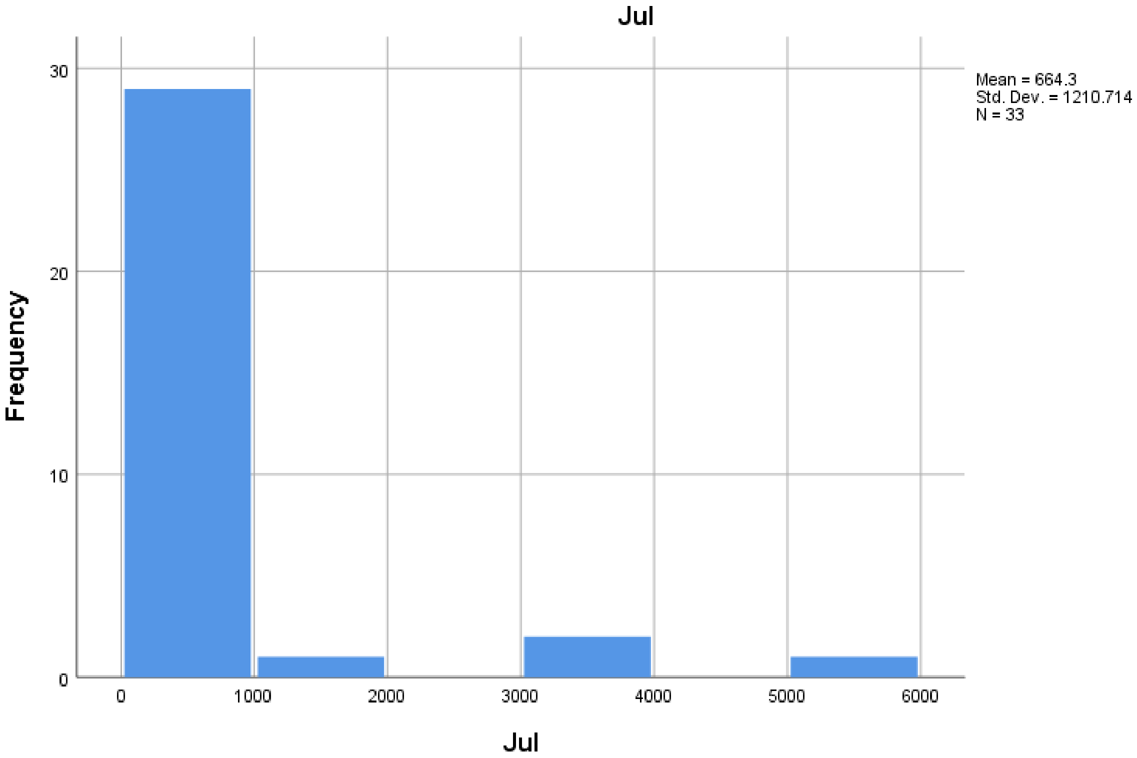

Sep
Mean = 428.6
Std. Dev. = 506.057
N = 25
Frequency
0
2
4
6
8
10
12
0
500
1000
1500
2000
2500
Sep

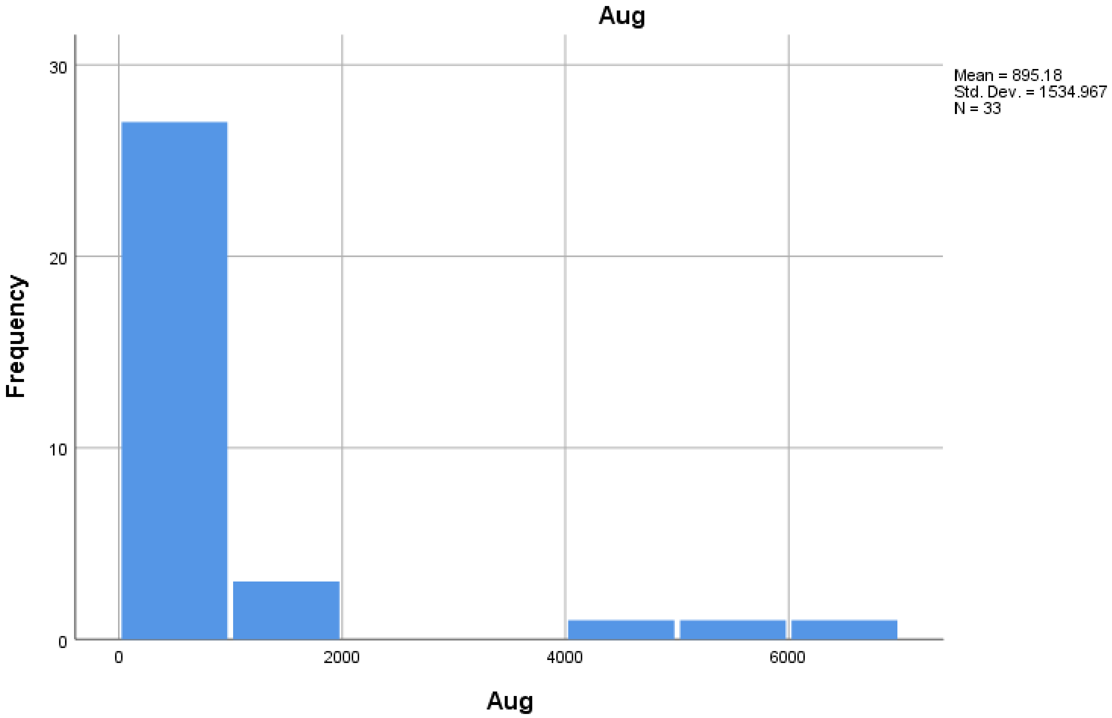

Oct
Frequency
0
1
2
3
4
5
6
0
200
400
600
800
1000
Oct
Mean = 313.6
Std. Dev. = 222.171
N = 25

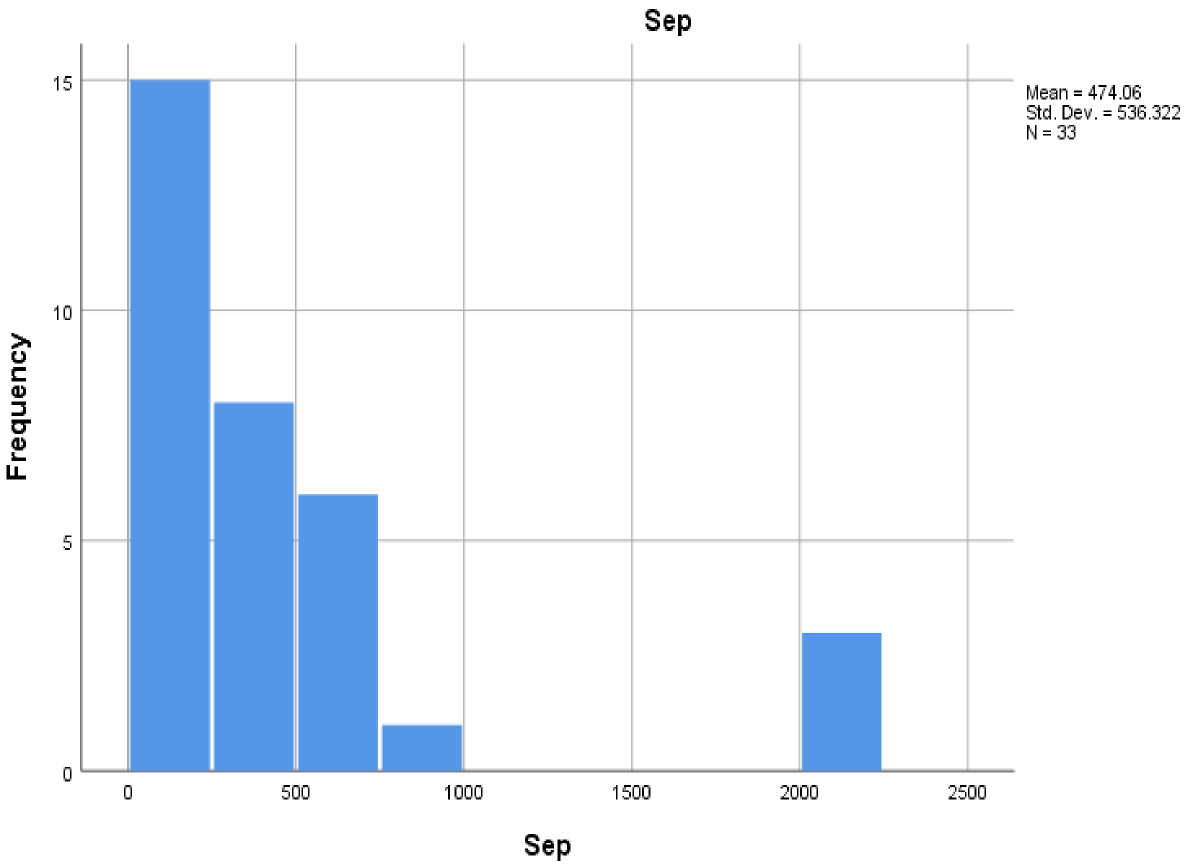

April
Frequency
0
5
10
15
20
25
0
2000
4000
6000
8000
10000
12000
14000
April
Mean = 1661.18
Std. Dev. = 2971.274
N = 33

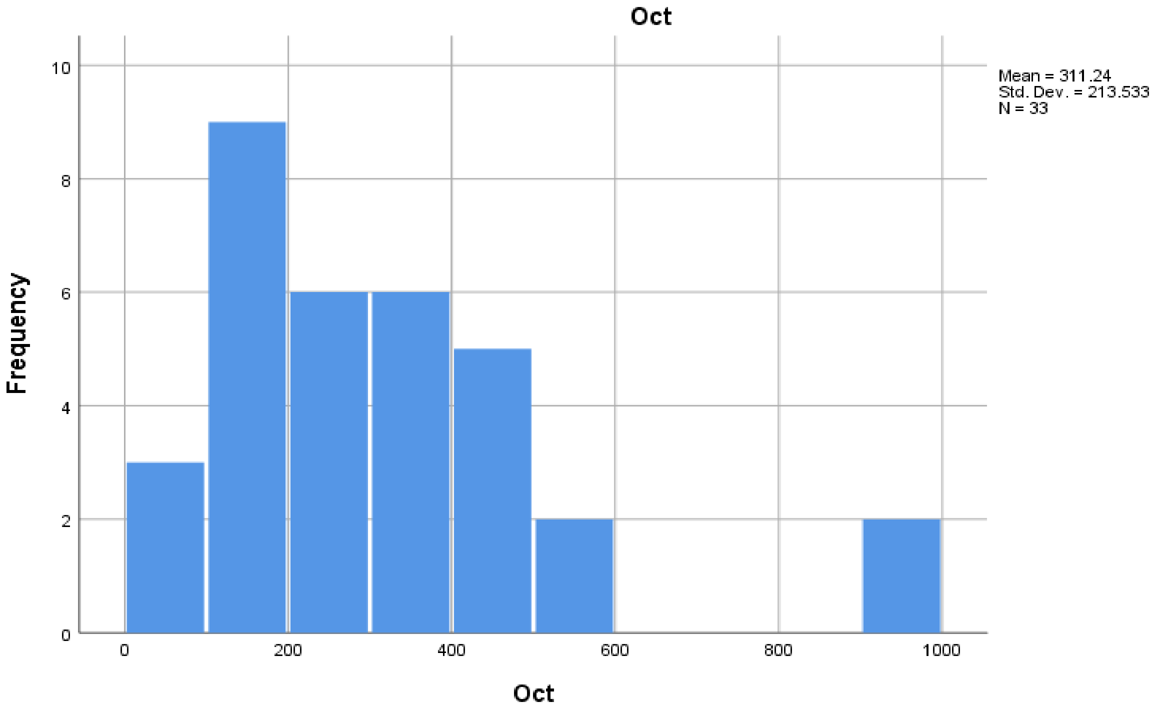

May
Mean = 1264.06
Std. Dev. = 1322.349
N = 33
Frequency
0.0
2.5
5.0
7.5
10.0
12.5
0
1000
2000
3000
4000
5000
May


Jun
Mean = 440.82
Std. Dev. = 398.915
N = 33
Frequency
0.0
2.5
5.0
7.5
10.0
12.5
0
500
1000
1500
2000
Jun


Jul
Frequency
30
20
10
0
0
1000
2000
3000
4000
5000
6000
Jul
Mean = 664.3
Std. Dev. = 1210.714
N = 33


Aug
Frequency
30
20
10
0
0
2000
4000
6000
Aug
Mean = 895.18
Std. Dev. = 1534.967
N = 33


Sep
Frequency
15
10
5
0
0
500
1000
1500
2000
2500
Sep
Mean = 474.06
Std. Dev. = 536.322
N = 33


Oct
Frequency
10
8
6
4
2
0
0
200
400
600
800
1000
Oct
Mean = 311.24
Std. Dev. = 213.533
N = 33

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ABSTRACT

ROBUSTNESS AND COMPARATIVE STATISTICAL POWER OF THE REPEATED MEASURES ANOVA AND FRIEDMAN TEST WITH REAL DATA

by
OPEOLUWA BOLU FADEYI

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Advisor: Dr. Shlomo Sawilowsky
Major: Educational Evaluation and Research
Degree: Doctor of Philosophy

Parametric statistical tests including repeated measure ANOVA have been largely employed in behavioral research. The justification is based on the fact that the tests are robust to violations of the assumptions underlying the parametric tests. Consequently, repeated measure ANOVA has been extensively applied in behavioral studies including scenarios where parametric tests may not provide the best fit. Such situations arise when the distribution under consideration is nonnormal and when the sample size is small. In these circumstances, nonparametric statistics such as the Friedman test which are based on assumptions that do not interfere with the validity of the tests' outcomes could provide a better fit in terms of statistical power. This study examines the comparative power of the parametric repeated measures ANOVA with the nonparametric Friedman test. The relative comparison is based on varying sample sizes with differing group combinations in both normal and nonnormal distributions using real-life data. The parametric and nonparametric alternatives are subjected to the same experimental conditions. The conditions include the same significant levels, hypotheses, and equal sample sizes. The results of the study indicate that Friedman's test outpowered and outperformed the repeated measures in all small sample sizes and across all the group combinations. Also,

the Friedman test demonstrated superiority in controlling the error rates that are either close or below the nominal alpha level. This showed that the rate at which the nonparametric Friedman's test gives non-accurate predictions is lower than that of the repeated measures ANOVA. The study concludes that the application of parametric repeated measures ANOVA when the fundamental assumptions are not satisfied should be replaced with the nonparametric Friedman test.

AUTOBIOGRAPHICAL STATEMENT

OPEOLUWA BOLU FADEYI

EDUCATION

Wayne State University	Detroit,Michigan
Ph.D. Educational Evaluation Research	March 2021
University of Ibadan	Ibadan, Nigeria
M.Ed. Educational Management (Personnel Administration)	Nov 2011
University of Ibadan	Ibadan, Nigeria,
B.Ed. Adult Education (with Geography)	April 2007

Special Training

Wayne State University	Detroit, Michigan
Broadening Experiences in Scientific Training (BEST)	Feb 2017- Feb 2018.

Publications

[1]. **Fadeyi, O.B.,** Sawilowsky, S. S., (2020). Robustness and comparative statistical power of the repeated measures ANOVA and Friedman test with real data (Dissertation).

[2]. **Fadeyi, O.B.,** (2011). Decision-making strategies in Nigerian organizations: A case study of Nigerian Postal Services (unpublished Master Thesis).

[3]. **Fadeyi, O.B.,** (2007). Salaries and wages as motivational factors for job satisfaction in Nigerian organizations (unpublished undergraduate project)

Presentations

- **Presentation:** Longitudinal study of developmental domains from childhood to 80 years (Fall, 2015)
- **Seminar paper:** A review of the relationship between temperament and adult personality
- **Seminar Paper**: Relationship between Social Psychology and Law
- **Seminar Paper:** Understanding and Creating Safe Environments for Sexual Minority Students

Academic and Community Services

- Volunteer, Braille Group of Buffalo, Buffalo, 2020
- Program Coordination, Wayne State University commencement, 2017
- Childcare and teaching (0-3, 4-6, 7-9), RCCG Winners Chapel, Detroit, 2014-2018